PROGRESSIVE MATHEMATICAL EXERCISES

SECOND SERIES

A. T. RICHARDSON

WITH ANSWERS

(GRADUATED) MATHEMATICAL EXERCISES

FOR HOME WORK.

SECOND SERIES.

BY

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WITH ANSWERS.

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PREFACE.

The present collection of Exercises, gathered from many sources, is one which has accumulated through several years; and consists of papers set weekly or bi-weekly to boys of all ages during that time. They serve to recall back work, and keep boys always ready for the examination. The First Series contains 261 papers, about half the total number, and commences with exercises in Arithmetic suitable to boys who have gone through the First Four Rules, Simple and Compound, and are beginning Fractions; and Algebraical Exercises consisting chiefly of Numerical Values, Addition, and Subtraction. From these onward, the exercises rise in difficulty by careful gradations, reaching Cube Root and Compound Interest in Arithmetic, and Quadratic Equations in Algebra, at the end of the First Series.

The Second Series is a continuation of the First, and includes problems in Higher Algebra, Logarithms, Trigonometry, and easy Mechanics, and Analytical Geometry.

The problems in the First Series are all arranged with a view to cover the whole of the back work up to the point where the student is working, in about four or five consecutive papers. Here and there a question may be found which appears to be beyond the average of the whole paper. These are inserted purposely, *and are always very simple*, in order that boys may be encouraged to look up methods they have not yet reached, and so to find that a little research enables them to do a new sort of question, which is not so hard as it looks. I have found a three-fold advantage in the plan. First, it points out the boys who have mathematical tastes, and a desire to excel in the science; secondly, a new rule so learned is rarely forgotten; and thirdly, it greatly increases the boys' interest, and gives a real zest to learning, when they find they can work out something new by their own efforts, without having been first shown the way. Instances of this occur in Exercises LVIII. 5; LXIII. 8; CIII. 7; CLXII. 5; CCXLV. 4, and elsewhere.

The relative standards of the different branches of Mathematics have been arranged chiefly with a view to the Local, and Army and Navy Examinations, but it will be found that the papers may be so selected as to be suitable for almost any standards that usually occur. Euclid Riders begin at Ex. CLXXIII., and Quadratic Equations at Ex. CCXVIII.

One of the most important things in Mathematics is accuracy. I have therefore indicated in the earlier papers

constantly, and occasionally in later ones, a simple method by which the working may be readily and shortly tested; and which, though not infallible, will usually detect mistakes. See Ex. LXIX. 7, *et passim.*

To facilitate the correcting of papers, answers are given with especial fulness, and as they have all been worked over at least twice, they will be found to contain few, if any, mistakes. I shall be very grateful to any one who will point out any I may have missed. In a collection of upwards of 6000 problems, there may easily be some that have escaped notice.

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Isle of Wight College,
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GRADUATED

MATHEMATICAL EXERCISES.

I.

1. If 20 thalers = 3 sovereigns, 4 sovereigns = 100 francs, each franc containing 20 sous, and each thaler 24 groschen, how many groschen are there in 600 sous?
2. Calculate to 6 places of decimals :—
$$\tfrac{1}{2} + (\tfrac{1}{2})^2 + (\tfrac{1}{2})^4 + (\tfrac{1}{2})^6 + (\tfrac{1}{2})^8 + (\tfrac{1}{2})^{10}.$$
3. Find the square root of
$$9x^4 + 30x^3 - 23x^2 - 80x + 64.$$
4. Divide
$$k^2 + 1 + \frac{1}{k^2} \quad \text{by} \quad k - 1 + \frac{1}{k}.$$
5. Simplify

(i.) $$\frac{1}{(x+y)^2} - \frac{1}{y^2 - x^2} - \frac{1}{(x-y)^2},$$

(ii.) $$\frac{(2k-3l)^2 - k^2}{4k^2 - (3l+k)^2} + \frac{4k^2 - (3l-k)^2}{9(k^2 - l^2)} + \frac{9l^2 - k^2}{(2k+3l)^2 - k^2}.$$

6. A sum of £20 14s. is divided among a certain number of persons; if each had received 5s. more, he would have received as many shillings as there were persons. How many were there?
7. Resolve
$$4(xy + ab)^2 - (x^2 + y^2 - a^2 - b^2)^2$$
into 4 factors.

8. Find a point such that the perpendiculars let fall from it on two given straight lines shall be of the same (given) length. How many such points are there?

II.

1. Find the 6th root of 326940373369.
2. If $3{\cdot}7\dot{9}$ of 45 guineas is the cost of 1 acre 3 roods, how much can be bought for $\cdot\dot{3}$ of £538 13s.?
3. In a mile race A starts at 270 paces of 48 inches per minute, and B at 300 paces of 44 inches per minute; after 4 minutes B quickens to 320. Which wins, by how much, and in what time?
4. Solve

$$\text{(i.)}\ \frac{x}{5}-\frac{y}{4}=1,\ \ 3x-4y=10,$$

$$\text{(ii.)}\ (x-1)^3+(2x+3)^3=(3x)^3+8.$$

$$\textit{N.B.}\ (x-1)+(2x+3)=3x+2.$$

5. A man walks a certain distance at a miles per hour, he then returns by train at b miles an hour. His whole time is c hours. Find the distance he goes.
6. Simplify

$$\frac{x^3+y^3+z^3-3xyz}{(x-y)^2+(y-z)^2+(z-x)^2}.$$

7. Find the square root of

$$16x^4-40x^3+89x^2-80x+64.$$

8. If the opposite sides of a quadrilateral are equal it is a parallelogram.

III.

1. A, B, and C could build a wall in $4\frac{32}{37}$ days; A and B in $6\frac{2}{3}$ days; B and C in $8\frac{2}{11}$ days. How many days would it take each separately to do it?

2. Simplify

$$2\tfrac{1}{2}+\frac{1}{3\tfrac{1}{3}+\dfrac{1}{4\tfrac{1}{4}}}+\left\{2\tfrac{3}{4}+\tfrac{5}{2}\text{ of }\frac{7}{3\tfrac{4}{5}}-\frac{1\tfrac{2}{3}}{2\tfrac{1}{2}}\right\}\div 1\tfrac{77}{228}.$$

3. Divide

$$32x^5+243 \quad\text{by}\quad 2x+3$$

by means of factors.

4. Find the L.C.M. of

$$a^3-b^3,\quad a^2-b^2,\quad a^6-b^6,\quad\text{and}\quad a^4-b^4.$$

5. Simplify

(i.) $\dfrac{1}{1+\dfrac{x}{y+z}}+\dfrac{1}{1+\dfrac{y}{z+x}}+\dfrac{1}{1+\dfrac{z}{x+y}}$,

(ii.) $\dfrac{(b-c)^2}{(a-b)(a-c)}$ + the two corresponding fractions,

(iii.) $\dfrac{3x^2-8x+5}{x^3-4x^2+5x-2}$.

6. Solve the equations :—

(i.) $ax-b=4ax-4b$,

(ii.) $ax^2+2bx+c=0$,

(iii.) $x^3+\dfrac{1}{x^3}=\dfrac{730}{27}$.

7. There is a number of two digits, of which the unit digit is 3 times the other: if 36 be added to the number the digits are reversed. Find it.

8. Given the perpendicular of an equilateral triangle, construct the triangle.

IV.

1. A quarter of an estate belongs to A, $\frac{2}{9}$ to B, and the remainder, which is worth £2500 more than A's share, to C. Find the worth of the whole.

2. If 7 men, working 10·6 hours a day, earn £4 15s. 3d. in $5\frac{1}{4}$ days, what will 28 men earn in $15\frac{3}{4}$ days, working $5\frac{3}{10}$ hours a day ?

3. Multiply

$$x^3 - \tfrac{1}{2}x^2 + 3x - \tfrac{3}{2} \quad \text{by} \quad 2x^2 + x + \tfrac{1}{2},$$

and test your answer by putting $x = \frac{1}{2}$.

4. Simplify

$$\text{(i.)} \quad \frac{x-1}{x^2-4x+4} \times \frac{x-2}{x^2-2x+1} \div \frac{x-3}{x^2-3x+2},$$

$$\text{(ii.)} \quad \left\{a^{\frac{5}{6}} \times \left(\frac{1}{ab}\right)^{\frac{1}{3}} \times \left(\frac{a^{-\frac{1}{3}}b}{ab^{-\frac{2}{3}}}\right)^{-\frac{1}{2}}\right\}^{\frac{2}{7}}.$$

5. Find the G.C.M. and L.C.M. of

$$12x^2 + 7xy - 10y^2,\ 15x^2 + 2xy - 8y^2, \text{ and } 15x^2 + 5xy - 10y^2.$$

6. Solve the equations :—

$$\text{(i.)} \quad \begin{cases} \frac{1}{2}x - \frac{1}{3}y = 1, \\ \frac{1}{5}y - \frac{1}{6}x = 1, \end{cases}$$

$$\text{(ii.)} \quad 3x^2 - \tfrac{1}{3}x = 26,$$

$$\text{(iii.)} \quad \sqrt{x^2 + 5x - 10} + \sqrt{x^2 + 5x - 13} = 3.$$

7. The area of a floor is 504 square feet; if it were 4 feet longer and 3 feet narrower, it would still have the same area. Find its dimensions.

8. If the diagonals of a quadrilateral bisect each other, it is a parallelogram.

V.

1. Simplify $\frac{1}{3}$ of $\frac{2}{3} + \frac{1}{4}$ of $\frac{5}{6} + \frac{7}{9}$ of $3 + \frac{1}{12} + \frac{2}{3}$ of $\frac{3}{7}$.

2. Find the value of

£·275 + 1·03125 of a guinea + ·00390625 of £1 12s.

3. Divide

$$x^3 + 2(a-2b)x^2 + 2(a^2 - ab + 4b^2)x + a^3 - 8b^3$$

by $x + a - 2b$, without removing the brackets.

4. Find the square root of

$$x^2 + \frac{4}{9} + \frac{9y^2}{4x^2} - \frac{4x}{3} + 3y - \frac{2y}{x}.$$

5. Find the G.C.M. of

$5x^3 + 9x^2y - 12xy^2 + 2y^3$, and $10x^3 - 7x^2y + 6xy^2 - y^3$.

6. Solve the equations :—

(i.) $\frac{x-3}{5} + \frac{2-x}{3} - \frac{1-2x}{15} = 0,$

(ii.) $3x - 4y = 25, \quad 5x + 2y = 7,$

(iii.) $\sqrt{x+y} = 6\sqrt{y}, \quad x - y^2 = 24\sqrt{y}.$

7. $\frac{1}{10}$ of a rod is red, $\frac{1}{20}$ orange, $\frac{1}{30}$ yellow, $\frac{1}{40}$ green, $\frac{1}{50}$ blue, $\frac{1}{60}$ indigo, and the remainder, which is 302 inches, white; what is its length?

8. Two straight lines AB and CD intersect at E, and the triangle AEC = the triangle BED; show that BC is parallel to AD.

VI.

1. An event occurred 40,000 days before January 1, 1871. Find the date, giving day of the week, and day of the month.

2. Prove without using the rule for the extraction of square root that $2 \cdot 7\dot{6}$ is the square root of $7 \cdot 65\dot{4}$.

3. A man has £576 6s. 8d. per annum after paying income tax at the rate of 4d. in the £. What is his income?

4. Given

$$x + y = \sqrt{m} \quad \text{and} \quad x - y = \sqrt{n},$$

express $x^3 + y^3$ in terms of m and n.

5. Divide

$$(m+1)(bx+an)b^2x^2 - (n+1)(mbx+a)a^2$$

by $bx - a$.

6. Subtract

$$\frac{x+3}{x^2+x-12} \text{ from } \frac{x+4}{x^2-x-12},$$

and divide the difference by

$$1+\frac{2(x^2-12)}{x^2+7x+12}.$$

7. Solve the equations :—

(i.) $(2x+1.5)(3x-2.25)=(2x-1.125)(3x+1.25)$,

(ii.) $x^2(3ax-2by)=ab^3,\ xy(3by-2ax)=a^2b^2$.

8. Show that twice the angle in a segment of a circle which is greater than a semicircle, together with the angle contained by the tangents drawn at the extremities of the base of the segment = two right angles.

VII.

1. Find the average of

$$12\tfrac{19}{25},\ 21,\ 7\tfrac{3}{4},\ {\cdot}034,\ 3\tfrac{1}{8},\ 0,\ 24\tfrac{1}{2}, \text{ and } 12\tfrac{7}{20}.$$

Give the answer in decimals.

2. Selling for £4162 10s. a man lost $7\frac{1}{2}$ per cent. What would he have gained or lost, selling at 4500 guineas ?

3. What must be expended in purchase of $3\frac{1}{2}$ per Cent. Consols at $93\frac{1}{8}$ to obtain an income of £644 ?

4. Find the L.C.M. of

$$6a^2(a^2-b^2),\ 18ab(a^3-b^3),\ 9b^2(a^3+b^3).$$

5. If α and β be the roots of

$$x^2-px+q=0,$$

find the value of $\alpha\beta$ and of $\alpha^2+2\alpha\beta+\beta^2$.

6. Solve

$$x^2-3x=3(9+x).$$

7. When will the hands of a clock be directly opposite to each other after 12 o'clock ?

8. If $x=4$, find the value of

$$\sqrt{2x+1}-\left(x+\frac{6}{\sqrt{x}}\right)-\left(3-\frac{x^2}{4-\sqrt[3]{2x}}\right).$$

VIII.

1. Find the present worth of £1842 15s. due in $\frac{1}{4}$ year at 5 per cent.
2. A mixture is formed from 3 sacks, containing equal quantities of wheat, barley, and oats, by taking 60 per cent. of the first, 40 per cent. of the second, and 70 per cent. of the third. What percentage of the whole is taken?
3. If $1\frac{3}{5}$ yds of cloth are worth $\frac{5}{12}$ bushel of corn, and 12 yds. of cloth cost $4\frac{1}{2}$ dollars, what is the value of 5 quarters of corn in £ sterling?
4. Solve

(i.) $\frac{1}{12}x^2-\frac{1}{3}x=\frac{1}{2}x-2$,

(ii.) $2x-y=4, \quad 4x^2+2xy+y^2=52$.

5. Find 3 numbers in the proportion of 1, 2, 3 such that the sum of their squares is 350.
6. If $$a:b::c:d,$$ prove that $a+b:a-b::c+d:c-d$.
7. An ordinary train, the average speed of which is 20 miles an hour less than the express, takes two hours longer than the express to go 150 miles. What is the average speed of each train?
8. Simplify

$$\frac{1}{1-\dfrac{x}{x-1}}-\frac{1}{\dfrac{x}{x+1}-1}.$$

IX.

1. Find, to the nearest inch, the diagonal of a square field of 2 acres.
2. A man has £576 6s. 8d. after paying income tax at 4d. in the £. What is his income?
3. If selling sugar at $5\frac{1}{4}$d. a lb. a man loses 12 per cent., at what price per cwt. must he sell it to gain 8 per cent.?
4. Simplify

$$\text{(i.)}\ \frac{x^6+y^6}{x^6-y^6}\times\frac{x-y}{x+y}\div\frac{x^4-x^2y^2+y^4}{x^4+x^2y^2+y^4},$$

$$\text{(ii.)}\ \frac{1}{2(x-1)}-\frac{x-5}{x^2-7x+10}+\frac{1}{2}\frac{x-6}{x^2-9x+18}.$$

5. Find the square root of

$$(a-b)^2\{(a-b)^2-2(a^2+b^2)\}+2(a^4+b^4).$$

6. If

$$\frac{a}{b}=\frac{c}{d},$$

prove that

$$\left(\frac{a}{b}\right)^2+\left(\frac{c}{d}\right)^2=\frac{2ac}{bd}.$$

7. Solve the equations :—

$$\text{(i.)}\ \frac{2x+1}{3}-\frac{3x-2}{4}=\frac{x-2}{6},$$

$$\text{(ii.)}\ \begin{cases} xy+\dfrac{x}{y}=10, \\ xy^2-x=6y. \end{cases}$$

8. Find two numbers which are in the ratio of $\frac{1}{2}$ to $\frac{2}{3}$; but which, if increased by 6 and 5 respectively, will be in the ratio of $\frac{2}{5}$ to $\frac{1}{2}$.

X.

1. If a piece of work can be finished in 45 days by 35 men, and if the men drop off by 7 at a time at the end of every 15 working days, how long will it be before the work is finished?

2. Simplify

$$\frac{3\frac{7}{8} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}.$$

3. Find (by practice) the value of 11 tons 6 cwt. 1 qr. 19 lbs. at £4 1s. $10\frac{1}{2}$d. per ton.

4. Simplify

$$\text{(i.)}\quad \frac{1}{x-1} + \frac{2x+1}{x^2+x+1} - \frac{3}{x}.$$

$$\text{(ii.)}\quad \frac{(x-y)^4 - xy(x-y)^2 - 2x^2y^2}{(x-y)(x^3-y^3) + 2x^2y^2}.$$

5. Solve the equations :—

$$\text{(i.)}\quad \frac{x-\frac{1}{2}}{x-1} - \frac{3}{5}\left(\frac{1}{x-1} - \frac{1}{3}\right) = \frac{23}{10(x-1)},$$

$$\text{(ii.)}\quad ax - by = 2ab, \quad 2bx + 2ay = 3b^2 - a^2.$$

6. Find the L.C.M. of

$9x^2 - 4$, $4x^2 - 36$, $3x^2 - 7x - 6$, and $3x^2 + 7x - 6$.

7. The first two terms of an Arithmetic Progression are 5 and 3. Find the next five terms.

8. Through the vertices A, B, C of a triangle, straight lines are drawn making the angles BAL, CBM, ACN all equal. If they intersect, two and two, in the points A', B', C', then the triangle $A'B'C'$ is equiangular to ABC.

XI.

1. Nine men were engaged to dig a trench 3 ft. deep, $4\frac{1}{2}$ ft. wide, and 120 yds. long in a certain time; but it was found that it must be 5 ft. deep, 4 feet wide, and 180 yds. long. How many more men must be engaged that it may be finished in the stated time?

2. Simplify

$$1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}+\left(\frac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\times 3\frac{1}{3}+\frac{13}{36}}-\frac{1}{2\frac{1}{2}}\right)\div\frac{1}{1\frac{2}{3}}.$$

3. Multiply 12·22361814 by 2·11529768, correct to 3 places of decimals.

4. How many minutes does it want to 10 o'clock, if three quarters of an hour ago it was twice as many minutes past 8?

5. There is a number of 2 digits which when doubled and 36 added, gives the same result as if the number were inverted, doubled, and 36 subtracted. Also 4 times the sum of its digits is 3 less than the number itself. Find it.

6. Write down an equation whose roots are 2 and -1.

7. Prove the accuracy of the following:—
"Take any number, double it, add 10, halve the result, subtract the original number, and 5 will remain."

8. Solve the equations:—

$$\text{(i)}\quad \frac{1}{x}+\frac{2}{y}=\frac{1}{2},\qquad \frac{3}{x}+\frac{4}{y}=\frac{1}{3},$$

$$\text{(ii.)}\quad \frac{1}{y}+\frac{2}{x}=\frac{2(x+1)}{xy},\qquad \frac{1-2x^2}{x}=\frac{y}{x}-(1+2x).$$

XII.

1. Find the amount of £8900 in 3 years at $2\frac{1}{2}$ per cent. compound interest.

2. Reduce 33 yds. to the decimal of a mile.

3. If the proportion of the diameter of a circle to its circumference be 113 to 355, find the circumference of a circle whose diameter is 132 ft.

4. If $x=3$, $y=1$, find the value of

$$\frac{\sqrt{3xy}-\{(3x^2-2y^2)^{\frac{1}{2}}-(6x^2-5y^2)^{\frac{1}{2}}\}}{xy-\{(x^2+y^2)-(x^2-y^2)\}}.$$

5. Simplify

$$\text{(i.)}\ ab \div \left(\frac{a^2}{b}\times\frac{b^2}{a}\right),$$

$$\text{(ii.)}\ \frac{1}{x}+\frac{2}{x-1}-\frac{3}{x+2},$$

$$\text{(iii.)}\ \frac{\dfrac{ax}{x^2-y^2}+\dfrac{b}{x-y}-\dfrac{a}{x+y}}{\dfrac{ax}{a^2-b^2}+\dfrac{y}{a-b}-\dfrac{x}{a+b}}.$$

6. Find the L.C.M. of

$$16a^2b(a-x),\quad 24b^2(a^2+ax)\quad \text{and}\quad 9ab(a^2-x^2).$$

7. Find the sum of the Arith. Progression

$$1+2\tfrac{2}{3}+4\tfrac{1}{3}+\ldots \text{ to 20 terms.}$$

8. Through 2 given points draw 2 straight lines forming with a straight line given in position, an equilateral triangle.

XIII.

1. Find the square root of $2{\cdot}5$.

2. Simplify

$$\frac{8\frac{2}{5}-4\frac{1}{2}+27\frac{3}{11}}{12\frac{3}{5}-6\frac{3}{4}+40\frac{10}{11}}.$$

3. A man buys a parcel of coffee and sells it at a loss of 3 per cent. Had he got £14 more for it he would have gained 4 per cent. What was the cost price?

4. Solve

$$\text{(i.)}\ (x-3a)^3-3(x-2a)^3+3(x-a)^3-x^3=9a^3-a^2x,$$

$$\text{(ii.)}\ 2x+3y=5,\quad 10x-6y=11.$$

5. Find the G.C.M. of

$$8x^3+6x^2-12x-9\quad \text{and}\quad 24x^4-20x^2-24.$$

6. Show that if a number of 3 digits is divided by 9, the remainder is the same as when the sum of the 3 digits is divided by 9.

7. Simplify

$$\text{(i.)}\quad \frac{3(5+2x)}{xy} - \frac{3ax+by+15a}{axy} + \frac{b}{ax},$$

$$\text{(ii.)}\quad \frac{25x(x^2-y^2)}{36(x-y)^2} \times \frac{12x(x-y)}{x^2+xy} \div \frac{5}{3x^2}.$$

8. If the diagonals of a parallelogram are equal all its angles are equal.

XIV.

1. A cube contains $18 \cdot \dot{9}6\dot{2}$ cub. yds. Find how many linear feet there are in an edge.

2. A man invests £6534 in the 3 per Cents. at 90, and on their rising to 91 transfers his stock to the $3\frac{1}{2}$ per Cents. at $93\frac{1}{2}$. Find change of income.

3. A cistern has 2 pipes which can fill it in 2 and 3 hours respectively, and another which will empty it in 5 hours. If all 3 are open, in what time will it be filled?

4. Multiply

$$x+1+\frac{1}{x} \quad \text{by} \quad x-1+\frac{1}{x}.$$

5. If

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \dots,$$

each of them $= \left\{\dfrac{a_1^2+a_2^2+a_3^2+\dots}{b_1^2+b_2^2+b_3^2+\dots}\right\}^{\frac{1}{2}}$.

6. Solve

$$\text{(i.)}\quad \tfrac{2}{5}x^2 + \tfrac{5}{2}(x-1) = 0,$$

$$\text{(ii.)}\quad x^3 - y^3 = 61, \quad x - y = 1.$$

7. Find a 3rd proportional to

$$\frac{x}{y} \quad \text{and} \quad x\left(\frac{y}{x} - \frac{x}{y}\right).$$

8. Find a square equal to the sum of two given squares.

XV.

1. A bankrupt's estates amount to £455 1s. 6¾d., and his debts to £937 10s. What can he pay in the £, and what will a creditor lose on a debt of £114?

2. Simplify

$$\frac{1+\frac{1}{3}}{1-\frac{1}{3}} \div \frac{1+\frac{3}{5}}{1-\frac{3}{5}} + \frac{1+\frac{1}{5}}{1-\frac{1}{5}} \div \frac{1+\frac{5}{13}}{1-\frac{5}{13}}.$$

3. Find the L.C.M. of

$$x^2+5x+10, \quad x^2-7x-30, \quad \text{and} \quad x^2+4x+3,$$

and the G.C.M. of

$$x^4+3x^2-10 \quad \text{and} \quad x^4-3x^2+2.$$

4. Find the value of

$$\frac{x^3+3x^2-20}{x^4-x^2-12} \text{ when } x=2.$$

5. Solve the equations :—

$$\text{(i.)} \quad x^2+\sqrt{4x^2+24x}=24-6x,$$

$$\text{(ii.)} \quad x^3+y^3=133, \quad x+y=7.$$

6. A man and a boy being paid for certain days' work, the man received 27s., and the boy, who had been absent 3 days out of the time, 12s.; had the man instead of the boy been absent those 3 days, each would have received the same. Find their wages per day.

7. Find a mean proportional between

$$\frac{x+y}{x-y} \quad \text{and} \quad \frac{x^2-y^2}{x^2y^2}.$$

8. Draw a straight line, equal to one straight line, parallel to another, and terminated by two given straight lines.

XVI.

1. Find the value of $\frac{2910}{6499}$ of £134.

2. What fraction of an angle of a regular pentagon is the angle of an equilateral triangle?

3. Express as a decimal
$$1\tfrac{1}{8} + 5\tfrac{5}{640} + \cdot 75 \text{ of } 7\tfrac{1}{2};$$
and find its value when the unit is £1000.

4. Solve
$$\text{(i.)}\quad \tfrac{1}{4}(x-1) + \tfrac{3}{35}(6x-1) = \tfrac{1}{5}(2x+7) + \tfrac{1}{7}(x+8),$$
$$\text{(ii.)}\quad \begin{cases}(a+b)x-(a-b)y=3ab,\\(a+b)y-(a-b)x=ab.\end{cases}$$

5. Simplify
$$\text{(i.)}\quad \left(1-\frac{x}{y}\right)y + \left(1-\frac{y}{z}\right)z + \left(1-\frac{z}{x}\right)x,$$
and (ii.) find the value of
$$1+\cfrac{1}{1-\cfrac{1}{a+1}},\ \text{when } a=3.$$

6. Divide
$$x^{4r} - \tfrac{1}{2}x^{2r}y^{2s} + \frac{y^{4s}}{16} \quad \text{by} \quad x^{2r} - x^r y^s + \frac{y^{2s}}{4},$$
and divide
$$a \quad \text{by} \quad a-b \quad \text{to four terms.}$$

7. Prove that a ratio of greater inequality is decreased if the same quantity be added to both terms of the ratio.

8. Straight lines bisecting two adjacent angles of a parallelogram intersect at right angles.

XVII.

1. Find the number of yards in the side of a square which contains 109 acres 3 roods 8 perches 9 sq. yds.

2. A circular plate of lead 2 inches in thickness, and 8 inches in diameter, is converted into shot, each ·05 inch radius. How many shot will it make?
3. A river 30 feet deep and 200 yards wide flows 4 miles an hour. How many cubic feet, and tons, fall into the sea every minute? (1 cub. ft. = 1000 oz.)
4. Simplify

$$\text{(i.)}\ \frac{2a-1}{2a} - \frac{2a}{2a-1} + \frac{\frac{1}{2}}{2a^2-a},$$

$$\text{(ii.)}\ \frac{4x^3+4x^2-7x+2}{4x^3+5x^2-7x-2}.$$

5. Find the square root of

$$a^{12} - 8a^9 + 18a^6 - 8a^3 + 1.$$

6. Solve

$$\text{(i.)}\ 12(x-3) - 3(2x-1) + 5x = 22,$$

$$\text{(ii.)}\ \tfrac{1}{3}(x-\tfrac{5}{2}) - \tfrac{3}{5}(x+\tfrac{4}{3}) + \tfrac{7}{2} = 0,$$

$$\text{(iii.)}\ 2x-3=y, \quad 3y+4=5x.$$

7. Find the 20th term and the sum of 20 terms of

$$2\tfrac{7}{9} + 1\tfrac{2}{3} + 1 + \ldots\ldots$$

8. Find two numbers in the proportion of 5 to 8, such that the one is 9 more than the other.

XVIII.

1. Two pipes together fill a cistern in 1 hour: one of them alone fills it in $1\frac{1}{2}$ hours; how long will the other take to fill it?
2. A street being 850 ft. long, and the width of the pavement on each side 5 ft. 3 in., find the cost of paving it at 1s. 3d. per sq. ft.
3. Multiply 57875 by 729819 in three lines of multiplication.

4. If
$$a^b = b^a,$$
show that
$$\left(\frac{a}{b}\right)^{\frac{a}{b}} = a^{\frac{a}{b}-1}.$$

5. Solve the equation
$$\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0.$$

6. A number consists of 2 digits, and is equal to 3 times the sum of its digits; if 45 be added to it the digits are reversed: find the number.

7. Simplify
$$\text{(i.)}\ x^{\frac{1}{2}}y^{\frac{1}{3}} \div \left(\frac{y^{-\frac{1}{2}}}{x^{\frac{1}{3}}} \times \frac{x^{\frac{2}{3}}}{y^{\frac{1}{3}}}\right).$$
$$\text{(ii.)}\ \frac{1}{x} + \frac{2}{x+1} + \frac{3}{x+2}.$$

8. A straight line drawn perpendicular to BC, the base of an isosceles triangle ABC, cuts AB at D, and CA produced at E. Show that the triangle AED is isosceles.

XIX.

1. Find the difference between the Simple and Compound Interest on £2000 for $1\frac{3}{4}$ year at 5 per cent.

2. Reduce $\frac{2}{9}$ of 1 oz. 13 dwt. to the decimal of $1\frac{1}{3}$ of 11 dwt. 6 gr.

3. If 40 men do a piece of work in 7 weeks, working 6 days of 8 hours each per week, in how many weeks will 30 men do it, working 4 days of 7 hours each per week?

4. Solve
$$\text{(i.)}\ \frac{3x-1}{3} + \frac{5}{12} = \frac{x}{4} + \frac{2x+1}{5},$$
$$\text{(ii.)}\ \frac{1}{x} + \frac{1}{y} = a, \quad \frac{1}{y} + \frac{1}{z} = b, \quad \frac{1}{z} + \frac{1}{x} = c.$$
$$\text{(iii.)}\ \sqrt{x+10} + \sqrt{x-6} = 8.$$

5. Find the cube root of

$$27x^6 - 108x^5 + 171x^4 - 136x^3 + 57x^2 - 12x + 1.$$

6. Express

$$a(a^2 - ab + b^2) - b(a^3 - ab + b^3) - 2(a^3 - b^4)$$

as the product of two factors.

7. Divide

$$a + 243x^{\frac{5}{3}} \quad \text{by} \quad a^{\frac{1}{5}} + 3x^{\frac{1}{3}}.$$

8. If a quadrilateral have two pairs of adjacent sides equal, one of its diagonals bisects the other at right angles.

XX.

1. A man invests £5000 in Turkish 6 per Cents. at 80. Find the rate of interest he gets for his money. When his stock has risen to 104 he sells out, and buys £20 railway shares at £18 which pay dividend at the rate of $4\frac{1}{2}$ per cent. Find the alteration in income.

2. Find the cube root of

$$59638983643.$$

3. Simplify

$$\frac{1}{7\frac{1}{4} \text{ of } 3\frac{3}{11} + 3\frac{3}{11}} \div \left(\frac{3}{13} - \frac{2}{9}\right) - \left(\frac{13}{3} + \frac{1}{6}\right) \div \frac{2}{3} \text{ of } \frac{3}{8} \text{ of } 63.$$

4. Solve

$$\text{(i.)} \quad \frac{2}{x+3} + \frac{x+3}{2} = \frac{10}{3},$$

$$\text{(ii.)} \quad \frac{x + \sqrt{2 - x^2}}{x - \sqrt{2 - x^2}} = \frac{4}{3}.$$

5. Sum to 10 terms

$$25 + 10 + 4 + \ldots\ldots$$

6. A cistern has two pipes which will fill it in $4\frac{1}{2}$ and 6 hours respectively; but it has a leak which will empty it in 5 hours. In how many hours will it be filled when all are working together?

7. Reduce to their simplest forms :—

$$\text{(i.)}\quad \frac{a^2 - ab}{a^2b - abc - ab^2 + b^2c}.$$

$$\text{(ii.)}\quad \frac{\dfrac{x^2}{y^2} - 2 + \dfrac{y^2}{x^2}}{2\left(\dfrac{x^2}{y^2} - \dfrac{y^2}{x^2}\right)}.$$

8. The four triangles into which a parallelogram is divided by its diagonals are all equal.

XXI.

1. If the 3 per Cents. be at $93\frac{1}{8}$, what must be expended in the purchase of stock to obtain an income of £552 per annum ?

2. Simplify

$$\frac{5{\cdot}1183}{{\cdot}0141} \text{ of } 22{\cdot}\dot{2} \text{ of } {\cdot}0\dot{9} \text{ of } {\cdot}2\dot{3}\dot{4}.$$

3. Find the difference between the amounts at Compound and Simple Interest of £1760 at 5 per cent. for $2\frac{1}{2}$ years, interest payable half-yearly.

4. Insert three geometrical terms between 3 and 243.

5. Find the value of

$$3a - \tfrac{1}{2}\{3b - 7(c - d)\}$$

when $a = 7,\ b = 4,\ c = -3,\ d = -5.$

6. Find the L.C.M. of

$$x^2 - 1,\ x^2 - 2x - 3,\ x^2 - 3x + 2.$$

7. Simplify

$$\frac{a^3 - x^3}{ax}\left(\frac{a + x}{a^2 - x^2} - \frac{a - x}{a^2 + ax + x^2}\right).$$

8. Find a square equal to the difference of two given squares.

XXII.

1. Find the cost of 19 yards 2 ft. 5 in. at 5s. 7d. a foot.
2. Multiply 1·92648 by ·007537 correct to four places.
3. Find the present worth of £5000 due eight years hence, at $4\frac{1}{2}$ per cent.
4. Find the G.C.M. of
$$2x^3 - 3ax^2 - 7a^2x + 4a^3 \quad \text{and} \quad 6x^3 - 7ax^2 - 4a^2x + 3a^3.$$
5. Solve the following equations :—
$$\text{(i.)} \quad \frac{2x}{3} + \frac{x+1}{4} - \frac{x-1}{2} = x - 8,$$
$$\text{(ii.)} \quad a\left(\frac{x}{2} - 1\right) + x = 3 + \frac{a}{2},$$
$$\text{(iii.)} \quad \frac{x}{2} + \frac{2}{y} = \frac{5}{4}, \quad \frac{x}{3} + \frac{3}{y} = \frac{5}{3}.$$
6. Twenty-eight tons of goods are to be carried in carts and waggons, and it will require either 15 carts and 12 waggons or else 24 carts and 8 waggons. What can each carry ?
7. In how many minutes after 1 o'clock will the minute hand be as far in front of the hour hand as it was behind at 1 o'clock ?
8. The square on a straight line drawn from the vertex of an isosceles triangle to the base, together with the rectangle under the segments of the base, is equal to the square on a side of the triangle.

XXIII.

1. Find the greatest number which will divide 17260 and 16039, leaving remainders of 5 and 2 respectively.
2. Simplify
$$\left\{\frac{3\frac{1}{2} - 2\frac{1}{6}}{\frac{1}{4} \text{ of } \left(\frac{1}{5} + \frac{1}{7}\right)} \div 15\tfrac{5}{9}\right\} \times \left\{\frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{2} + \frac{1}{3}} + \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3} + \frac{1}{4}} + \frac{\frac{1}{4} - \frac{1}{6}}{\frac{1}{4} + \frac{1}{6}} - \frac{\frac{1}{6} - \frac{1}{8}}{\frac{1}{6} + \frac{1}{8}}\right\}.$$

3. Multiply ·05656 by 1145·269007 correct to two places of decimals.
4. A is twice as old as B; and seven years ago their united ages amounted to as many years as now represent A's age. Find their present ages.
5. Which term of the series
$$\tfrac{7}{6},\quad \tfrac{4}{3},\quad \tfrac{3}{2},\quad \text{etc.}$$
is 18 ?
6. Given the difference of the reciprocals, and the difference of the squares of the reciprocals of two numbers, find the numbers.
7. Find the G.C.M. of
$$2x(x-3)+3(x-6\tfrac{2}{3})+15$$
and $$2x^3-5x^2-6x+15.$$
8. Solve the equations :—

(i.) $1+\dfrac{x}{2}-\dfrac{2x}{3}=\dfrac{3x}{4}-4\tfrac{1}{2}$,

(ii.) $\dfrac{x-5}{7}+\dfrac{x^2+6}{3}=\dfrac{x^2-2}{2}-\dfrac{x^2-x+4}{6}+3$,

(iii.) What are the two smallest positive *integral* values of x, which will make $2x^3$ a perfect square ?

XXIV.

1. Reduce 6s. $1\frac{1}{2}$d. to the decimal of £1 ; and 18s. $4\frac{1}{2}$d. to the decimal of £1000.
2. A boy copied ·55 of £5, instead of 5·5 of £5. What was the amount of his mistake ?
3. What sum must be insured to cover £2922 and premium and commission in case of loss ? Premium 2 guineas per cent., and commission $\frac{1}{2}$ per cent.
4. If the numerator of a fraction be increased by 1, and the denominator diminished by 1, the result will be 1. If

the numerator be increased by the denominator, and the denominator diminished by the numerator, the result will be 4. Find the fraction.

5. In the equation

$$ax^2 + bx + c = 0,$$

Show that $-\frac{b}{a}$ is the sum, and $\frac{c}{a}$ the product of the two roots.

Hence find the condition that the roots may be equal.

6. Solve the equations :—

$$\text{(i.)}\quad \frac{2-x}{3} + \frac{3-x}{4} + \frac{4-x}{5} + \frac{5-x}{6} + \tfrac{3}{4} = 0,$$

$$\text{(ii.)}\quad \frac{2+x^2}{3} - \frac{x-x^2}{2} = 1 - x + x^2.$$

7. Simplify

$$\frac{\dfrac{a}{b^2} - \dfrac{b}{a^2}}{\dfrac{b}{a} - \dfrac{a}{b}} \div \frac{\dfrac{a}{b} + \dfrac{b}{a+b}}{ab}.$$

8. If one angle of a triangle be triple another, the triangle may be divided into two isosceles triangles.

XXV.

1. What ready money will discharge a debt of £1056 18s. due four months hence at $4\frac{7}{8}$ per cent. ?
2. A man invests £6104 in Consols at $95\frac{3}{8}$, and sells out at $97\frac{1}{4}$. What does he gain ?
3. Find the cubic content, and the area of the faces, of a block of stone, 3 ft. 4 in. by 4 ft. 3 in. by 1 ft. 1 in.
4. Simplify

$$45{\cdot}\dot{2} \text{ of } {\cdot}\dot{4}0\dot{4} \text{ of } {\cdot}3\dot{7}20\dot{3} \text{ of } £3\ 0s.\ 9d.$$

5. Simplify

$$\text{(i.)}\ \frac{3x+2}{(x-1)^2}-\frac{6}{x^2-1}-\frac{3x-2}{(x+1)^2},$$

$$\text{(ii.)}\ \frac{1}{1+\frac{1}{a}}\times\frac{1}{1-\frac{1}{a}}\div\frac{1}{a-\frac{1}{a}}.$$

6. Sum

(i.) $3\frac{1}{2}+2\frac{1}{2}+1\frac{1}{2}+\ldots$ to 15 terms,

(ii.) $3\frac{1}{2}+2\frac{1}{2}+1\frac{11}{14}+\ldots$ to 5 terms.

7. Find the L.C.M. of

$$(a^2-x^2),\quad (a^2+3ax+2x^2),\quad (a^2-4x^2),$$

and the G.C.M. of

$$2x^3-16x+6 \quad\text{and}\quad x^4+3x^3+x+3.$$

8. If one side of a triangle be trisected, and lines drawn parallel to the base through the points of trisection, they will trisect the other side.

XXVI.

1. Divide ·827134 by 10·567 correct to four places, by Contracted Division.

2. Simplify

$$\cdot 3125 \text{ cwt.} + \cdot 615625 \text{ ton} + 2\cdot 375 \text{ qrs.}$$

3. Calculate the value, to six places of decimals, of

$$\frac{1.2}{2.3}+\frac{1.2.3}{2.3.4}+\frac{1.2.3.4}{2.3.4.5}+\frac{1.2.3.4.5}{2.3.4.5.6}+\frac{1.2.3.4.5.6}{2.3.4.5.6.7}.$$

4. There is a penny difference in the price of a dozen oranges at two shops, one giving two more for a shilling than the other. Find the prices.

5. Find the G.C.M. of

$$2x^2-14x+20 \quad\text{and}\quad 4x(x^2+10)-25(x+1)(x-1)-37.$$

6. Find the square root of

$$9x^4+12x^3+10x^2+4x+1,$$

and prove that the result is true when $x=10$.

7. Find the fraction such that if you quadruple the numerator and add 3 to the denominator the fraction is doubled; but if you add 2 to the numerator, and multiply the denominator by 4, it is halved.
8. Two rectangles have equal areas and perimeters; show that they are equal in all respects. (Prove this geometrically and algebraically.)

XXVII.

1. Find the cube root of 994011992.
2. Which is the better investment, the $4\frac{3}{4}$ per Cents. at 95 or the 5 per Cents. at 104?
3. A man sold a horse for £41 17s. and lost 7 per cent. Find the cost of the horse.
4. At what time are the hands of a watch together between 7 and 8?
5. Sum to n terms, and to infinity
$$\tfrac{1}{2}-\tfrac{1}{4}+\tfrac{1}{8}-\tfrac{1}{16}+\ldots\ldots$$
6. Find the value of
$$\left\{a^{\frac{2}{3}}-b^{-\frac{2}{3}}\right\}^{3}.$$
7. Solve the equations:—
$$\text{(i.) } x^2-8x=33,$$
$$\text{(ii.) } \begin{cases} \dfrac{x}{5}-\dfrac{y}{3}=3, \\ \dfrac{x}{7}-\dfrac{y}{6}=3. \end{cases}$$
8. The side BC of a triangle ABC is bisected at E, and AB at G. AE is produced to F, and CG to H, so that $EF=AE$ and $GH=CG$. Show that FB, HB are in one straight line.

XXVIII.

1. Reduce 32·16 pennyweights to the decimal of 1 lb. Troy.
2. Find, by Practice, the value of
   2 tons 5 cwt. 3 qrs. $3\frac{1}{2}$ lbs. at £3 13s. 6d. per cwt.
3. If the area of a field be 87 ac. 1 ro. 26 p., and the breadth 925 links, find the length.
4. Solve the equations :—

$$\text{(i.)}\ 7x^2 + 13x = 2.$$

$$\text{(ii.)}\ x^6 - 7x^3 = 8.$$

$$\text{(iii.)}\ \frac{3x-9}{7} - \frac{x+1}{11} = \frac{3x-14}{8}.$$

5. Simplify

$$\text{(i.)}\ \sqrt[3]{a^5b^3},$$

$$\text{(ii.)}\ \sqrt{180},$$

$$\text{(iii.)}\ \left(a^{\frac{1}{n+1}} \times a^{\frac{1}{n-1}}\right)^{\frac{1}{n}},$$

and $$\text{(iv.)}\ (a^3 \times a^4)^{\frac{1}{2}}.$$

6. Divide

$$a^2 + (2ac - b^2)x^2 + c^2x^4 \quad \text{by} \quad a - bx + cx^2$$

   without removing the bracket.
7. Reduce to its simplest form

$$\frac{x^2 + 3x - 4}{x^3 + x^2 - 4x + 2}.$$

8. If from any point of one side of an equilateral triangle straight lines be drawn parallel to the other sides, the perimeter of the parallelogram thus formed will be constant (*i.e.*, will be of the same length, wherever the point is taken).

XXIX.

1. If a cubic foot contains 343 cubic units, how many square units are there in a rectangle whose sides are 147 feet and 40 feet respectively?

2. How many revolutions will be made by a wheel revolving 151 times in 3 minutes, while another wheel, revolving 241 times in 7 minutes, makes 723 revolutions?
3. Find the discount on £3073 19s. 2d. in $4\frac{5}{6}$ years at $4\frac{3}{4}$ per cent.
4. If
$$a+b+c+d=2s,$$
then
$$4(ab+cd)^2-(a^2+b^2-c^2-d^2)=16(s-a)(s-b)(s-c)(s-d).$$
5. Show that if
$$8r=p(4q-p^2), \quad \text{and} \quad 64s=(4q-p^2)^2,$$
then
$$x^4+px^3+qx^2+rx+s$$
is a perfect square.
6. Solve
$$\text{(i.)} \quad \frac{x+1}{x-1}+\frac{x+2}{x-2}-\frac{2x+6}{x-3}=0,$$
$$\text{(ii.)} \quad 3x-7y=7, \quad 11x+5y=87.$$
7. Transform 1234567 into the duodenary scale.
8. If straight lines be drawn bisecting two exterior angles of a triangle, their point of intersection is equidistant from all the sides of the triangle.

XXX.

1. Find the value of 1 qr. 7 bus. 3 pks. of wheat, if 5 qr. 3 bus. 1 pk. cost £9 17s. 6d.
2. Find the cube root of 674526133.
3. Find the amount of £1256 10s. for two years at $3\frac{1}{2}$ per cent. Compound Interest.
4. By what must
$$x^7+1$$
be divided, that the remainder may be seven times the square root of the divisor?

5. Solve the equations :—

$$\text{(i.)}\quad \frac{x+6}{4}-\frac{3x-16}{12}-1=\frac{x+3}{6},$$

$$\text{(ii.)}\quad 3x+2\sqrt{x}=1.$$

6. Reduce

$$\frac{a^4+3a^3-7a^2-21a-36}{a^4+2a^3-10a^2-11a-12}$$

to its lowest terms.

7. If
$$s=\frac{a+b+c}{2},$$
show that
$$\frac{1}{s-a}+\frac{1}{s-b}+\frac{1}{s-c}-\frac{1}{s}=\frac{abc}{s(s-a)(s-b)(s-c)}.$$

8. If straight lines be drawn from the vertices of a triangle, perpendicular to the opposite sides, prove that they meet in a point.

XXXI.

1. At what time between five and six are the hands of a watch at right angles?
2. By selling wine at 15s. a gallon, a man loses 6 per cent.; what must he sell it at, to gain $17\frac{1}{2}$ per cent.?
3. Find the difference between the interest and discount on £2016 for 73 days at 4 per cent.
4. Solve

$$\text{(i.)}\quad 16x-y=4x+2y=6,$$

$$\text{(ii.)}\quad x^3+y^3=737,\quad x+y=11.$$

5. Divide
$$x^{4m}-y^{4m}\quad\text{by}\quad x^m-y^m.$$

6. A number consists of six digits. The first digit is 1, and if it is removed, and put at the end, the number is three times as great as before. Find the number.

7. Sum the series

$$1,\ 2,\ 3,\ 4, \text{ etc., to } 100 \text{ terms.}$$

8. The two sides of a triangle are together greater than twice the line joining the vertex to the mid point of the base.

XXXII.

1. Find the cost of building a wall 54 yards long, 7 ft. 4 in. high, and 9 inches thick, at £22 per rod. (A rod of brickwork is $272\frac{1}{4}$ sq. feet of brickwork, $13\frac{1}{2}$ inches thick.)

2. Simplify

$$\frac{5\cdot 1183}{\cdot 00705} \text{ of } 11\cdot\dot{1} \text{ of } \cdot 2\dot{9} \text{ of } \cdot 1\dot{1}\dot{7}.$$

3. Solve

$$\text{(i.)}\ (c+x)(x-a)-x(x-b)=\frac{ac}{b}(c-a),$$

$$\text{(ii.)}\begin{cases} 7x-3y=1, & 4x-z=0, \\ 3z-2u=-2, & 3y-u=2. \end{cases}$$

4. Prove that if

$$a:b=c:d=e:f=g:h,$$

then each of these ratios $=$

$$\sqrt{a^2+c^2-e^2+g^2} : \sqrt{b^2+d^2-f^2+h^2}.$$

5. Simplify

$$\frac{a^4+10a^3+35a^2+50a+24}{a^3+9a^2+26a+24}.$$

6. What number is that, whose square and cube added together are nine times the next higher number?

7. Multiply

$$\frac{x^{2r}}{y^{2s}}+1+\frac{y^{2s}}{x^{2r}} \quad \text{by} \quad \frac{x^r}{y^s}-\frac{y^s}{x^r}.$$

8. Given three sides of a quadrilateral, and the angles adjacent to the fourth, construct it.

XXXIII.

1. A rectangular field containing 10 acres is 176 yards wide. How far is it across the field from corner to corner?

2. Simplify

$$\frac{(423 \div \frac{3}{5}) + (1\frac{4}{5} \div 3) + (4\frac{1}{2} \div 1\frac{2}{3})}{\frac{3}{7} \text{ of } 41 + 32 \times \frac{2}{3} - 1 \div \frac{7}{10}}.$$

3. A cistern is filled by two pipes in 9 and 10 minutes respectively, and emptied by another in 20 minutes; how much will be filled in 5 minutes, if all are open?

4. If $$\frac{x-y}{1+xy} + \frac{k-l}{1+kl} = 0,$$

prove that $$\frac{x-l}{1+lx} - \frac{y-k}{1+ky} = 0.$$

5. Factorize

$$x^3 + y^3 + z^3 - 3xyz, \quad \text{and} \quad x^4 + x^2y^2 + y^4.$$

6. Solve

$$\text{(i.)} \ \frac{x+2}{3} + 2 = \frac{x+4}{5} + \frac{x+6}{7},$$

$$\text{(ii.)} \ 2x - 15y = 3x - 24y = 1,$$

$$\text{(iii.)} \ (\sqrt{1+x} - 1)(\sqrt{1-x} + 1) = 2x.$$

7. What must be added to

$$\frac{4x}{(x-1)^2(x+1)} - \frac{x+1}{(x-1)^2} + 1$$

to make it equal to 2?

8. If the opposite angles of a quadrilateral are equal, it is a parallelogram.

XXXIV.

1. How many yards of carpet $2{\cdot}1\dot{6}$ yards wide will cover a room $25{\cdot}\dot{3}$ ft. by $28{\cdot}\dot{8}$ ft.?
2. If the discount on a sum due at the end of three years be $\frac{200}{221}$ of the simple interest, find the rate per cent.
3. If 12 men and 4 boys can reap 13 acres in a day; and 21 men and 15 boys can reap 99 acres in 4 days; how long will it take 8 men and 8 boys to reap 30 acres?
4. Solve

   (i.) $31\left\{\dfrac{24-5x}{x+1}+\dfrac{5-6x}{x+4}\right\}+370=29\left\{\dfrac{17-7x}{x+2}+\dfrac{8x+55}{x+3}\right\}$,

   (ii.) $7x+3y=10, \quad 35x-6y=1.$
5. Multiply

$$a^{\frac{1}{2}}+b^{\frac{1}{2}} \quad \text{by} \quad a^{\frac{1}{2}}-b^{\frac{1}{2}}.$$

6. Find what number in the scale of 5 represents 123456.
7. One thousand copper coins were found to be worth £3. There were as many pence as halfpence; how many farthings were there?
8. Divide a straight line into two parts, so that the square on the one part may be nine times the square on the other.

XXXV.

1. If a litre is ·52 gallons, and 1200 francs = £49, find to the nearest penny in English money the value of a pint of liquid, which is worth 20 francs a litre.
2. Find the cost of painting the four walls of a room, 20 ft. $7\frac{3}{4}$ in. long, 15 ft. $4\frac{1}{4}$ in. broad, and 12 ft. 4 in. high, at 9d. a square yard.
3. Find the amount of £10 in 4 years at $4\frac{1}{2}$ per cent. Compound Interest; neglecting fractions of a penny.

4. Solve

$$\text{(i.)} \begin{cases} -\dfrac{l}{x}+\dfrac{m}{y}+\dfrac{n}{z}=a, \\ \dfrac{l}{x}-\dfrac{m}{y}+\dfrac{n}{z}=b, \\ \dfrac{l}{x}+\dfrac{m}{y}-\dfrac{n}{z}=c, \end{cases}$$

$$\text{(ii.)} \quad \frac{a-2x}{abx}=\frac{1}{b^2}-\frac{1}{ab},$$

$$\text{(iii.)} \quad \frac{2}{x}=\frac{3}{x-1}-\frac{1}{x-2}.$$

5. Simplify

$$\text{(i.)} \quad \frac{a}{x(a-x)}-\frac{x}{a(a-x)},$$

$$\text{(ii.)} \quad \frac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}},$$

$$\text{(iii.)} \quad \{(x^3)^{\frac{1}{2}}\}^{\frac{1}{3}} \times \{-(-x)^3\}^{\frac{1}{3}}.$$

6. Find the square root of

$$x^4-2ax^3+5a^2x^2-4a^3x+4a^4.$$

7. Divide

$$a^3-\frac{1}{a^3} \quad \text{by} \quad a-\frac{1}{a}.$$

8. Prove Euclid I. 17 without producing a side.

XXXVI.

1. Reduce 75·0125 cwt. to lbs. and the decimal of a lb.
2. Find the cube root of 9555119848.
3. Divide £2025 among A, B, C, D, and E, so that A's share is to B's as 1 : 2; C's : B's :: 5 : 4; D's : C's :: 6 : 5; and E's : D's :: 4 : 3.

4. Find the square root of
$$4x^8 - 4x^6 - 7x^4 + 4x^2 + 4.$$

5. Divide
$$x^{\frac{2}{3}} - y^{\frac{2}{3}} \quad \text{by} \quad x^{\frac{1}{3}} - y^{\frac{1}{3}}.$$

6. Simplify
$$\text{(i.) } \sqrt{x^3} \div \sqrt{x^{-1}},$$
$$\text{(ii.) } \frac{1}{1 - \dfrac{1}{1+x}} + \frac{1}{1 - \dfrac{1}{1-x}}.$$

7. Solve
$$\text{(i.) } \frac{3+2x}{2-x} - \frac{2-3x}{2+x} + \frac{16x - x^2}{x^2 - 4} = \frac{1}{3}.$$
$$\text{(ii.) } 5x - 3y = 13, \quad 3x + 2y = 4.$$

8. Given the sum of a side and diagonal of a square, construct it.

XXXVII.

1. A grocer mixes seven parts of coffee at £8 10s. per cwt. with five parts of chicory at £2 10s. per cwt.: at what rate per lb. must he sell the mixture, to gain $16\frac{2}{3}$ per cent.?

2. On what days of the week will the 10th of February fall in the years 1960 and 2060?

3. Find the value of
$$5{\cdot}4\dot{9} \text{ of } {\cdot}03\dot{1}\dot{8} \text{ of } {\cdot}\dot{0}4761\dot{9} \text{ of 25 guineas.}$$

4. Find the value of
$$\sqrt[3]{51271550875}.$$

5. Sum to eight terms
$$\text{(i.) } 2 + 9 + 16 + \ldots$$
$$\text{(ii.) } 64 + 32 + 16 + 8 + \ldots$$

6. Find three numbers, whose sum is 21, and of which the greatest exceeds the least by 4, and the remaining one is half the sum of the other two.

7. Solve

$$\text{(i.)}\ \tfrac{1}{2}(x-1)+\tfrac{2}{3}(x+2)=\tfrac{9}{4}(x-3),$$

$$\text{(ii.)}\ 3x+\tfrac{1}{2}y=21,\quad 2y+\tfrac{1}{3}x=14,$$

$$\text{(iii.)}\ \frac{x}{3}-\frac{6}{x}=\frac{x^2-6}{3(x+4)}.$$

8. Find the square root of

$$\frac{4x^2}{y^2}+\frac{4y^2}{x^2}-8.$$

XXXVIII.

1. Find the cube root of 149721291.

2. Simplify

$$\text{(i.)}\ \frac{1}{8(x+5)}-\frac{1}{4(x+3)}+\frac{1}{8(x+1)},$$

$$\text{(ii.)}\ \frac{1+\sqrt{1-2x}}{1-\sqrt{1-2x}}+\frac{x-\sqrt{1-2x}}{x};$$

$$\text{(iii.)}\ \frac{a^{\frac{1}{2}}b^{\frac{1}{3}}}{c^{\frac{1}{6}}}\div\left\{\frac{c^{\frac{1}{2}}}{a^{-\frac{1}{3}}b^{-\frac{1}{3}}}\times\frac{a^{-\frac{5}{6}}c^{-\frac{2}{3}}}{b^{\frac{5}{6}}}\right\}.$$

3. If a be the first term, d the common difference, s the sum of n terms of an A.P., prove that

$$s=\frac{n}{2}\left\{2a+\overline{n-1}\,.\,b\right\}.$$

Hence sum to eight terms

$$1\tfrac{1}{2},\quad 4\tfrac{1}{2},\quad 7\tfrac{1}{2},\ldots$$

4. Find the square root of

$$x^4+8x^3+24x^2+32x+16.$$

5. Solve

$$\text{(i.)}\ \frac{x^2-5x+4}{x-4}+\frac{x^2-5x+6}{x-3}=3.$$

$$\text{(ii.)}\ \frac{x}{2}+\frac{y}{3}=8,\quad \frac{x}{4}-\frac{y}{12}=1.$$

6. Multiply

$$a^{\frac{1}{3}} + b^{\frac{1}{3}} \quad \text{by} \quad a^{\frac{1}{3}} - b^{\frac{1}{3}}.$$

7. Divide

$$a^4 - a^2b^2 - a^2c^2 + b^2c^2 \quad \text{by} \quad a^2 - ab + ac - bc.$$

8. Find the cost of 1 ton 2 qrs. 20 lbs. at £9 6s. 8d. per ton.

XXXIX.

1. Simplify

$$\left(\tfrac{3}{5} \text{ of } 7\tfrac{1}{2} - \tfrac{8}{17}\right) \div 1\tfrac{2}{9}; \quad \text{and} \quad \frac{2\frac{3\frac{3}{4}}{12}}{20}.$$

2. Divide 12·2574 by 5·3568 to three places, by Contracted Division.

3. Calculate to six places of decimals

$$1 + \frac{2^2}{3.4} + \frac{2^2.4^2}{3.4.5.6} + \frac{2^2.4^2.6^2}{3.4.5.6.7.8} + \frac{2^2.4^2.6^2.8^2}{3.4.5.6.7.8.9.10}.$$

4. The seventh term of an A.P. is 1, and the sum of twenty-five terms is zero. Find the Progression.

5. Find the value of

$$x - (\sqrt{x+1} + 2) - \frac{x - \sqrt[3]{x}}{\sqrt{x-4}}$$

when $x = 8$.

6. A and B can do a certain task in 30 days, working together. After 11 days B is called off, and A finishes it in 28 days. How long would each take alone?

7. Solve

$$\text{(i.)} \quad (mx - a)(nx - b) = 0,$$

$$\text{(ii.)} \quad \frac{x+2}{x+1} + \frac{x+1}{x+2} = \frac{13}{6}.$$

Find the value of a, that

$$\frac{x+3}{x+a} + \frac{x-3}{x-a} = \frac{2x-3}{x-1}$$

may be satisfied by $x = 4$.

8. Construct a triangle of given area, with two of its sides of given lengths.

XL.

1. In a town of 15,000 electors, ·72 voted for one candidate, and ·125 for another. How many did not vote?
2. Find the discount on £459 18s. due three years hence, at four per cent.
3. If 20 scudi = 105 francs; 4 shillings = 5 francs; what are 45 scudi worth in English money?
4. If $a:b::c:d$ prove that
$$ab+b^2 : ac-ad :: ac+2bc+bd : \frac{c^2a}{b} - cd.$$
5. A man bought a certain number of sheep for £210. He lost 10, and then sold the rest at 10s. profit per head. He thus got just as much as he gave for them. How many did he buy?
6. Find the G.C.M. of
   (i.) $(a-b)^2, \quad a^3-b^3.$
   (ii.) $x^3-3a^2x-2a^3$, and $x^3-ax^2-4a^3$.
7. Show how to find the sum of a series of n terms in A.P. whose first term is a, and last l.
   Sum the series
$$6, -2, \tfrac{2}{3}, -\tfrac{2}{9}, \text{ etc., to infinity.}$$
8. Show that of all rectangles with the same perimeter, the square has the largest area.

XLI.

1. Find to five places of decimals the difference between the cube of 4·791288, and the square of 10·487655.

2. Find the rate, in miles per hour, of a train which goes a mile in 48 secs., and the number of revolutions made in a minute by a wheel of the engine 8 ft. 2 in. in diameter. (Circumference $= 3\frac{1}{7}$ times diameter.)
3. Find the values of

$$\frac{x - \left(1 + \dfrac{x+2}{3x-1}\right)}{x^2+x+1}$$

when $x=1$, $x=0$, and $x=-1$.
4. Find the G.C.M. of

$$x^4 - 6x^2 + 25 \quad \text{and} \quad x^2 - 4x + 5.$$

5. Solve the equations:—

(i.) $\dfrac{3-x}{2} - \frac{1}{3}\left(\dfrac{3-2x}{4}\right) = \dfrac{2x+3}{7} + \left(\frac{1}{4} - \dfrac{3x+1}{2}\right),$

(ii.) $\begin{cases} x + \frac{1}{2}y - \frac{1}{3}(x+y) = y + \frac{1}{2}, \\ 2y - x + 1 = \frac{1}{3}(2x + y + 3). \end{cases}$

6. Boxes of two sizes are used for packing balls of two colours. Each large box can hold 20 black, or 23 red balls; and each small box, 15 black or 17 red. When the larger boxes are filled with black balls, and the smaller with red, the total number of balls is 410. But if the red balls are put in the large boxes, and the black in the small ones, the total number is 16 more. How many boxes are there?
7. Simplify

$$\frac{1}{x+2} + \frac{1}{x-2} + \frac{2+4x-2x^2}{8-2x^2}.$$

8. If α and β are the roots of the equation

$$x^2 - px + q = 0,$$

prove that the expression $x^2 - px + q$ is identical with

$$(x-\alpha)(x-\beta).$$

XLII.

1. A can do a piece of work in 20 days, B in 12 days; A and B work at it for 6 days, and then C finishes it in 2 days. How long would C have taken alone?

2. Simplify

$$\frac{2}{3+\dfrac{1}{5+\dfrac{2}{7+\frac{1}{4}}}}+\frac{4\frac{11}{13}\text{ of }\frac{8}{95}\text{ of }7\frac{3}{7}}{12\frac{1}{35}-2\frac{3}{7}}+\frac{2\frac{1}{3}+1\frac{35}{48}}{9\frac{2}{7}-3\frac{39}{112}}.$$

3. Find $(\cdot 04173)^2$ correct to 4 places of decimals.

4. Solve the equations :—

$$\text{(i.)}\quad 2x^2-7x=72.$$

$$\text{(ii.)}\quad x+y=\frac{15}{4},\quad x-y=xy.$$

$$\text{(iii.)}\quad (\sqrt{x+2}+1)^2-4(\sqrt{x+2}+1)=5.$$

5. Prove that if α and β be the two roots of the quadratic equation

$$x^2-px+q=0,$$

then $\alpha+\beta=p$ and $\alpha\beta=q$.

6. If

$$\frac{x}{y}=\frac{a}{b},$$

prove that each of these fractions is equal to

$$\frac{mx+na}{my+nb}.$$

7. Find the sum of n terms of an Arithmetical Series. Sum

(i.) $13+16+19+\ldots\ldots$ to 10 terms,

(ii.) $13+10+7+\ldots\ldots\ldots$ to 10 terms,

(iii.) $2+\frac{5}{4}+\frac{25}{32}+\ldots\ldots\ldots$ to n terms, and to infinity.

8. Prove that $a^m\times a^n=a^{m+n}$,

hence deduce the interpretations of a^0 and a^{-m}.

XLIII.

1. Simplify $687\frac{7}{999} \times 1000$; and $999\frac{862}{863} \times 753$.
2. Divide $\cdot 2846537$ by $\cdot 32856$ correct to 3 places by Contracted Division.
3. How much does a banker gain, who discounts a bill for £745 payable in 3 months at 5 per cent?
4. Define Proportion, and prove that if

$$a : b :: c : d,$$

then $$ad = bc,$$

and $$a^2 : b^2 :: c^2 : d^2.$$

5. Find the sum of a geometrical series of n terms, whose first term is a, and ratio r.

   Find the sum when $r = a + 1$, and $a = \sqrt[n]{p} - 1$.
6. Find the G.C.M. of

$$6x^3 + x^2 - 11x - 6 \quad \text{and} \quad 6x^4 - x^3 - 14x^2 - x + 6.$$

7. Simplify

$$\text{(i.)}\ \frac{1}{a+b} + \frac{2b}{a^2 - b^2} + \frac{1}{a-b},$$

$$\text{(ii.)}\ \left\{a - \frac{2a}{x + \frac{1}{x}}\right\} \div \left\{\frac{x}{2} + \frac{1}{2x} - 1\right\}.$$

   Test your results by putting $x = a = 2$, $b = 1$.
8. The sum of the squares on the sides of a parallelogram is equal to the sum of the squares on its diagonals.

XLIV.

1. Simplify

$$\frac{2\frac{3}{4} - 1\frac{2}{9}}{\frac{19}{36} + \frac{3}{2} \text{ of } \frac{1}{4}} \div 1\frac{9}{13}.$$

2. Divide $73\cdot 12$ by $\cdot 5284679$ correct to 2 places of decimals.

3. To ·275 of a bushel add ·725 of a quarter, and find the value of the whole at $6\dot{6}$s. per bushel.
4. Prove that the quotient of the difference of the cubes of any two numbers, by the difference of the numbers, is always equal to the sum of the squares and the product of the numbers.
5. Solve the equations :—

$$\text{(i.)}\quad \frac{ax-b}{c}+\frac{bx-c}{a}+\frac{cx-a}{b}=0,$$

$$\text{(ii.)}\quad \begin{cases} \dfrac{x-y}{2}+\dfrac{x+y}{3}=2\tfrac{1}{2}, \\ \dfrac{x+y}{2}+\dfrac{x-y}{3}=4\tfrac{1}{6}. \end{cases}$$

6. A man possesses £5000 stock, some at 3 per cent., four times as much at $3\frac{1}{2}$ per cent., and the rest at 4 per cent. His income is £176. Find the amount of each stock.
7. A railway train, after travelling an hour, is detained 15 minutes, and then goes on at $\frac{3}{4}$ of its former speed, and arrives 24 minutes late. Had the detention taken place 5 miles further on, it would have arrived 3 minutes sooner than it did. Find the original speed, and distance travelled.
8. Describe a circle with given radius, to touch a given circle, and a given straight line.

XLV.

1. If *A* can do a piece of work in 6 days, *B* in 8 days, and *C* in 9 days, in how many days will they finish it, all working together?
2. If a man selling sugar at $4\frac{3}{4}$d. a lb. loses 10 per cent., what price did he give per cwt.?

3. What Principal put out at Compound Interest for 3 years at 5 per cent. would amount to £144 14s. $0\frac{3}{4}$d.?
4. Find two numbers whose sum is 15, and the sum of their cubes 855.
5. If α and β be the roots of

$$x^2+px+q=0,$$

find the value of $\alpha+\beta$.

6. Sum

$$\tfrac{2}{3}+\tfrac{7}{15}+\tfrac{4}{15}+\ldots\ldots \text{ to 21 terms,}$$

and $2-\frac{2}{3}+\frac{2}{9}-\ldots\ldots\ldots$ to 5 terms, and to infinity.

7. Solve

(i.) $\frac{2}{3}x^2+\frac{3}{2}(x-1)=0$,

(ii.) $\begin{cases}5(x+y)-7(x-y)=26,\\ (3x+7y)\div 4=(6x-y)\div 3.\end{cases}$

8. Factorize $y^2z-yz^2+z^2x-zx^2$.

XLVI.

1. A vessel has a supply pipe which will fill it in an hour, and three waste pipes which empty it in 6, 4, and 2 hours respectively. If all are open, how soon will it be filled?
2. Four bells begin to toll simultaneously, and toll at intervals of 4, 6, 8, and 10 seconds. After what time will they again toll simultaneously?
3. Multiply ·26927 by ·0015051 correct to four places of decimals.
4. Solve the equations :—

(i.) $$\frac{1}{x-5}+\frac{2}{2x-6}=\frac{3}{(x-5)(x-3)}.$$

(ii.) $$\frac{x+\frac{y}{2}-3}{x-5}+7=0=\frac{3y-10(x-1)}{6}+\frac{x-y}{4}+1.$$

Test your results.

5. A sculls at the rate of $6\frac{2}{3}$ miles an hour. He leaves X at the same time that B leaves Y. A spends 20 minutes in Y, and reaches X 3 hours after B. B sculls 6 miles an hour, and there is no stream. Find the distance from X to Y.

6. Solve the equation

$$9x^2 - 63x + 68 = 0,$$

and find for what values of m the equation

$$m^2x^2 + (m^2 + m)ax + a^2 = 0,$$

will have equal roots.

7. If $$\sqrt{5} = 2{\cdot}23606,$$

find the value of $\dfrac{6}{\sqrt{5} - 1}$ to five places.

8. On a given straight line as base, construct a triangle, having given the length of one side, and a point through which the other passes.

Show that the above is only possible under a certain condition, and state what it is.

XLVII.

1. Divide ·04273 by 58·19 to six places of decimals, by Contracted Division.

2. Simplify

$$2{\cdot}3475 \text{ of } £5 + 1{\cdot}2125 \text{ of } £2 + {\cdot}625 \text{ of } 13s.\ 4d.$$

3. Find the value of

$$\frac{1}{1\,.\,2} - \frac{1}{3\,.\,2^3} + \frac{1}{5\,.\,2^5} - \frac{1}{7\,.\,2^7}$$

to six places of decimals.

4. Solve the equations :—

$$\text{(i.)}\ \frac{x-2}{3} + \frac{4x+5}{6} - \frac{7x-8}{9} = 0,$$

(ii.) $x + \frac{3}{y} = \frac{7}{2}$, $3x - \frac{2}{y} = \frac{26}{3}$,

(iii.) $5x + 2 = 3y$; $6xy - 10x^2 + \frac{y - 2x}{a} = 8$.

Test your results.

5. A and B travel together 120 miles. B takes a return ticket, and pays half as much again as A. They find that B travels cheaper than A by 4s. 2d. for every 100 miles. Find the cost of A's ticket.

6. Show how to make a quadratic equation whose roots shall be 3 and -2.

What relation must exist between the coefficients of the equation

$$ax^2 + bx + c = 0$$

that one root may be double the other?

7. Find the sum of a G.P. of n terms, whose first term is a, and ratio r.

Find also the sum of the reciprocals of the terms of the same series.

8. In a right-angled triangle the square on the hypotenuse is equal to the square on the difference of the sides, together with four times the area of the triangle.

XLVIII.

1. What is the difference between the true and the commercial discount on a sum of £200 due six months hence at $4\frac{3}{4}$ per cent.?

2. Express as a decimal

$$5\tfrac{5}{640} + \cdot75 \text{ of } \tfrac{6}{5} \text{ of } 7\tfrac{1}{2},$$

and find its value when the unit is £1000.

3. Calculate to six places of decimals

$$\frac{1}{1\,.\,3} + \frac{1}{1\,.\,2\,.\,3\,.\,5} + \frac{1}{1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,7} + \text{etc.}$$

4. Solve the equations :—

$$\text{(i.)}\ 6x^2 - 7x - 20 = 0,$$

$$\text{(ii.)}\ x^{\frac{1}{2}} + x^{-\frac{1}{2}} = (1+x)^{\frac{1}{2}} + (1+x)^{-\frac{1}{2}}.$$

5. A man starts to walk up a mountain. He walks half a mile per hour faster in the first half of the distance than in the second, and reaches the top in $5\frac{1}{2}$ hours. Walking down a mile an hour quicker than the first half of the ascent, he descends in $3\frac{3}{4}$ hours. Find the distance to the top, and his rates of walking.

6. If
$$\frac{a+b}{1-ab} + \frac{c+d}{1-cd} = 0,$$
prove that
$$a+b+c+d = abcd\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right);$$
and hence show that
$$\frac{b+c}{1-bc} = \frac{d+a}{1-ad}.$$

7. Simplify

$$\text{(i.)}\ \frac{b^2-c^2}{b+c} + \frac{c^2-a^2}{c+a} + \frac{a^2-b^2}{a+b},$$

$$\text{(ii.)}\ \frac{x^4+x^2+1}{x^3-1},$$

$$\text{(iii.)}\ \frac{20x^2+27x+9}{15x^2+19x+6} + \frac{20x^2+27x+9}{12x^2+17x+6}.$$

8. If four points E, F, G, H be taken in the sides of a square $ABCD$ so that $AE = BF = CG = DH$, prove that $EFGH$ is a square.

XLIX.

1. Supply the term wanting in this proportion,
$$204 : (\quad) :: 238 : 336.$$

2. Simplify

$$2\tfrac{1}{2} + \cfrac{1}{3\tfrac{1}{3} + \cfrac{1}{4\tfrac{1}{4}}}\ ; \text{ and } \left(2\tfrac{3}{4} + \tfrac{5}{2} \text{ of } \frac{7}{3\frac{4}{5}} - \frac{1\frac{2}{3}}{2\frac{1}{2}}\right) \div 1\tfrac{77}{228}.$$

3. Find the value of $(\cdot 0176325)^3$ to eight places of decimals.

4. Solve the equations :—

$$\text{(i.) } x^2 - \frac{5}{2}x - 6 = 0,$$

$$\text{(ii.) } \frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{x-5}{x^2-6x+8},$$

$$\text{(iii.) } \frac{x+3+\frac{1}{x}}{x-3+\frac{1}{x}} + \frac{1}{3} = x - 3 + \frac{1}{x} + 19.$$

5. Prove that a quadratic equation cannot have more than two roots.

If the equations

$$x^2 - 3x = a, \quad \text{and} \quad x^2 - 4x = 5$$

have a common root, prove that

$$a^2 - 14a + 40 = 0.$$

6. Sum the following series :—

(i.) $1 + 3 + 9 + 27 + \ldots\ldots\ldots\ldots$ to 8 terms, and to n terms,

(ii.) $1 + 3 + 7 + 15 + 31 + \ldots\ldots$ to 8 terms, and to n terms.

(Add 1 to each term.)

Test your results by assuming $n = 1$, and 2.

7. Find the value, when $b = 3$, $c = 7$, of

$$\sqrt{\frac{3b+c}{c-b}} - \frac{\sqrt[3]{9b+7c-12}}{c-b}.$$

8. Prove that the sum of the squares of any two numbers is always greater than twice their product.

What does this become in Geometry?

L.

1. What will be the cost of papering a room, 21 ft. long, 15 ft. broad, and 11 ft. high, which has two windows, each 9 feet high and 3 ft. wide, a door 7 ft. by 3 ft. 6 in., and the fireplace 4 ft. by 4 ft. 6 in.; with paper 2 ft. 3 in. wide, at 9s. a piece? The price of putting it on is 6d. a piece, and each piece contains 12 yards.
2. A bankrupt owes £4608, and pays 13s. $10\frac{1}{4}$d. in the £. How much do all his creditors lose?
3. Gunpowder being composed of 33 parts nitre, 7 charcoal, and 5 sulphur, find how many lbs. of each will be required to make 30 lbs. of powder.
4. Solve the equations
$$\frac{x}{a+b}+\frac{y}{a-b}=2a, \qquad \frac{x-y}{4ab}=1.$$
5. Find the value of
$$\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1+\sqrt{1-x}},$$
when $x=\frac{\sqrt{3}}{2}$.
6. Sum to 10 and to n terms
$$1+2\tfrac{1}{2}+4+\ldots\ldots$$
7. Reduce
$$\frac{x^4+ax^3-9a^2x^2+11a^3x-4a^4}{x^4-ax^3-3a^2x^2+5a^3x-2a^4}$$
to its lowest terms.
8. Inscribe a square in a right-angled triangle.

LI.

1. If the length of a square reservoir be 12 ft. 8 in., what weight of water must be drawn off that it may sink 3 ft. 9. in.? (1 cub. ft. of water weighs 1000 oz.)

2. What fraction of 2 miles 5 fur. 9 po. 1 ft. 6 in., is $\frac{5}{11}$ of 7 furlongs?
3. A man by selling out of Three Per Cent. stock at 99, gains 10 per cent. on his investment. At what price did he buy, and what was his income supposing that he realised £15345?
4. Solve the equation

$$\frac{x^2}{\sqrt{x^2+5}} = 1 + \frac{1}{\sqrt{x^2+5}}.$$

5. Simplify

$$\left\{-(x^3)^{\frac{1}{2}}\right\}^{-\frac{1}{3}} \times \left\{-(-x)^{-3}\right\}^{\frac{1}{2}}.$$

6. Divide

$$1 \text{ by } 1 - x + x^2 \text{ to 4 terms.}$$

7. Continue the Harmonic Progression

6, 3 ... for 6 terms.

8. The lines bisecting the angles at the base of an isosceles triangle meet the sides at D and E. Show that DE is parallel to the base.

LII.

1. Find the square root of 78783376.
2. A level reach in a canal 14 miles 6 furlongs long, and 48 ft. broad, is kept up by a lock 80 ft. long, and 12 ft. broad, and having a fall of 8 ft. 6. in. How many barges can pass through the lock before the water in the upper canal is lowered 1 inch?
3. Add together

$\cdot 60625$ of £1, $+ \cdot\dot{1}4285\dot{7}$ of 14s. $10\frac{1}{2}$,

and $\frac{23}{11}$ of $\frac{3}{71}$ of £3 5s. 1;

and express the result as the decimal of 27s.

4. Simplify

$$3x^2\sqrt{\frac{2}{3x^2}} + 5\sqrt{\frac{27x^2}{50}}.$$

5. Sum the series

$$1+\tfrac{5}{6}+\tfrac{2}{3}+\ \ldots \text{ to 18 terms.}$$

6. Solve

(i.) $x^2-y^2=24, \quad xy=35.$

(ii.) $(a-b)(x-c)-(b-c)(x-a)-(c-a)(x-b)=0.$

7. A number consisting of two digits has one decimal place. The difference of the squares of the digits is 20, and if the digits be reversed, the sum of the two numbers is 11. Find the number.

8. Trisect a right angle.

LIII.

1. By what must 1350 be multiplied to make it a perfect cube?

2. Simplify

$$\frac{5\frac{5}{8}\div\frac{2}{3}}{1\frac{1}{5}\text{ of }\frac{5}{9}\div 10\frac{1}{3}}\times\frac{2}{5}\text{ of }\frac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{13\frac{7}{8}\text{ of }5\frac{1}{3}}.$$

3. A pond with vertical sides, whose area is 1 ac. 3 ro. 22 po. 14 yd. $8\frac{1}{2}$ feet, has a tower in the middle whose sides are 18 ft. and 16 ft.; how many gallons must be pumped out to reduce the depth 18 inches?

(N.B.—1 gall. $=277\frac{1}{4}$ cub. in.)

4. Sum to 10 terms

$$2-2^2+2^3-2^4+\ \ldots.$$

5. Find the square root of 33224 in the scale of 6.

6. Solve

(i.) $x^4-2x^3+3x^2-2x+1=1.$

(ii.) $(a+x)(b+x)=(c+x)(d+x).$

7. Find the L.C.M. of

$$12x^2+5x-3, \quad\text{and}\quad 6x^3+x^2-x.$$

8. The sum of the diagonals of a quadrilateral is less than the sum of any four lines drawn to the four angles of the quadrilateral from any point within it, except the intersection of the diagonals.

LIV.

1. Make out a bill for 8 dozen knives at 35s. per dozen; 6 dozen at 25s. per dozen; 12 pairs carvers at 12s. per pair; 5 pairs do. at 13s. 6d. per pair; 10 pairs do. at 12s. per pair; and 5 steels at 4s. 6d. each.

2. Simplify

$$\frac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}+\frac{7}{12}\text{ of }\frac{9\times 10}{14\times 3}-\frac{22\frac{1}{2}}{30}.$$

3. Resolve into factors :—

(i.) $15x^2-7x-2$,

(ii.) $14x^2-37ax+5a^2$,

(iii.) $3x^4+2x^3y-2x^2y^2-2xy^3-y^4$.

4. Simplify

(i.) $\dfrac{a+x}{2(a-x)}+\dfrac{a-x}{2(a+x)}-\dfrac{2ax}{a^2-x^2}$,

(ii.) $\dfrac{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}}{\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}}$.

5. Find the cube root of

$$64x^3+96x^2y+48xy^2+8y^3.$$

6. Solve

(i.) $\frac{1}{2}(x-7)-\frac{2}{5}(11-x)=\frac{3}{4}(x+5)-7\frac{1}{2}$,

(ii.) $2x^2-11x=21$,

(iii.) $xy=30,\quad x-y=3\frac{1}{2}$.

7. The wages of 24 men and 16 boys amount to £5 16s. per diem: half that number of men with 21 more boys would earn the same money. Find the wages of each man and boy.

8. Trisect a given straight line.

LV.

1. Calculate to six places of decimals

$$\frac{1}{3}-\frac{1}{3\,.\,3^3}+\frac{1}{5\,.\,3^5}-\frac{1}{7\,.\,3^7}.$$

2. Reduce 3 qr. 14 lb. to the decimal of 1 cwt.

3. Find the coefficient of x^5 in

$$(1+2x+3x^2+4x^3+5x^4)\times(1+3x+5x^2+7x^3+9x^4).$$

4. Simplify

(i.) $\dfrac{x^4+(a+b)^2x^2+(a+b)^4}{x^3+(a+b)^3}\times\dfrac{1}{x^2+(a+b)x+(a+b)^2},$

(ii.) $\sqrt{\dfrac{x^2}{a^2}+\dfrac{y^2}{4}+9a^2+3ay+6x+\dfrac{xy}{a}}.$

5. Solve

(i.) $\dfrac{1}{x-3}+\dfrac{1}{x-5}=\dfrac{2}{x-7},$

(ii.) $x^2+y^2=18\frac{1}{2},\quad x+y=6.$

6. Two men start at the same time from opposite ends of a piece of road, and meet in 6 minutes; after passing, one gets to the end of the road 5 minutes before the other. How long does each take?

7. Sum the series

$3\frac{1}{2}+1+\frac{2}{7}+\ldots\ldots$ to n terms, and to infinity.

8. Construct a right-angled triangle, having given the hypotenuse, and one of the sides.

LVI.

1. Simplify

$$\frac{\dfrac{\frac{2}{5}\text{ of }\frac{7}{8}\text{ of }6\frac{3}{7}\div 7\frac{7}{11}+19\frac{13}{20}+8\frac{19}{22}}{3\frac{5}{8}+\frac{2}{7}+4\frac{1}{10}-\frac{11}{15}\text{ of }1\frac{6}{7}}-\dfrac{221}{680}}{\dfrac{\frac{39}{152}+\frac{6}{95}}{\frac{9}{38}}+39\frac{11}{12}-24\frac{3}{5}}.$$

2. If 16 darics make 17 guineas, 19 guineas make 24 pistoles, 31 pistoles make 38 sequins, how many sequins are there in 1581 darics?

3. Express

$$\frac{x+y}{xy}\left(\frac{1}{x}-\frac{1}{y}\right)-\frac{y+z}{yz}\left(\frac{1}{z}-\frac{1}{y}\right)$$

as the difference of 2 squares;
and evaluate it when $x=1$, $z=\frac{1}{2}$.

4. Factorize
   (i.) $x^2-3x(a+b)-4(a+b)^2$,
   (ii.) $(x+y)^4-8(x+y)^2(k-l)^2+16(k-l)^4$.

5. Find the G.C.M. of
$x^4-2x^3-4x^2-55x$ and $x^5+8x^4+25x^3+52x^2-11x$.

6. The sum of two numbers is 2572, and their G.C.M. is 643. Find the numbers.

7. Multiply

$$x^{\frac{2}{p}}-(y+z)x^{\frac{1}{p}}+yz \quad \text{by} \quad x^{\frac{1}{p}}-z.$$

8. The sides AB, AC of a triangle are bisected at E and F respectively; a perpendicular is let fall from A on BC, meeting it at D; show that the angle FDE is equal to the angle BAC, and that $AFDE$ is equal to half the triangle ABC.

LVII.

1. Find the change in income, when £4800 stock is transferred from the Three Per Cents. at $87\frac{7}{8}$, to a 4 per cent. stock at $116\frac{7}{8}$. Brokerage $\frac{1}{8}$ per cent.

2. If 6 men and 2 boys can reap 13 acres in 2 days, and 7 men and 5 boys can reap 33 acres in 4 days, how long will it take 2 men and 2 boys to reap 10 acres?

3. Simplify

$$\text{(i.)}\quad \frac{x^3 - xy^2}{(xy + y^2)^2} \times \frac{x^2y + y^3}{(x^2 - xy)^2} \div \frac{x^2 + y^2}{x^2y^2},$$

$$\text{(ii.)}\quad \sqrt{4 + \sqrt{16x^2 + 8x^3 + x^4}},$$

$$\text{(iii.)}\quad \left\{\left(a^{\frac{2}{3}}b^{-\frac{1}{3}}\right)^6\left(a^{-\frac{2}{3}}b^{\frac{1}{3}}\right)^{-6}\right\}^{-\frac{1}{4}}.$$

4. If two numbers differ by 2, twice the square of the number between them, is less by 2 than the sum of the squares of the numbers.

5. Evaluate

$$\frac{(a+b)^2}{c+d} \times \frac{(a+d)^2}{b+c} \div \frac{(c-d)^2}{a+b},$$

when $a = -4$, $b = 3$, $c = -2$, $d = 1$.

6. Find the factors of

$$\text{(i.)}\quad k^8 + k^4l^4 + l^8,$$

$$\text{(ii.)}\quad 2abc + a^2(b+c) + b^2(c+a) + c^2(a+b).$$

7. The first term of an Arithmetical Series is 3, the last term 21, and the sum 156; find the number of terms.

8. From a given point draw two straight lines, making the same (given) angle with two given straight lines, which intersect. How many solutions are there?

LVIII.

1. If 5 men and 7 women earn £7 13s. in 6 days, and 2 men and 3 women earn 3 guineas in the same time, in what time will 6 men and 12 women earn £60?

2. Simplify

$$\frac{15\frac{1}{3} - 7\frac{1}{6}}{12\frac{1}{5} + 3\frac{1}{4}} \div \frac{16\frac{1}{3} + 2\frac{5}{12}}{9\frac{1}{4} - 3\frac{1}{3}} \times \left(9\tfrac{12}{49} - 7\tfrac{1}{7}\right).$$

3. If two numbers differ by 2, prove that the difference of their squares is twice their sum.

4. Find the G.C.M. of

$$3x^4 + x^3y - 4x^2y^2 - xy^3 + y^4$$

and $$3x^4 + 2x^3y - 2x^2y^2 - 2xy^3 - y^4.$$

What is meant by the L.C.M. of two numbers?

5. If α and β are the roots of

$$x^2 + px + q = 0,$$

find the values of $\alpha + \beta$ and $\alpha\beta$.

6. Solve

$$\text{(i.)}\quad \frac{1}{x+a+b} = \frac{1}{x} + \frac{1}{a} + \frac{1}{b},$$

$$\text{(ii.)}\quad \begin{cases} 3x^2 + 2xy - y^2 = 180, \\ \dfrac{x+y}{3x-y} = \dfrac{5}{9}. \end{cases}$$

[*N.B.*—$(x+y)(3x-y) = 3x^2 + 2xy - y^2$.]

7. The first term of a G.P. is $\frac{3}{4}$, the sixth is 24; find the common ratio, and sum of six terms.

8. If the sides of an equilateral and equiangular hexagon are produced to meet, the angles formed by these lines are together equal to four right angles.

LIX.

1. Simplify

$$\tfrac{3}{2} - [\tfrac{1}{3} + \{1 - (\tfrac{1}{3} - \tfrac{1}{4})\}] + \tfrac{1}{2}\left(\frac{\frac{3}{4} - \frac{1}{2}}{\frac{2}{3} - \frac{1}{4}}\right) + \frac{19}{20}.$$

2. If 1 cwt. 2 qrs. 12 lbs. of lead are rolled into a sheet 18 feet long, and 6 feet wide, find its thickness. (A cubic foot of lead weighs 720 lbs.)

3. Find the square root of

$$\frac{k^4}{9} + 3k^2l^2 + 9l^2 - \frac{2k^3l}{3} - 6kl^3.$$

4. Two lengths of cloth are bought for £5 6s. One is 4 yards longer than the other, and each costs as many shillings per yard as it is yards long. Find the lengths.

5. If $x=16$, $y=9$, $p=2$, $q=3$, find the value of

$$\frac{x^{\frac{1}{p}}+\frac{1}{x^p}}{x^{-\frac{q}{p}}+x^{\frac{1}{p}}+x^{-p}}+\frac{1}{343}\left(\frac{4x^{\frac{1}{p}}y^q}{3x^q y^{\frac{1}{p}}}\right)^{-\frac{1}{2p}}.$$

6. Solve the equations :—

(i.) $(3x-6)^2+7x^2-256=0.$

(ii.) $x^2+y^2=20=10(x+y-\sqrt{2xy}).$

7. Form the equations whose roots are

(i.) $3,\quad 2,\quad -1.$

(ii.) $0,\quad k,\quad -l.$

8. Prove that the three straight lines which bisect the angles of a triangle meet in a point.

LX.

1. Find the length of the edge of a cube, which contains 3605 cub. ft. 64 cub. in.

2. Find the price of 3 acres 1 rood 7 perches, 88 sq. yds. at £161 6s. 8d. per acre.

3. When the Three Per Cents. are at $87\frac{1}{2}$, and shares paying 5 per cent. are at $130\frac{1}{4}$, which is the better investment? What sum does a man invest equally in each when the difference between the two incomes is £561?

4. Find the square root of $33-20\sqrt{2}$,

and the cube root of $6\sqrt{3}+10$.

5. Simplify

$$\left[a^{\frac{1}{3}}\left\{a^{-\frac{1}{2}}b^{-\frac{1}{3}}(a^2b^2)^{\frac{2}{3}}\right\}^{-\frac{1}{2}}\right]^6.$$

6. Solve the equations :—

$$\text{(i.)}\ x^2 - 6x + 8 = 0,$$

$$\text{(ii.)}\ \begin{cases} xy = 24, \\ 2x - 5y = 8. \end{cases}$$

7. If the work done by $x - 1$ men in $x + 1$ days : work done by $x + 2$ men in $x - 1$ days : : 9 : 10, find the numbers of men and days in each case.

8. Prove that the greatest rectangle which can be inscribed in a circle is a square.

LXI.

1. The sum of £533 6s. 8d. is received for £560 due two years hence, what is the rate of discount ?

2. If 30 cub. in. of gunpowder weigh 1 lb., what weight of powder will be required to fill a cylinder of 8 inches internal diameter, and with a length of $2\frac{1}{2}$ feet ?

   (If r be the radius of a circle, its area $= \pi r^2$; $\pi = \frac{22}{7}$.)

3. Find the square root of 13 to three places ot decimals.

4. Divide

$$nm(x^5 + 1) + (n^2 + m^2)(x^4 + x) + (n^2 + 2mn)(x^3 + x^2)$$

   by $nx^2 + mx + n$.

5. Simplify

$$\text{(i.)}\ \left\{\frac{a^{-2}}{b^{-2}}\left(\frac{a}{b}\right)^{\frac{1}{2}}\right\}^{\frac{1}{3}},$$

$$\text{(ii.)}\ (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})(\sqrt{2} + 1)(\sqrt{2} - 1).$$

6. Solve

$$\text{(i.)}\ \frac{x-4}{x-5} - \frac{x-5}{x-6} = \frac{x-7}{x-8} - \frac{x-8}{x-9},$$

$$\text{(ii.)}\ \frac{x+y}{2} + \frac{x-y}{3} = 11, \quad \frac{x}{4} - \frac{x+y}{9} = 1.$$

7. Find the G.C.M. of

$$2x^3 - 9x^2y + 11xy^2 - 3y^3 \quad \text{and} \quad 4x^3 - 4x^2y - 5xy^2 + 3y^3.$$

8. If two sides of a triangle be given, its area will be greatest when they contain a right angle.

LXII.

1. A runs a two-mile race with B and loses. Had he run $\frac{1}{3}$ quicker, he would have won by 22 yards. Compare the rates at which each ran.
2. Find the cube roots of 512768384064 and $42\frac{7}{8}$.
3. Two clocks are at 12 at the same instant. One loses 7 seconds, and the other gains 8 seconds in 24 hours. When will one be half an hour in front of the other, and what time will each show ?
4. A shilling weighs 3 dwt. 15 grs., of which 3 parts out of 40 are alloy and the rest pure silver. If the value of silver rises 8 per cent., what must be the reduction of pure silver in a shilling ?
5. Find the value of
$$(2\sqrt{2}+\sqrt{3})(3\sqrt{2}-\sqrt{3})(3\sqrt{3}-\sqrt{2}).$$
6. Sum
$$3+3\tfrac{3}{4}+4\tfrac{1}{2}+\ldots\ldots \text{ to 8 terms.}$$
7. Find the mean proportional between 360 and 250; and if $x-4 : x-2 :: x-1 : x+3$, find the value of x.
8. Solve
$$\text{(i.) } x^2-3x-28=0,$$
$$\text{(ii.) } \frac{3x^2-2y^2}{5}-\frac{x^2-y^2}{2}=1, \quad \frac{x^2}{3}+\frac{y^2}{2}=4.$$

LXIII.

1. What decimal of £4 3s. 4d. is $\frac{1}{2600}$ of £10 16s. 8d. ?
2. A and B together do a piece of work in 7 days, B alone in 13 days. If B works 3 days, how long will A take to finish ?

3. Shares in a railway pay £3 5s. dividend per annum; how much ought I to give for them to get 5 per cent. for my money?

4. If ax^2+bx+c is divisible by $x-h$ without remainder, then $x=h$ will satisfy the equation

$$ax^2+bx+c=0.$$

5. Find a third proportional to

$$x^2-a^2 \quad \text{and} \quad ax-a^2;$$

and show that if

$$a:b::c:d,$$

then $$a^2:b^2::(a-c)^2:(b-d)^2.$$

6. Sum to 7 terms

$$11\tfrac{2}{3}+10\tfrac{1}{2}+9\tfrac{1}{3}+\ldots\ldots,$$

and find the 20th term.

Sum

$$6+3+1\tfrac{1}{2}+\ldots\ldots \text{ to } n \text{ terms.}$$

7. Solve

$$\text{(i.)} \quad \frac{1}{x-2}-\frac{1}{x-3}+\frac{1}{x+1}=0,$$

$$\text{(ii.)} \quad \begin{cases} 5(x+y)-7(x-y)=26, \\ \dfrac{3x+7y}{4}=\dfrac{6x-y}{3}. \end{cases}$$

8. Find two numbers whose sum is 15, and the sum of their squares 173.

LXIV.

1. Three debts of £200, £300, £400 are due respectively in 3, 4, and 5 months. If the interest is 5 per cent., what amount will pay them all off at the end of 4 months?

2. Two cogged wheels work together, there being 32 cogs on one, and 36 on the other. The larger wheel makes 64 revolutions per second. How often will the same cogs come together in 6 days of 10 hours?

3. Find the length of the edge of a cube whose contents are 120066 cub. yds. 10 cub. ft.
4. Form the equation whose roots are 4, 5, and 0.
5. If $a : b :: c : d$, prove that
$$a^2 + c^2 : a^2 :: b^2 + d^2 : b^2.$$
6. Insert 4 Harmonic means between
$$\tfrac{2}{3} \text{ and } \tfrac{2}{13}.$$
7. In an A.P. the first term is 2, the last 29, and the sum 155. Find the difference.
8. Solve
$$\text{(i.) } x - a = \sqrt{2a^2 - 2ax - x^2},$$
$$\text{(ii.) } \frac{x^2 + y^2}{17} + \frac{x + y}{10} = 0, \quad xy = 1.$$

LXV.

1. A rectangular grass plot, the sides of which are as 2 : 3, costs £14 8s. for turfing, at 4d. a sq. yd. Find the length of its sides.
2. Transform 30763451 into the scale of 12.
3. Between 1841 and 1851 the population of England increased 14·2 per cent. In the latter year it was 21,121,290. What was it in the former year?
4. Solve
$$\text{(i.) } \frac{9}{x} - \frac{4}{y} = 2, \quad \frac{18}{x} + \frac{8}{y} = 10,$$
$$\text{(ii.) } 2x^3 - 3x^2 + 5 = 0,$$
$$\text{(iii.) } 2x^2 - 1 = 5x + 2.$$
5. If $x : y :: m : n$, then
$$\frac{1}{x} - \frac{1}{2y} - \frac{1}{3m} + \frac{1}{4n} = \frac{1}{xn}\left\{\frac{x}{4} - \frac{y}{3} - \frac{m}{2} + n\right\}.$$
6. Any number which, on being added to a number of two digits, reverses the digits, must be a multiple of 9.

7. Sum

$$100 + 101 + 102 + \dots + n$$

without using any formula.

8. Prove that a ratio of greater inequality is diminished by adding the same quantity to each of its terms.

LXVI.

1. Simplify

8·5 of 2s. 6d. + 1·25 of 5s. + 3·0625 of 6s. 8d.

2. Find the sum of money which should be accepted in payment of a debt of £150 due 146 days hence. Interest 5 per cent.

3. The length of $\frac{1}{360}$ of the earth's circumference is about $69\frac{1}{22}$ miles. The circumference of a circle $= 3\frac{1}{7}$ times the diameter. Find the earth's diameter.

4. A man starts to walk from A to B, a distance of 29 miles, at $3\frac{1}{2}$ miles an hour. Another starts some time afterwards from B to A at $4\frac{1}{2}$ miles an hour. They meet a mile nearer A than B. Find what time the second started.

5. Write down the two smallest *integral* values of a which would make the remainder in the division of

$$x^3 + ax^2 - 3a^2x - 3a^3 \quad \text{by} \quad x - 2a$$

a perfect square.

6. Solve the equations :—

$$\begin{cases} x + \dfrac{y}{x} = 5, \\[2ex] \dfrac{3\dfrac{y}{x} - x + 1}{x} - \dfrac{y}{x^2} = \dfrac{5}{2}. \end{cases}$$

7. There are three numbers in G.P. whose sum is 70. If the extremes be each multiplied by 4, and the mean by 5, they will be in A.P. Find them.

8. Circles described on the equal sides of an isosceles triangle as diameters, will intersect at the middle point of the base.

LXVII.

1. Which of the quantities

$$x^3+y^3, \quad x^4-y^4, \quad x^5-y^5, \quad x^6+y^6,$$

is exactly divisible only by $x-y$;
which by $x+y$, and which by x^2-y^2?

2. Solve the following equations :—

$$\text{(i.)}\ \frac{1}{x^2-1}+\frac{1}{x+1}=\frac{7}{8}+\frac{1}{1-x},$$

$$\text{(ii.)}\ x-y+\sqrt{\frac{x-y}{x+y}}=\frac{20}{x+y}, \quad x^2+y^2=34.$$

3. A number of posts are placed at equal distances in a straight line. If to twice their number we add the distance between every two, measured in feet, the sum is 68; if from four times the distance between every two we subtract half the number of posts, the result is the same. Find the distance between the extreme posts.

4. A ship has on board 366 emigrants, equivalent to 326 statute adults, infants not being reckoned and children counting as half. There was 1 infant to every 23 adults. Find how many there were of each class?

5. Insert 6 Arithmetic means between

$$1 \text{ and } \tfrac{1}{7}.$$

6. In the Geometrical series

$$\tfrac{2}{5}, \quad \tfrac{1}{3}, \quad \tfrac{5}{18}, \quad \text{etc.},$$

find the 5th term, the sum of n terms, and the sum to infinity.

7. If $a : b :: c : d$, prove that

$$\frac{a}{c} = \frac{a+b}{c+d} = \sqrt{\frac{a^2+b^2}{c^2+d^2}}.$$

8. Of all parallelograms, with diameters of given lengths, the rhombus is the greatest.

LXVIII.

1. Find the square root of 173889 and of $134\frac{155}{169}$.
2. Find the cost of carpeting a room 10 yds. 2 ft. long, 7 yds. 1 ft. broad, with carpet $\frac{3}{4}$ yd. wide, at 4s. 6d. a yard.
3. A and B walk a race of 25 miles. A gives B 45 mins. start, and walking uniformly a mile in 11 mins., catches B at the 20th milestone. Find B's rate, and by how much he lost in time and space.
4. Prove that

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{80}-\sqrt{5}} = \sqrt{5}(1+\sqrt{2}).$$

5. Solve

(i.) $5x - 3y - 28 = 0, \quad 3x - 2y - 13 = 0,$

(ii.) $a^2x^2 - 2a^3x + a^4 - 1 = 0.$

6. In what scale is the denary number 59535 represented by 40803?
7. Divide

$$a^{-\frac{3}{2}} - b^{-\frac{3}{2}} \quad \text{by} \quad a^{-\frac{1}{2}} - b^{-\frac{1}{2}}.$$

8. Find a point which shall be the same given distance from a given point and from a given line.

LXIX.

1. In a compound metal the proportion of tin to copper is 7·75 to 92·25. Find to the nearest penny the value of 8 cwt. 3 qr. of it. Tin costs £140; copper £80 per ton.

2. Simplify

$$\frac{1\frac{3}{4}-\frac{7}{6}\text{ of }\frac{18}{28}}{\frac{5}{6}\text{ of }\frac{12}{20}+5\frac{1}{2}}\div\frac{1}{6}-\left\{\frac{3\frac{1}{3}}{7}+\frac{2}{10\frac{1}{2}}-\frac{5}{18}\text{ of }\frac{4}{7}\right\}\div\frac{4}{7}.$$

3. A rectangular court is 50 yds. long and 30 yds. broad. It has paths joining the middle points of the opposite sides of 6 ft. in breadth, and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 1s. 8d. per sq. foot, and the turf 3s. per sq. yd., find the cost of laying out the court.

4. Solve the equation :—

$$\sqrt{x+4ab}=2a+\sqrt{x}.$$

5. Transform 1331 from the quinary to the senary scale. Show that it is a cube in any scale.

6. Simplify

$$\frac{x^2+y^2}{xy}-\frac{x^2}{xy+y^2}-\frac{y^2}{x^2+xy}.$$

7. Sum the series :—

(i.) $5\frac{1}{2}+4+2\frac{1}{2}+\ldots\ldots.$ to 5 terms,

(ii.) $\sqrt{5}+1+\frac{1}{\sqrt{5}}+\ldots\ldots$ to 8 terms.

8. Prove that the rectangle contained by the sum and difference of two straight lines, is equal to the difference of the squares on the lines.

LXX.

1. If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12; find how long 13 men, 14 children, and 15 women will take to do it.

2. Find the discount on £2034 5s. due in $6\frac{6}{17}$ months, at $5\frac{2}{3}$ per cent.

3. Find the value of 27 yds. 2 ft. 9 in. of one material, if 17 yds. 1 ft. 11 in. of another, worth $\frac{1}{7}$ as much again, cost £5 5s. 10d.

4. Sum

   (i.) $\frac{5}{6}$, $\frac{1}{2}$, $\frac{1}{6}$, ... to 12 terms,

   (ii.) $1^2 + 2^2 + 3^2 + \ldots$ to n terms.

5. Multiply 57264 by 675 in the scale of 8, and find the square root of 20461 in the scale of 7.

6. Solve

   (i.) $$1{\cdot}2x - \frac{{\cdot}18x - {\cdot}05}{{\cdot}5} = {\cdot}4x + 8{\cdot}9.$$

   (ii.) $$(x-9)(x-7)(x-5)(x-1) = (x-2)(x-4)(x-6)(x-10).$$

7. Prove that if x be a real positive quantity, then $x^4 + 4x^2 + 1$ is not less than $3x^3 + 3x$.

8. A is a given point, and B a given point in a given straight line: it is required to draw from A a straight line AP to the given straight line, so that the sum of AP and PB may equal a given length C.

LXXI.

1. Find the amount of £6500 in 3 years at 5 per cent. Compound Interest.

2. By selling goods for £817 19s. a man lost 9 per cent. At what price should he have sold to gain $10\frac{1}{2}$ per cent.?

3. Find the value of 12345 things at £2 8s. $11\frac{3}{4}$ each.

4. At a cricket match the contractor provided dinner for 24, and fixed the price to gain $12\frac{1}{2}$ per cent. on his outlay. Three were absent. The remaining 21 paid the fixed price for dinner, and the contractor lost 1s. What was the charge?

5. Solve

$$\text{(i.)}\quad \frac{x+8}{x+4}+\frac{x-3}{x-1}=2.$$

$$\text{(ii.)}\quad 3x^2+29x-10=0.$$

6. Form the equation whose roots are the doubles of those of the equation

$$(x+5)^2-4x=160.$$

7. Square $\sqrt{2}+\sqrt[3]{3}-\sqrt[4]{4}.$

8. Describe a circle to pass through two given points and have its centre in a given straight line.

LXXII.

1. The sum of £9 0s. 10d. is allowed for papering a room 27·7 feet long, 19·55 feet wide, and 12·4 feet high. How much per yard must be given for a paper 2·7 ft. wide ?

2. Distinguish between Interest and Discount.
What ready money will discharge a debt of £1056 18s. due 4 months hence, at $4\frac{7}{8}$ per cent. ?

3. Simplify

$$(a+b+c)^2-(a+b)^2-(b+c)^2-(c+a)^2+a^2+b^2+c^2.$$

4. Find the square root of

$$x^2y^2+a^2b^2+z^4+2abxy+2xyz^2+2abz^2.$$

5. If you subtract 1 from any number, and multiply the result by n, then subtract 1 and add the original number, prove that the original number is 1 more than the one $(n+1)^{\text{th}}$ part of the final result.

6. Solve

$$\text{(i.)}\quad x^2+10=13(x+6),$$

$$\text{(ii.)}\quad \begin{cases} x^2+xy=a^2, \\ y^2+xy=b^2. \end{cases}$$

7. Find the quadratic equation whose roots are

$$\sqrt{2} \quad \text{and} \quad 2^{\frac{3}{2}}.$$

8. Find a triangle equal in area to a given quadrilateral.

LXXIII.

1. When £100 stock in the Three per Cents is worth $89\frac{1}{2}$, at what rate may the same quantity of stock be purchased in the Three-and-a-half per Cents with equal advantage?
2. Selling at 2s. 8d. a man gains $\frac{1}{8}$ of his outlay. If he sells at 3s. 1d., what is his gain per cent.?
3. Solve

   (i.) $3x^2 - 4x = 35$.

   (ii.) $x^2 + y^2 = \dfrac{5}{2}, \quad x^2 - xy + y^2 = \dfrac{7}{4}$.
4. Find a number, the square of which shall exceed the square of a given number by three halves of the product of the numbers themselves.
5. Define an Arithmetical Series, and give some examples, constructed by yourself.
6. Find the square root of
$$4 + 2\sqrt{3}$$
   in the form $\sqrt{x} + \sqrt{y}$.
7. Break up into factors
$$x^4 + x^2y^2 + y^4, \quad \text{and} \quad x^3 + y^3 + z^3 - 3xyz.$$
8. Which quantity is the greater,
$$\frac{x}{y^2} + \frac{y}{x^2} \quad \text{or} \quad \frac{1}{x} + \frac{1}{y},$$
   x and y being both positive?

LXXIV.

1. Find the cost of carpeting a room 48 ft. 9 in. by 20 ft. 3 in., at 3s. 4d. per sq. yd.
2. Find the value of $\cdot 267\dot{1}87\dot{5}$ of £3 in shillings, pence, and decimal of a penny.

3. What must be the dimensions of a rectangular cistern, of which the base is square, and the depth equal to a quarter of a side of the base, in order that it may contain 19321 gallons of water, each gallon containing 278 cubic inches?

4. If a be the 1st term, and b the 2nd term of an A.P. what is the p^{th} term?

5. Sum

$$\text{(i.)}\ \tfrac{2}{3}+\tfrac{5}{6}+1+\ldots\ldots \text{ to 12 terms,}$$
$$\text{(ii.)}\ 1+\tfrac{1}{4}+\tfrac{1}{16}+\ldots \text{ to } n \text{ terms,}$$

and show that the sum of (ii.) is always less than $\frac{4}{3}$, whatever n may be.

6. Solve

$$(x+1)(x+3)-2(x+2)=0.$$

7. Shew that $x=2$ satisfies the equation

$$x^3+8=2x^2+11x-14,$$

and find the other values of x.

8. Prove Euc. I. 26 by superposition.

LXXV.

1. Simplify

$$\frac{4\cdot\dot{2}8571\dot{4} \text{ of } 3\cdot 4}{1\frac{3}{10} \text{ of } 2\cdot\dot{4}2857\dot{1}} \times \frac{\cdot 4\dot{3} \text{ of } \cdot 625}{\cdot 24}.$$

2. Which is the better investment, stock paying $3\frac{1}{2}$ per cent. at 88, or 4·4 per cent. at $93\frac{1}{3}$?

3. If the discount on a bill due 5 months hence at $3\frac{3}{4}$ per cent. is £774 12s. 6d., what is the amount of the bill?

4. Solve

(i.) $ax-by=a-b,\quad a^2x+b^2y=a^2-ab+b^2,$

(ii.) $x+6y+5z=0,\quad 2x-9y+3z=0,\quad x+3y+z=3.$

5. Find the square root of

$$\frac{9x^2}{a^2}+\frac{a^2}{9x^2}-6\frac{x}{a}-\frac{2a}{3x}+3.$$

6. If the sum of p terms of an A.P. be p^2, and of q terms q^2, prove that the sum of n terms is n^2.
7. If 2 women can do the work of 3 boys, and 3 men the work of 5 women, find a woman's share of £44, which is paid to 5 boys, 3 women, and 5 men.
8. B requires n times as long as A for a piece of work. They worked together for $m-1$ days, and then B finished it alone in $n-m+1$ days. In what time could each do it ?

LXXVI.

1. The amount is £304 10s. $11\frac{1}{4}$d., time 3 years, rate $3\frac{3}{4}$ per cent. ; what is the principal ?
2. Find to 6 places of decimals the sum of
$$\frac{2}{5}+\frac{4}{5\times 10}+\frac{8}{5\times 10\times 15}+\frac{16}{5\times 10\times 15\times 20}+\text{etc.}$$
3. Find the cube root of
$$\frac{x^3}{y^3}-\frac{y^3}{x^3}-3\left(\frac{x}{y}-\frac{y}{x}\right).$$
4. Simplify (i.) $\dfrac{\sqrt{p^2+q^2}-\sqrt{p^2-q^2}}{\sqrt{p^2+q^2}+\sqrt{p^2-q^2}}+\dfrac{\sqrt{p^4-q^4}}{q^2}$.

(ii.) $\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x}}}+\dfrac{1}{x-\dfrac{1}{x-\dfrac{1}{x}}}$.

5. Solve the equations :—

(i.) $(x+5)(x-3)+(x-4)(x+3)=2(x+2)(x-3)$,

(ii.) $\begin{cases}\dfrac{1}{2x+3y}=\dfrac{7}{3x-y}, \\[2ex] \dfrac{3}{(x-1)(y-2)}=\dfrac{4}{x(1-x)}-\dfrac{6}{x(y-2)}.\end{cases}$

(iii.) $\dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}=\dfrac{xyz}{x+y+z}$.

6. If I buy oranges at 1s. 6d. a dozen, and $3\frac{1}{2}$ times as many apples at 4d. a dozen, and after mixing them, sell them at 1s. a dozen and thereby gain 11s., how many dozen of each do I buy?

7. Sum the series

$$6 + 5\tfrac{2}{3} + 5\tfrac{1}{3} + \dots \text{ to 37 terms.}$$

8. Describe a square equal to the sum, and another equal to the difference, of two given squares.

LXXVII.

1. If 16 miles of railway are laid by 130 men in 6 days, working 10 hours a day, how many hours a day must men work when 125 men lay 40 miles in 13 days?

2. Resolve 25012 into its simplest factors.

3. If α and β are the roots of the equation

$$3x^2 + 7x + 2 = 0,$$

find the equation whose roots are

$$\frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha}.$$

4. Simplify

$$\text{(i.)}\ \frac{x^5 - x^4y - xy^4 + y^5}{x^4 - x^3y - x^2y^2 + xy^3},$$

$$\text{(ii.)}\ \frac{a^{\frac{3}{4}}b^{\frac{1}{2}}}{c^{\frac{1}{3}}} \times \frac{a^{-\frac{1}{4}}c^{-\frac{1}{3}}}{b^{-\frac{1}{3}}} \div \frac{a^{-\frac{1}{2}}b^{-\frac{1}{6}}c^{-\frac{2}{3}}}{c}.$$

5. Solve

$$\text{(i.)}\ \frac{3-x}{3+x} - \frac{2-x}{2+x} + \frac{1-x}{1+x} = 1,$$

$$\text{(ii.)}\ \begin{cases} 2x^2 - 5xy + 3y^2 = 1, \\ 3x^2 - 5xy + 2y^2 = 4, \end{cases}$$

$$\text{(iii.)}\ x^2 + \frac{1}{x^2} - \frac{25}{6}\left(x - \frac{1}{x}\right) + 2 = 0.$$

6. What is the price of eggs per dozen, when 2 less in the shilling's worth raises the price 1d. per dozen?

7. Find the limits between which x must lie,

   if $\frac{4x}{3} - 1$ is not > 11, and $\frac{3x}{2} + 3$ not < 15.

8. $ABCD$ is a quadrilateral, having BC parallel to AD: show that its area is the same as that of the parallelogram, which can be formed by drawing through the middle point of DC a straight line parallel to AB.

LXXVIII.

1. Multiply 2·1825 by ·0046, and divide the result by ·0002425.

2. Find the value of 17 cwt. 1 qr. 18 lb. at £7 11s. 8d. per cwt.

3. Find the G.C.M. of

   $a^5 - a^4 + a^3 + 2a^2 - 6a - 4$ and $a^5 - a^4 + a^3 - 2a^2 - 2a$.

4. Find the cube root of

   $$27x^6 + 135x^5 + 171x^4 - 55x^3 - 114x^2 + 60x - 8.$$

5. Simplify

   $$\sqrt[7]{\sqrt{a^{\frac{20}{3}}b^8c^4}\sqrt[3]{a^{11}b^9c}}.$$

6. Solve

   (i.) $\frac{x-a}{b-2a} + \frac{2(x-b)}{a-2b} = 1$,

   (ii.) $x - \frac{a^2 - b^2}{x} = \frac{b^2}{a}$,

   (iii.) $xyz = a^2(y+z) = b^2(z+x) = c^2(x+y)$.

7. A man sells a acres more than the m^{th} part of his estate, and there remain b acres more than the n^{th} part. Find the size of the whole.

8. If the middle points of two sides of a triangle be joined, the triangle thus formed is a quarter of the whole triangle.

LXXIX.

1. Calculate to 6 places of decimals :—

$$\frac{1}{2}+\frac{1 . 3}{2 . 4}+\frac{1 . 3 . 5}{2 . 4 . 6}+\frac{1 . 3 . 5 . 7}{2 . 4 . 6 . 8}+\frac{1 . 3 . 5 . 7 . 9}{2 . 4 . 6 . 8 . 10}.$$

2. If a model of the earth were made with a diameter of 20 yards, express as a fraction of an inch the height which would represent a mountain 25,000 feet high. (Earth's diameter $=7900$ miles.)

3. Find the continued product of

$$a+x, \quad a+\tfrac{1}{2}y, \quad \text{and} \quad a-\tfrac{1}{2}z.$$

Hence deduce the value of $(a+b)^3$.

4. Find the G.C.M. of

$$x^7-3x^6+x^5-4x^2+12x-4 \quad \text{and} \quad 2x^4-6x^3+3x^2-3x+1.$$

5. Simplify

$$\left\{\frac{x-y}{2(x+y)}-\frac{x+y}{2(x-y)}-\frac{2y^2}{x^2-y^2}\right\}\frac{y-x}{2y},$$

and express

$$(a^2+b^2)(c^2+d^2)$$

as the sum of two squares.

6. Solve

(i.) $\dfrac{4x-3}{2x-1}=\dfrac{4x-7}{2x-5}$,

(ii.) $(x-2)^2(x-1)+2(x-2)(x-1)+x-9=0$.

7. A and B can do a piece of work in 2 days, A and C in $2\frac{1}{4}$ days, B and C in $1\frac{7}{11}$ days. How long would it take each separately ?

8. If $\dfrac{a+b}{2}$, b, $\dfrac{b+c}{2}$ are in H.P., show that

(i.) $\dfrac{2}{a+b}$, $\dfrac{1}{b}$, $\dfrac{2}{b+c}$ are in A.P.,

(ii.) a, b, c are in G.P.

LXXX.

1. Supply the term omitted in the proportion—

$$(\quad) : 3344 :: 2832 : 3648.$$

2. A garrison of 1000 men was victualled for 30 days; after 10 days it was reinforced, and then the provisions were exhausted in 5 days. How many men were there in the reinforcement?

3. Find the G.C.M. and L.C.M. of

$x^2+3xy+2y^2$, $x^2+xy-2y^2$, $x^3+2x^2y-xy^2-2y^3$.

4. Find the square root of

$$\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{25}-\frac{xy}{6}+\frac{2xz}{15}-\frac{yz}{10},$$

and if $a^2d=c^2$, find the condition that

$$x^4+ax^3+bx^2+cx+d$$

may be a perfect square, in terms of a, b, and c.

5. Solve the equations :—

$$\text{(i.)}\ x(x^2-8x-65)=0,$$

$$\text{(ii.)}\ \begin{cases} 5x+3y+4z=2, \\ x+y+z=1, \\ 6x-5y-z=0. \end{cases}$$

6. Form the equations whose roots are

(i.) -3, 4,

(ii.) ± 2, ± 3,

(iii.) $a \pm b$.

7. Show that

$$(p^2-q^2-2pq)^2, \quad (p^2+q^2)^2, \quad \text{and} \quad (p^2-q^2+2pq)^2$$

are an A.P.; and find what values of p and q, will make them equal to 1, 25, 49.

8. $ABCD$ is a quadrilateral, having BC parallel to AD; E is the middle point of DC. Show that the triangle AEB is equal to half $ABCD$.

LXXXI.

1. Multiply 608·191358 by ·0251575 correct to 3 places of decimals.

2. Find the Simple Interest on £357 15s. 9d. for 3 years 5 months at 3 per cent.

3. If $a:b::3:5$; $b:c::7:9$; and $c:d::15:16$, find the ratio $a:d$.

4. Divide

$$\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}-2\right)^2 \text{ by } \frac{y}{x}-\frac{x}{y}.$$

5. Simplify

(i.) $$\left\{\frac{a}{b}-\frac{(a^2-b^2)x}{b^2}\right\} \div \left\{\frac{b}{a}-\frac{b^2-a^2}{ab+a^2x}\right\}+\frac{a(a^2-b^2)x^2}{b^2(a+bx)},$$

(ii.) $$\left\{\frac{a+b}{(a-b)^2}-\frac{a-b}{(a+b)^2}\right\} \times \left\{\frac{(a+b)^2}{a-b}+\frac{(a-b)^2}{a+b}\right\} \div \left\{\frac{1}{(a-b)^2}-\frac{1}{(a+b)^2}\right\}.$$

Test these results by making $a=2$, $b=1$.

6. Prove that

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}-\frac{1}{\sqrt{3}+\sqrt{2}-1}=\tfrac{1}{2}(2-\sqrt{2}).$$

7. If $\frac{2}{3}$ of A's money with £8 be equal to B's money, and $\frac{3}{4}$ of B's money with £18 be equal to A's money, how much has each?

8. From a given point P draw a straight line to a given straight line AB, so that it shall be bisected by another given straight line AC.

LXXXII.

1. £1640 invested from 13th January to 1st November amounts to £1705 12s. Find the rate of interest.
2. Find the Compound Interest on £3245 6s. for 3 years at 5 per cent.
3. Find the cube root of

$$8a^3 - 36a^2x + 48a^2y + 54ax^2 + 96ay^2 - 144xy^2 + 108x^2y - 144axy - 27x^3 + 64y^3.$$

4. Solve

$$\text{(i.)}\quad (kx+l)x(mx-l)=0,$$

$$\text{(ii.)}\quad \frac{7}{16}+\frac{3x}{2}=(3x-1)^2,$$

$$\text{(iii.)}\quad \begin{cases} x-y=3, \\ x^2-3xy+y^2=-1. \end{cases}$$

5. Divide

$$a_1a_2x^2+b_1b_2y^2+c_1c_2z^2+(a_1b_2+a_2b_1)xy+(a_1c_2+a_2c_1)xz + (b_1c_2+b_2c_1)yz$$

by $a_2x+b_2y+c_2z$.

6. If $s=\frac{1}{2}(a+b+c)$, show that

$$(s-a)(s-b)(s-c)=s^3-\frac{s(a^2+b^2+c^2)}{2}-abc.$$

7. If a, b, c are in G.P., prove that

$$a+b, \quad 2b, \quad b+c$$

are in H.P.

8. If from the right angle of a triangle, a straight line be drawn perpendicular to the hypotenuse, and another bisecting it, the angle between them will be equal to the difference of the two acute angles of the triangle.

LXXXIII.

1. A room, 3 times as long as broad, is carpeted at 4s. 6d. per sq. yd., and the walls coloured at $4\frac{1}{2}$d. per sq. yd., the respective costs being £8 5s. $4\frac{1}{2}$d. and 2 guineas. Find the dimensions.
2. Find within an inch the diameter of a circle whose area is 100 sq. yds. ($\pi = 3{\cdot}14159$.)
3. The specific gravities (or relative weights) of spirit and water are ·923 and 1. From a gallon jar of spirit, a certain quantity is drawn off, and then the jar filled with water. The mixture has a specific gravity of ·992. How much spirit has been taken away ?
4. Solve

$$\text{(i.)}\ 2\sqrt{x^2+x} + 2x = 1 - \sqrt{x} - \sqrt{1+x},$$

$$\text{(ii.)}\ x + 1 - \frac{x^2+3}{x+2} = 2.$$

5. What is the $(r+4)^{\text{th}}$ term of

$\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{8}$, etc., and the sum to infinity ?
6. Prove that no real value of x can make

$$3x^2 + 7x + 5 = \text{zero}.$$

(Solve as a quadratic equation.)
7. If $a+x : a-x$ is the duplicate of $a+b : a-b$, find x.
8. The sum of two numbers = 6 times their difference, and their product exceeds their sum by 23. Find them.

LXXXIV.

1. A cube contains 11 cub. ft. 675 cub. in. Find the length of its diagonal.
2. Given $\log 2 = {\cdot}30103$ and $\log 7 = {\cdot}845098$, find $\log 2{\cdot}45$ and $\log 22{\cdot}4$.

3. Divide

$$a^n - b^m \quad \text{by} \quad a^{\frac{n}{5}} - b^{\frac{m}{5}}.$$

4. Find the value of x which will make

$$x^3 + 3cx^2 + 2c^2x + 5c^3$$

a perfect cube.

Show that there can only be one such value.

5. If
$$x + \sqrt{x^2 - 1} = y,$$
find the value of x in terms of y.

6. If α, and β, are the roots of
$$ax^2 + bx + c = 0,$$
find the value of
$$\alpha^2\left(\frac{\alpha^2}{\beta} - \beta\right) + \beta^2\left(\frac{\beta^2}{\alpha} - \alpha\right).$$

7. Find the sum of all numbers from 1 to 1000, which are not divisible by 2, or 5.

8. The straight lines which bisect an angle of a quadrilateral inscribed in a circle, and the opposite exterior angle, meet on the circumference.

LXXXV.

1. Solve

(i.) $x + 3 = \dfrac{4}{\sqrt{x}}$,

(ii.) $\dfrac{1}{x} + \dfrac{1}{y} = 5, \quad \dfrac{x}{y} + \dfrac{y}{x} = 2\frac{1}{6}$,

(iii.) $4x + 5y = 39$ in positive integers.

2. If $\dfrac{A+B}{2}$, B, $\dfrac{B+C}{2}$ be in H.P., prove that
$$A, \quad B, \quad C \quad \text{are in G.P.}$$

3. If
$$a : b :: b : c,$$
then
$$b^4 = \frac{a^2 - b^2 + c^2}{a^{-2} - b^{-2} + c^{-2}}.$$

4. In what scale will 4161 be written 10101?
Find the square root of *eet*001 in the duodenary scale.

5. Sum to n terms

$$1+3+6+10+15+\ldots,$$

and write down the n^{th} term.

6. Prove that

$$a^2+b^2>2ab.$$

7. If the number of permutations of n things, two together, is thrice as great as the number of combinations of n things, three together, find n.

8. Sum to n terms the series whose n^{th} terms are

$$\text{(i.) } na+a^n,$$
$$\text{(ii.) } na^{n-1}.$$

LXXXVI.

1. Find the value of

$$\frac{1}{1\times 2}+\frac{1}{1\times 2\times 3}+\frac{1}{1\times 2\times 3\times 4}+\text{etc.}$$

correct to seven places of decimals.

2. Find the length of the edge of a cube containing 41063 cub. ft. 1080 cub. in.

3. Simplify

$$\frac{2\frac{1}{2}+1\frac{1}{4}-2\frac{1}{8}}{3\frac{3}{4}+2\frac{1}{3}-4\frac{1}{4}} \times \frac{\frac{9}{49}-\frac{4}{25}}{\frac{75}{121}-\frac{16}{27}} \div \frac{\frac{3}{7}+\frac{2}{5}}{\frac{5}{11}+\frac{4}{9}}.$$

4. After £12 has been equally divided among a number of men, an additional shilling a-piece is given them, and each has then as many shillings as there are men. Find the number.

5. Sum to 6 terms

$$\tfrac{1}{2}+1\tfrac{1}{4}+3\tfrac{1}{8}+\ldots$$

and insert 12 arithmetical means between $-\frac{1}{5}$ and 5.

6. Solve

(i.) $9x-8y=1, \quad 12x-10y=1,$

(ii.) $ax^2-\dfrac{6c^2}{a+b}=cx-bx^2,$

(iii.) $\dfrac{x^2+y^2}{17}+\dfrac{x+y}{10}=0, \quad xy=1.$

7. If $\quad 2x+3y : 2x-3y :: 2a^2+3b^2 : 2a^2-3b^2,$ then x has to y the duplicate ratio that a has to b.

8. What is meant by $\sin x$?

LXXXVII.

1. Find the difference between the Simple and Compound Interest on £378 15s. for 2 years at $3\frac{1}{3}$ per cent.

2. I gave away a dole at 3s. 6d. a week till the number of shillings I had left was the same as the number of weeks I had been giving it. I then raised the rate to 4s. a week, and the money was given away in 35 weeks. How much had I to give away?

3. Solve the equations :—

(i.) $\sqrt{x+5}+\sqrt{x}=\sqrt{6x+1},$

(ii.) $\dfrac{1}{x-a}+\dfrac{1}{y}=\dfrac{2}{b}, \quad 2x-y=2a+b.$

4. If of two numbers of two digits the sum is $\frac{11}{7}$ of the difference, and the numbers consist of the same digits reversed, find the numbers.

5. Distinguish between Ratio and Proportion.

If $a : b = c : d$, prove that

$$pa^2 + qb^2 : pa^2 - qb^2 = pc^2 + qd^2 : pc^2 - qd^2.$$

6. What is π?

What is "the angle whose circular measure is π"?

Show that π has the same meaning in both cases.

7. The area of a circle is 400 square yards. Find its diameter to the nearest inch.

8. To a given straight line apply a rectangle whose area shall be equal to the difference of the squares on its sides. (See Euc. II. 11.)

LXXXVIII.

1. Two men A and B play a match of four games, on condition that when A wins, B pays him half the money B has, and a shilling over; and when B wins, A pays him a shilling less than half the money A has. A having won the first and third, B the second and fourth, A has lost ten shillings, and B's money exceeds A's by two shillings more than half what A had at first. What did they start with?

2. Prove that if

$$ax_1^2 + 2bx_1 + c = 0, \quad \text{and} \quad ax_2^2 + 2bx_2 + c = 0,$$

then
$$ax^2 + 2bx + c = a(x - x_1)(x - x_2)$$

for all values of x.

3. If a, b, c, d be proportionals, prove that

$$a + d = b + c + \frac{(a-b)(a-c)}{a};$$

and hence if four positive quantities are proportionals, the sum of the greatest and least will be less than the sum of the other two.

4. Find the middle term of an A.P. whose first term is 1, last term 39, and the number of terms odd.

5. Write down the sum of 10 terms of a G.P. whose first term is 1, and common ratio 10.

   If $2a$, $2b$, $2c$ be in H.P., and b be subtracted from each term, the remainders will be in G.P.

6. What is the circular measure of the angle $a^\circ . 6a' . 9a''$?

7. AB, the diameter of a semicircle $APQB$, is 2 inches long; AP, AQ make angles of 60° and 30° respectively with AB. Find the lengths of BQ, BP, PQ.

8. On the side AB of a triangle ABC as diameter, a circle is described, EF is a diameter parallel to BC; show that EB, FB bisect the interior and exterior angles at B.

LXXXIX.

1. If $A \propto B$ when C is constant, and $A \propto C$ when B is constant; then $A \propto BC$ when B and C both vary.

   A beam 64 feet long, $2\frac{3}{4}$ broad, $1\frac{1}{2}$ thick, costs £11; what will be the cost of a beam 159 ft. long and $1\frac{1}{2}$ sq. ft. in section?

2. What is meant by $a : b :: c : d :: e : f$?

   If $a : b :: c : d$, may we always say that
   $$a : c :: b : d?$$
   If $a + x : a - x$ is the subduplicate of the ratio
   $$a + b : a - b,$$
   find x.

3. Multiply $ax + a^2x^2 + a^3x^3$ by $1 - ax$,

   and find the value of the result, when $x = \dfrac{2}{a}$.

4. Two men A and B own together 175 shares. They agree to divide, and A takes 85 shares, while B takes 90 shares and pays A £100. What is the value of a share?

5. A woman finding that as apples were cheaper, she could sell 60 more than she used to do for 5s., lowered her price a penny a dozen. Find their original price per dozen.

6. Solve the equation

$$\text{(i.)}\ \frac{m}{x+a}+\frac{m}{x-a}=\frac{n}{x+b}+\frac{n}{x-b},$$

and discuss the case in which $m=n$.

Solve also :—

$$\text{(ii.)}\ \frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}=x^4+x^2-1,$$

$$\text{(iii.)}\ (x^2+x+1)^{-1}=\frac{3}{2}-x-x^2.$$

7. Reduce $36^\circ\ 48'\ 54''$ to circular measure.

8. Solve the equation

$$\sin x+2\cos(90^\circ+x)=\frac{3}{2}.$$

XC.

1. Find the square root of

$$4x^4-20x^3+13x^2+30x+9.$$

2. Two rectangular fields each contain one acre. One is 4 poles shorter and 2 poles wider than the other. Find their dimensions.

3. Prove that

$$(a+b)(a+x)(b+x)-a(b+x)^2-b(a+x)^2=(a-b)^2x,$$

and divide

$$a^3-b^3 \quad \text{by} \quad a^{\frac{3}{2}}-2ab^{\frac{1}{2}}+2a^{\frac{1}{2}}b-b^{\frac{3}{2}}.$$

4. Resolve into factors :—

(i.) $63x^3y - 28xy^3$,

(ii.) $a^5 - a^4b - ab^4 + b^5$.

5. Find the G.C.M. of

$$x^4 - 6x^3 + 13x^2 - 12x + 4 \quad \text{and} \quad x^4 - 4x^3 + 8x^2 - 16x + 16,$$

and the L.C.M. of

$$x^3 - y^3,\ x^3 + y^3,\ x^3 - xy^2, \quad \text{and} \quad x^2y + xy^2 + y^3.$$

6. Find all the Trigonometrical ratios for an angle of 585°. What is the meaning of $\pi = 180°$?

7. The altitude of a tower is found to be 60° at a distance of 160 feet from its base. Find its height and the distance of the observer from the top of the tower.

8. Prove Euc. II. 1, 2, 3, 4, 5, 6, 7, 8 algebraically.

XCI.

1. Define Proportion, and show that if

$$a : b :: c : d,$$

then $ad = bc$.

Prove also that

$$a^2 + ab + b^2 : a^2 - ab + b^2 :: c^2 + cd + d^2 : c^2 - cd + d^2.$$

2. Prove the rule for finding the G.C.M. of two expressions. Find that of

$$x^4 + 67x^2 + 66 \quad \text{and} \quad x^4 + 2x^3 + 2x^2 + 2x + 1.$$

3. Find the L.C.M. of

$$x^3 - a^3, \quad x^3 + a^3, \quad x^4 + a^2x^2 + a^4, \quad x^3 - ax^2 - a^2x + a^3,$$
$$\text{and} \quad x^3 + ax^2 - a^2x - a^3.$$

4. Solve the equations :—

(i.) $\dfrac{x+5}{6} + \dfrac{1}{9}\left(\dfrac{x}{2} + \dfrac{2}{5}\right) - \dfrac{2}{3}(3 + 2x) = \dfrac{4x - 14}{3} + \dfrac{x + 10}{10}$,

(ii.) $3xy + x^2 = 10, \qquad 5y - 2x = \dfrac{2}{x}$.

5. Simplify

$$\text{(i.)}\ \frac{1}{2(x-1)} - \frac{1}{x-2} + \frac{1}{2(x-3)},$$

$$\text{(ii.)}\ \frac{\frac{x}{a}+\frac{a}{x}-2}{x-a} + \frac{\frac{x}{a}+\frac{a}{x}+2}{x+a},$$

$$\text{(iii.)}\ (x^2+x+1)(x^2-x+1)(x^4-x^2+1).$$

6. Two men start together from C to E. A stops half an hour for refreshments after 4 miles; B goes on, at a faster rate than before. When A starts again, he goes as much slower than he did, as B goes faster, and gets to E 2 hours and 6 minutes after B, and $1\frac{1}{2}$ hours later than they would have arrived had both gone on together at the first rate. B arrives 12 minutes later than he would have done, had he gone at his faster pace the whole way. Find the distance, and their rates.

7. Prove that

$$\sin(180^\circ + \theta) = -\sin\theta$$

by two methods.

8. Construct a figure to find the Trigonometrical ratios of an angle of 750°, and find them.

XCII.

1. A sells a house to B for £4860, thus losing 19 per cent.; B sells it to C at a price which would have given A 17 per cent. profit. Find B's gain.

2. A purse of sovereigns is divided among three persons, the first receiving half of them and one more, the second half the remainder and one more, and the third 6. Find the whole number of sovereigns.

3. Find the square root of

$$x^2(x-5a)(x-a)+a^2(3x-a)^2-3a^2x^2.$$

4. Solve the equations :—

$$\text{(i.)}\quad \frac{12}{x} + 6x = 17,$$

$$\text{(ii.)}\quad \sqrt{2x+7} + \sqrt{3x-18} = \sqrt{7x+1},$$

$$\text{(iii.)}\quad x^3 - y^3 = x^2 + xy + y^2 = 19.$$

5. If $a : b :: c : d$, prove that

$$b + c > 2\sqrt{ad}.$$

Also prove that the sum of the greatest and least is greater than the sum of the other two.

6. Sum the series :—

(i.) $1 + 2 + 3 + \ldots\ldots\ldots\ldots\ldots\ldots$ to n terms,

(ii.) $\dfrac{1}{\sqrt{2}+1} + \sqrt{2} + \dfrac{1}{\sqrt{2}-1} + \ldots$ to 7 terms.

7. If $\sin A = \frac{1}{2}$, and $\sin B = \frac{1}{3}$, find $\sin(A+B)$.

8. Find the values of

$$\text{(i.)}\quad \sin(270° - A),$$

$$\text{(ii.)}\quad \cos(270° + A),$$

$$\text{(iii.)}\quad \tan(360° + A).$$

XCIII.

1. Find the value of 83827 sq. miles at £11 7s. 6d. per acre.

2. Solve

$$\text{(i.)}\quad \frac{x-1}{x-2} - \frac{x-2}{x-3} = \frac{x-5}{x-6} - \frac{x-6}{x-7},$$

$$\text{(ii.)}\quad \begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = c, \\[2mm] \dfrac{1}{y} + \dfrac{1}{z} = a, \\[2mm] \dfrac{1}{z} + \dfrac{1}{x} = b, \end{cases}$$

$$\text{(iii.)}\quad x^2 + 2x + 2\sqrt{x^2 + 2x + 1} = 47.$$

3. The volume of a triangle varies as its height and base conjointly. Find the ratio of the areas of two triangles, if the base of the first is double that of the second, and the altitude of the second three times that of the first.
4. The length of a rectangular field is to its breadth as 6 : 5; $\frac{1}{6}$ is planted, and the remainder, 625 yards, ploughed; find its dimensions.
5. Divide

$$x^4+1 \quad \text{by} \quad x^2-x\sqrt{2}+1.$$

6. Simplify $$\frac{1}{\sqrt{3}+\sqrt{2}}.$$
7. If two square numbers be added together, the double of the result is also the sum of two square numbers.
8. If d, g, r, are the number of degrees, grades, and units of circular measure in an angle, respectively, prove that

$$\frac{d}{90}=\frac{g}{100}=\frac{2r}{\pi}.$$

XCIV.

1. Given $3^x=71753\cdot7$, find x, by taking logs. of both sides.
2. If $a:b::b:c::c:d$ show that

$$\frac{c}{a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}-c}=\frac{b+c+d}{a-d}.$$

3. Find the square roots of

$$\frac{x^2}{9}+\frac{y^2}{4}-\frac{xy}{3}+\frac{2xz}{15}-\frac{yz}{5}+\frac{z^2}{25}$$

and $47\pm6\sqrt{60}$.

4. Solve

(i.) $\frac{x}{a}-\frac{y}{b}=0, \quad lx+my=k^2,$

(ii.) $\sqrt{9x+1}-1=3\sqrt{-x}.$

5. A sum of £9 10s. is to be distributed among men, women, and children—100 people in all. Each man receives 12s., each woman 8s., each child 1s. How many of each are there?

6. If A be the 1st, and B the $\overline{r+1}$th, terms of an A.P., what is the nth?

7. If the number of degrees in an angle be equal to the number of grades in the complement of the angle, find its circular measure.

8. Solve the equation

$$\sin\theta + \cos\theta = \frac{1}{\sqrt{2}}$$

XCV.

1. Find that decimal of a shilling which differs from a penny by less than the millionth part of a shilling.

2. Solve the equations:—

(i.) $$\frac{x+2y+3z-10}{10} = \frac{y+2z+3x-3}{11} = \frac{z+2x+3y+1}{12} = \frac{x+y+z}{6},$$

(ii.) $$\begin{cases} \left(\dfrac{3x}{x+y}\right)^{\frac{1}{2}} + \left(\dfrac{x+y}{3x}\right)^{\frac{1}{2}} = 2, \\ xy - (x+y) = 54. \end{cases}$$

3. What quantity must be subtracted from each term of the ratio $a : b$, to make it equal to $k : l$?

4. Sum the series:—

(i.) a, $5a+4b$, $9a+8b$, etc. to 20 terms,

(ii.) 9, -6, 4, etc. to 6 terms, and to infinity,

(iii.) Insert 4 Harmonic means between $\frac{2}{11}$ and $\frac{2}{13}$.

5. Three watches are set together. The first gains 5 minutes a week, the second 8 minutes a week, and the third loses 4 minutes a week. When will they be all together again?

6. Find the circular measure of $2^\circ\ 48'\ 45''$.

7. Solve the equation

$$\sin\theta + \cos\theta = 1.$$

8. Find all the Trigonometrical ratios for an angle of 675°.

XCVI.

1. Show that Compound Interest, reckoned quarterly at £1 4s. $6\frac{1}{2}$d. per cent., is nearly equal to Interest reckoned yearly at 5 per cent.

2. Multiply

$$\left(x^{\frac{1}{2}} + y^{\frac{3}{5}}\right)^3 \quad \text{by} \quad x^{\frac{1}{2}} - y^{\frac{3}{5}};$$

and divide

$$k - l \quad \text{by} \quad \sqrt[3]{k} - \sqrt[3]{l}.$$

3. Simplify $\sqrt{4 - \frac{3}{2}\sqrt{7}}$;

and prove that if

$$\sqrt{a + \sqrt{b}} = x + \sqrt{y},$$

where a, x are rational, and $\sqrt{b}$, $\sqrt{y}$ surds, then

$$\sqrt{a - \sqrt{b}} = x - \sqrt{y},$$

4. Solve

(i.) $8x - \dfrac{12}{x} = 5$, to 2 places of decimals,

(ii.) $$\begin{cases} \dfrac{4}{(x+y)^2} + \dfrac{3}{(x-y)^2} = \dfrac{7}{x^2 - y^2}, \\ x^2 - xy + y^2 = 43, \end{cases}$$

(iii.) $x - 4 : x + 2 :: x + 2 : x + 11.$

5. A's income : B's income :: 5 : 7;
A's expenditure : B's expenditure :: 7 : 5;
B saves £300 a year, and A gets £60 into debt.
Find their incomes and expenditure.

6. Find the circular measure of $10^g\ 10'$.

7. Solve the equation
$$2 \tan \theta = \sec \theta.$$

8. Trace the changes in sign and value of
$$\cos \theta - \sin \theta,$$
as θ changes from 0 to 2π.

XCVII.

1. What is the interest on 30029 rupees 4 annas 6 pice for 1 year at $4\frac{1}{2}$ per cent.? Reduce *the result* to English money, £ s. d. and decimal of a penny. 1 rupee = 2s. $4\frac{1}{2}$d. 16 annas = 1 rupee, 12 pice = 1 anna.

2. Find the sum of all the numbers between 250 and 1000 that are divisible by 7.

3. Find if any value of x can make both the expressions
$$x^5 - x^4 - 2x^3 + 2x^2 - 3x + 3 \quad \text{and} \quad x^3 + x^2 - 3x - 3$$
equal to zero.

4. Reduce to simplest forms :—

(i.) $$\frac{1}{x-y} + \frac{1}{y-z} + \frac{1}{z-x},$$

(ii.) $$\frac{6x^2y^2}{m+n} \div \left[\frac{3(m-n)x}{7(r+s)} \div \left\{\frac{4(r-s)}{21xy^2} \div \frac{r^2-s^2}{4(m^2-n^2)}\right\}\right].$$

5. Solve the equations :—

(i.) $$\frac{x-1}{2} + \frac{4x-2}{3} = \frac{3-10x}{4},$$

(ii.) $$\frac{x}{2} + \frac{y}{3} + \frac{1}{4} = \frac{x}{4} + \frac{y}{5} + \frac{1}{6} = \frac{x}{6} + \frac{y}{7} + \frac{1}{8}.$$

6. Express $\frac{5}{8}$ of a right-angle in degrees, grades, and circular measure.

7. If $\sin(x-y)=\frac{1}{2}$ and $\cos(x+y)=\frac{1}{2}$, find x and y.

8. Write down the general value of θ when
$$\tan\theta=1.$$

XCVIII.

1. A solid metal sphere, 6 inches in diameter, is formed into a tube 10 inches in external diameter and 4 inches in length. Find the thickness of the tube.

2. Find the cube root of $6\frac{479}{729}$.

3. Simplify

(i.) $\sqrt{\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}}$ to 5 places of decimals,

(ii.) $\dfrac{1}{6y^2-5xy+x^2}-\dfrac{2}{3y^2-4xy+x^2}+\dfrac{1}{x^2-3xy+2y^2}$,

(iii.) $\dfrac{x^4-8x^2y^2+16y^4}{x^3-6x^2y+12xy^2-8y^3}$.

4. If α and β are the roots of
$$x^2+px+q=0,$$
prove that
$$\alpha^3+\beta^3=3pq-p^3;$$
and find the value of $\dfrac{1}{\alpha^3}+\dfrac{1}{\beta^3}$.

5. Solve

$$\text{(i.)}\quad \begin{cases} \dfrac{xy}{c}=x+y, \\[2mm] \dfrac{yz}{a}=y+z, \\[2mm] \dfrac{zx}{b}=z+x, \end{cases}$$

(ii.) $$\begin{cases} \left(\dfrac{x-y}{x+y}\right)^2 - \dfrac{16}{21}\left(\dfrac{x-y}{x+y}\right) + \dfrac{1}{7} = 0, \\ 5(x-y) = 2(x+y+1). \end{cases}$$

6. Find the L.C.M. of

$$2x^3(a-x)^3, \quad 8ax(ax-x^2)^2, \quad \text{and} \quad 4a^3(a^2-x^2).$$

7. If $\sin\theta = \frac{1}{2}$, and $\sin\phi = \frac{1}{3}$,

find $\sin(\theta+\phi)$, and $\sin 2(\theta+\phi)$.

8. Show that an angle whose circular measure is of the form $\dfrac{n\pi}{20}$, where n is an integer, can be expressed by an integer in both degrees and grades.

XCIX.

1. A railway goes from A to C. A goods train starts from A at 12 o'clock, and a passenger train at 1 o'clock. After going $\frac{2}{3}$ of the distance, the goods train breaks down, and can only go at $\frac{3}{4}$ its former rate. At 2.40 p.m. a collision takes place 10 miles from C. The rate of the passenger train being double the reduced rate of the goods train, find the distance from A to C, and the rates of the trains.

2. One factor of

$$18(bc^2+ca^2+ab^2) - 12(b^2c+ca^2+a^2b) - 19abc$$

is $2a-3b$, find the others.

3. Find the value of

$$(mx+n)(nx-m) - c^2,$$

when

$$x = \frac{m^2-n^2+\sqrt{(m^2+n^2)^2+4mnc^2}}{2mn}.$$

4. Find the L.C.M. of

$$a^3,\ ab^2,\ a(a-b), \quad \text{and} \quad a^2-b^2$$

and the G.C.M. of

$$3x^3-3x^2y+xy^2-y^3 \quad \text{and} \quad 4x^2y-5xy^2+y^3.$$

5. Solve the equations :—

$$\text{(i.)}\quad \frac{x-1}{x^2+x}-\frac{x+1}{x^2-x}=\frac{1}{4-2x},$$

$$\text{(ii.)}\begin{cases}(a-b)x+y=2a(a-b),\\ \quad a^2x+by=a^3-b^3.\end{cases}$$

$$\text{(iii.)}\begin{cases}x+y+z=0, \quad x+2y+3z=1,\\ \qquad x+3y+4z=2.\end{cases}$$

6. Prove that

$$\tan A+\tan B=\sin(A+B)\sec A\sec B.$$

7. In a triangle ABC,

$$A=x^{\circ},\ B=x^{g},\ C=\frac{\pi x}{9}\ \text{(circular measure)}.$$

Express all three angles in degrees.

8. If the inscribed circle touch the sides AB, AC of a triangle at D and E, and the straight line from A to the centre meet the circumference in G, then G is the centre of the circle inscribed in the triangle ADE.

C.

1. Expand the continued product
$$a^2x^2(1+a^{-1}x)(1+a^{-1}x-a^{-1})(1-x^{-1}a)(1-x^{-1}a+x^{-1}).$$

2. Resolve into factors :—

$$\text{(i.)}\quad a^{\frac{3}{2}}+ab^{\frac{1}{3}}-a^{\frac{1}{2}}b^{\frac{2}{3}}-b,$$

$$\text{(ii.)}\quad 6x^2-5x-6;$$

and find the G.C.M. and L.C.M. of

$$30x^3-45a^2x+15a^3 \quad\text{and}\quad 60x^3+40ax^2-100a^3.$$

3. Simplify

$$\text{(i.)}\quad \frac{1+\frac{2}{x}}{1-\frac{3}{x}}\times\frac{\frac{x}{2}-1}{\frac{x}{2}+1}-\frac{1-\frac{2}{x}}{1+\frac{3}{x}}\div\frac{\frac{x}{2}-1}{\frac{x}{2}+1},$$

$$\text{(ii.)}\quad \frac{(a+b\sqrt{c})^3+(a-b\sqrt{c})^3-(2b\sqrt{ac})^2}{(a+b\sqrt{c})^2+(a-b\sqrt{c})^2}.$$

4. Solve the equations :—

$$\text{(i).}\quad 3-\frac{x-1}{5}+\frac{2x-7}{15}-\frac{x+4}{12}=0,$$

$$\text{(ii.)}\begin{cases} b(a+b)x=a(a-b)y, \\ \dfrac{a-bx}{a^2}-\dfrac{b-ay}{b^2}=\dfrac{x}{a}+\dfrac{y}{b}. \end{cases}$$

5. A man bought a number of pieces of cloth for £32, and sold them at £4 10s. each, thus gaining as much as one piece cost him: how many pieces were there?

6. If $m=\dfrac{\sqrt{17}+1}{2}$, $n=\dfrac{\sqrt{17}-1}{2}$,

find the value of m^2+n^2.

7. Find the size in degrees of the angle which is subtended at the centre of a circle of 3 feet radius, by an arc 10 feet long.

8. Prove that

$$(1-\tan\theta)(\cot\theta+1)=\sec\theta\operatorname{cosec}\theta\sqrt{1-4\sin^2\theta\cos^2\theta}.$$

CI.

1. Multiply

$$x^2+x(a^{\frac{1}{2}}-b^{\frac{1}{2}})-a^{\frac{1}{2}}b^{\frac{1}{2}} \quad\text{by}\quad x^2-x(a^{\frac{1}{2}}-b^{\frac{1}{2}})-a^{\frac{1}{2}}b^{\frac{1}{2}}$$

and divide

$$\frac{x^3}{27}+\frac{a^3}{2\sqrt{2}} \quad\text{by}\quad \frac{x}{3}+\frac{a}{\sqrt{2}}.$$

2. Find the G.C.M. of

$$8x^3-8x^2-4x-3$$

$$\text{and}\quad 2x^4+3x^3-3x^2-7x-3,$$

and L.C.M. of

$$x^2+4x+4,\quad x^2-4,\quad (x-2)^3.$$

3. Solve

$$\text{(i.)}\quad \tfrac{1}{2}\left(x-\frac{a}{5}\right)-\tfrac{1}{3}\left(x-\frac{a}{4}\right)=\tfrac{1}{7}(x+a),$$

(ii.) $\dfrac{x-5y}{4}+\dfrac{2x-y}{7}=2y+10, \quad \dfrac{4x-y}{9}=\frac{1}{2}(x-3y),$

(iii.) $7x^2-48x-7=0,$

(iv.) $x+y-z=5=y+z-x=z+x-y.$

4. If A, G, H are respectively the Arithmetic, Geometric, and Harmonic Means between a and b, prove that

$$G^2=AH.$$

5. Prove that $$\cot\frac{A}{2}=\frac{\sin A}{1-\cos A}.$$

6. Find the length of the circumference of the circle inscribed in a square whose area is 5499025 square feet. $(\pi=\frac{22}{7}.)$

7. Find $\tan 1125°$.

8. Any equilateral figure inscribed in a circle is equiangular.

CII.

1. Multiply

$$a+b+c^{\frac{1}{2}}-2b^{\frac{1}{2}}c^{\frac{1}{4}}+2c^{\frac{1}{4}}a^{\frac{1}{2}}-2a^{\frac{1}{2}}b^{\frac{1}{2}}$$

by $$a+b+c^{\frac{1}{2}}+2b^{\frac{1}{2}}c^{\frac{1}{4}}-2c^{\frac{1}{4}}a^{\frac{1}{2}}-2a^{\frac{1}{2}}b^{\frac{1}{2}}.$$

2. Prove that

$$a(b+c)(b^2+c^2-a^2)+b(c+a)(c^2+a^2-b^2)$$
$$+c(a+b)(a^2+b^2-c^2)\equiv 2abc(a+b+c).$$

3. Simplify

(i.) $\dfrac{a^3-a^2b-ab^2+b^3}{a^4-2a^3b+2ab^3-b^4},$

(ii.) $\left\{\dfrac{x+\sqrt{y}}{x-\sqrt{y}}-\dfrac{x^2+y-x\sqrt{y}}{x^2+y+x\sqrt{y}}\right\}\left\{\dfrac{x-\sqrt{y}}{x+\sqrt{y}}-\dfrac{x^2+y+x\sqrt{y}}{x^2+y-x\sqrt{y}}\right\}.$

4. Solve the equations :—

(i.) $\sqrt{x(3-x)}=\sqrt{x+1}+\sqrt{2(x-1)},$

(ii.) $3x^4-7x^3+4x^2+x-1=0,$

of which 1 is a root.

(iii.) $\dfrac{x^2}{y}+\dfrac{y^2}{x}=4\frac{1}{2}, \quad x+y=3.$

5. Three towns A, B, C, are connected by railways; from A to B is 4 miles further than from A to C, but a train which leaves A for B arrives at B 8 minutes before another train which left A at the same time arrives at C. When the first train has run 51 miles, the second has run $\frac{5}{9}$ of the distance from A to C; and if the first train had passed through B without stopping, it would have gone $\frac{2}{25}$ AB beyond B, when the second train reached C. Find the distances from A to B and C, and the rates of the trains.

6. Prove that a ratio of greater inequality is diminished, and a ratio of less inequality increased, by adding the same quantity to both terms.

   A number is added to each term of the ratio $5:9$, and also subtracted from each. The first ratio is double the second; find the number.

7. Prove that

$$\text{vers}\,(A+B)\,\text{vers}\,(A-B) = (\cos A - \cos B)^2.$$

8. Find the sine and cosine of an angle in terms of the cotangent.

   If $2\sec\theta = \tan\alpha + \cot\alpha$, find $\tan\theta$.

CIII.

1. The sums of the alternate terms of an A.P. are 24 and 30 respectively, and the last term exceeds the first by $10\frac{1}{2}$. Find the number $(2n)$ of terms, and the common difference.

2. Show that in the quadratic equation

$$x^2 - px + q = 0$$

   p is the sum, and q the product of the roots.

   Given that 4 is a root of the quadratic

$$x^2 - 5x + q = 0,$$

   find the value of q, and the other root.

3. A and B start at the same time from opposite ends of a road 28 miles long; had A not been delayed an hour at P, 9 miles on the road, he would have met B half way; on leaving P he goes a mile an hour faster, and meets B 4 miles from P. Find his original rate.

4. If $a : b :: c : d$, prove that

$$a+b : c+d :: a-b : c-d;$$

and if $a+b : a-b :: c : d$,
then
$$(a+b)(c+d) = 2ac.$$

5. If
$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \ldots,$$
each of these fractions is equal to
$$\frac{a_1 + a_2 + a_3 + \text{etc.}}{b_1 + b_2 + b_3 + \text{etc.}}$$

6. Express the angle of a regular hexagon in the three systems.

7. Trace the changes of $\cot A$, as A changes from 90° to 270°.

8. Prove that

(i.) $\sec^2 A - \sec^2 B = \tan^2 A - \tan^2 B$.

(ii.) $\sin(A+B)\sin(A-B) = \cos^2 B - \cos^2 A$.

CIV.

1. Simplify

(i.) $\dfrac{y}{2(x-y)} - \dfrac{y}{2(x+y)} - \dfrac{y^2}{x^2} - \dfrac{y^4}{x^2(x^2-y^2)}$,

(ii.) $(a+b+c)(a^{-1}+b^{-1}+c^{-1}) - a^{-1}b^{-1}c^{-1}(b+c)(c+a)(a+b)$.

2. Extract the square roots of

$$a^2x^{-2} + 2ax^{-1} + 3 + 2a^{-1}x + a^{-2}x^2 \quad \text{and} \quad 11 - 4\sqrt{6}.$$

3. Find the G.C.M. of

$$2x^4 - 2x^3 - 3x^2 + 5x - 5 \quad \text{and} \quad x^3 + 2x^2 - 2x + 3,$$

and the L.C.M. of

$$x^2 + x - 6, \quad x^4 - 3x^3 + 3x^2 - 3x + 2, \quad x^3 + x.$$

4. Solve

(i.) $\frac{a-b}{2a}x - \frac{2a}{b-a} = \frac{a+b}{b}x + \frac{b}{a+b}$,

(ii.) $7x - 11y - 3 = 0, \quad 5y - 6x + 7 = 0$,

(iii.) $\frac{3x-1}{7-x} - \frac{5-4x}{2x+1} = 3$,

(iv.) $x^2 + 3xy - y^2 = 3, \quad 3x - y = 1$.

5. The sum of an A.P. to n terms is $8\frac{1}{7}$; the first term is $3\frac{5}{7}$, and the difference is -1; find n.

6. Express in English measure $25^g\, 4^{``}$; and $\left(\frac{\pi}{12}\right)^r$.

7. If $\tan^2\theta = \frac{5}{4}$, find versin θ.

8. Prove that

$$\cos^2\alpha + \cos^2\left(\alpha + \frac{2\pi}{3}\right) + \cos^2\left(\alpha - \frac{2\pi}{3}\right) = \frac{3}{2}.$$

CV.

1. Given $\log_{10} 20 = 1{\cdot}3010300$, find $\log_{10}{\cdot}000125$. Find also $\log_3 243$.

2. Solve

(i.) $3x^2 - 4x - 4 = 0$,

(ii.) $x + y = \frac{1}{2}, \quad y - z = \frac{1}{3}, \quad z - x = \frac{1}{4}$,

(iii.) $11x + 7y = 90$, in positive integers.

3. Find an expression for the sum of a Geometric Series.

Sum

(i.) $\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \ldots$ to 6 terms,

(ii.) $\frac{3}{4} + \frac{2}{3} + \frac{7}{12} + \ldots\ldots$ to 19 terms.

4. Divide

$$\frac{x^4}{81} - \frac{ax^3}{27\sqrt{2}} + \frac{a^3x}{6\sqrt{2}} - \frac{a^4}{4} \text{ by } \frac{x^2}{9} - \frac{a^2}{2}.$$

5. Simplify

$$\frac{\sqrt{a+x}-\sqrt{a-x}}{a+x+\sqrt{a^2-x^2}}\times\frac{\sqrt{a^2-x^2}}{\sqrt{a}+\sqrt{x}}\times\frac{x}{a-\sqrt{a^2-x^2}}.$$

6. Express in degrees, grades, and circular measure the angle between the hands of a clock at 6.45.

7. Given

$$\cos(x+y)=\tfrac{1}{2}, \quad \text{and} \quad \cot(x-y)=1,$$

find the values of x and y, between $0°$ and $90°$.

8. Prove that

$$\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A.$$

CVI.

1. Simplify

$$\text{(i.)}\ \tfrac{1}{3}(x-3y)+\frac{2x-y}{2}-\tfrac{1}{12}(7x-9y),$$

$$\text{(ii.)}\ \frac{\dfrac{1}{x}-\dfrac{x+a}{x^2+a^2}}{\dfrac{1}{a}-\dfrac{a+x}{a^2+x^2}}+\frac{\dfrac{1}{x}-\dfrac{x-a}{x^2+a^2}}{\dfrac{1}{a}-\dfrac{a-x}{a^2+x^2}},$$

and show that the difference between

$$\frac{x}{x-a}+\frac{x}{x-b}+\frac{x}{x-c} \quad \text{and} \quad \frac{a}{x-a}+\frac{b}{x-b}+\frac{c}{x-c}$$

is the same whatever value be given to x.

2. Solve the equations :—

$$\text{(i.)}\begin{cases} 8x-7y=12, \\ \dfrac{x-2y}{4}+\dfrac{2x-y}{3}=1, \end{cases}$$

$$\text{(ii.)}\ x+\frac{ay}{a-b}=b=\frac{ax}{a+b}+y.$$

3. The price of a diamond varies as the square of its weight. Three rings of equal weight, composed each of a diamond set in gold, have values £a, £b, £c,

respectively, the diamonds in them weighing 3, 4, and 5 carats respectively. The cost of workmanship is the same for each ring. Show that a diamond of 1 carat is worth £$\left(\frac{a+c}{2} - b\right)$.

4. After walking at $3\frac{1}{2}$ miles an hour till the number of miles left of my journey was the same as the number of hours I had been walking, I quickened my pace to 4 miles an hour, and accomplished the distance in 2 hrs. 55 mins. Find the whole distance.
5. Find an expression for all angles with a given sine.
6. Solve
$$\sin x + \cos x = \sqrt{2} \cos A.$$
7. Given $\log 2 = \cdot 3010300$, $\log 3 = \cdot 4771213$, $\log 13 = 1 \cdot 1139434$; find $\log \cdot 78$.
8. Describe a circle about a given rectangle.

CVII.

1. Simplify
$$\text{(i.)}\ \frac{x+2a}{2b-x} + \frac{x-2a}{2b+x} - \frac{4ab}{4b^2-x^2},$$
$$\text{when} \quad x = \frac{ab}{a+b},$$
$$\text{(ii.)}\ \frac{13\sqrt{15} - 7\sqrt{21}}{13\sqrt{1\frac{2}{3}} - \sqrt{114\frac{1}{3}}}.$$
2. Find the cube root of $571 \cdot 787$,
and of
$$1728x^6 + 1728x^4y^3 + 576x^2y^6 + 64y^9.$$
3. Solve the equations :—
$$\text{(i.)}\ (x-a)^2 - 4a(x+2a) = 0,$$
$$\text{(ii.)}\ \frac{1}{x+\sqrt{2-x^2}} + \frac{1}{x-\sqrt{2-x^2}} = x.$$

4. A bag contains some sovereigns, three times as many half sovereigns, and some florins and half crowns. The value of the gold coins is double that of the silver, and the half crowns are worth half as much again as the florins. Compare the ratios of the numbers of the coins.

5. Show that in any triangle

$$\text{(i.)}\quad a = b\cos C + c\cos B,$$

$$\text{(ii.)}\quad \frac{\sin A}{\sin B} = \frac{\cos A\cos C + \cos B}{\cos B\cos C + \cos A}.$$

6. Prove that the angle subtended at the centre of a circle by an arc of constant length, varies inversely as the radius.

7. Show that

$$\cos(90° + A) = -\sin A,$$

and *hence* that

$$\sin(90° + A) = -\cos(180° + A).$$

8. AB is a diameter of a circle, CD a chord perpendicular to it, and E a point in CD. AE and BE produced cut the circle in F and G: show that $CF : FD :: CG : GD$.

CVIII.

1. Solve the equations :—

$$\text{(i.)}\quad 2x^2 + \tfrac{3}{4}x = \tfrac{5}{16},$$

$$\text{(ii.)}\quad x + \frac{2}{1 + \dfrac{1}{x}} = 3,$$

$$\text{(iii.)}\quad \left(\frac{1}{\sqrt{x}} + 5\right)^2 - 6\left(\frac{1}{\sqrt{x}} + 5\right) = 16.$$

2. A sum of £119 is divided among 10 men, 32 women, and 48 children; each man has as much as 2 women, and the 32 women get twice as much as the 48 children. Find what each receives.

3. The illumination from a source of light varies inversely as the square of the distance. How much farther away must a speck, now 8 inches from a candle, be placed to receive half as much light?

4. Find the sum of 17 terms, and the last term of an A.P. whose first term is 49, and second 44.

   The sum of 15 terms of an A.P. is 600, and the common difference 5, find the first term.

5. Prove that

   $$(\tan 4A + \tan 2A)(1 - \tan^2 3A \tan^2 A) = 2\tan 3A \sec^2 A.$$

6. Define the *tangent* and *secant* of an angle and obtain a formula connecting them.

   Deduce the corresponding formula for cosecant and cotangent.

7. Show geometrically that

   (i.) $\sin A = \sin(180° - A)$,

   (ii.) $\cos(180° + A) = \cos(180° - A)$.

8. In the figure of Euclid IV. 10, show that the angle ACD is three times the angle at the vertex of the triangle.

CIX.

1. Find the G.C.M. of

   $$y^3 - 2y^2 + 3y - 6 \quad \text{and} \quad y^4 - y^3 - y^2 - 2y.$$

2. A can do a piece of work in 1 hour, B and C each in 2 hours. How long would they all take together?

3. Solve the equations :—

   (i.) $x^2 - 5x + 4 = 0$,

   (ii.) $(x-5)(x-3) + (x-4)^2 = x + 1$,

   (iii.) $x + 3 + \sqrt{x+3} = 6$.

4. Find the sum of a series in A.P.

   Sum the series :—

   (i.) $1 + 3 + 5 + \ldots\ldots$ to n terms,

(ii.) $1 + \frac{3}{5} + \frac{1}{5} + \ldots \ldots$ to n terms,

(iii.) $7 + 5 + 3\frac{4}{7} + \ldots\ldots$ to n terms.

How many terms of (ii.) would amount to 0?

5. Show that

$$\sin 4A = 4\cos A(\sin A - 2\sin^3 A).$$

6. Express all the trigonometrical ratios in terms of the cosecant.

Find all the ratios of the angle whose secant is $\frac{37}{35}$.

7. Find $\operatorname{cosec} 1380°$, $\cot 1000^g$, and $\tan\dfrac{13\pi}{12}$.

8. C is the centre of a given circle, CA a straight line less than the radius; describe a circle to touch the given circle at P, and pass through A and C; and prove that P is the point on the circumference of the given circle at which CA subtends the maximum angle.

CX.

1. What is the weight of a hollow sphere of metal whose inside diameter is 18 inches, and thickness 2 inches, if 1 cub. ft. of the metal weighs 7776 oz.?

2. Sum $\sqrt{5} + 1 + \dfrac{1}{\sqrt{5}} + \ldots\ldots$ to 5 terms.

Find the n^{th} term, and sum to n terms of

$$1 . 9 + 9 . 17 + 17 . 25 + \ldots\ldots$$

3. Solve the equations :—

(i.) $x^2 + \dfrac{1}{x^2} + x + \dfrac{1}{x} = 4,$

(ii.) $\begin{cases} x^2 - xy = 3, \\ x^2 + y^2 = 13. \end{cases}$

4. Prove that a, b, c are in A.P., G.P., or H.P. according as

$$\frac{a-b}{b-c} = \frac{a}{a}, \quad \text{or} \quad = \frac{a}{b}, \quad \text{or} \quad = \frac{a}{c}.$$

5. Show that

   the number of combinations of n things, r together
   $=$ number of combinations of n things, $n-r$ together.

   How many words can be made of the letters of the word "rotation," all together?

6. Find the values of $(-1)^{\frac{1}{3}}$.

7. One solution of the equation

$$\sin x \cos^2 x = \cos \alpha \sin^2 \alpha$$

   is $\sin x = \cos \alpha$; find the others.

8. If

$$\sin \alpha = \frac{9}{41}, \quad \text{and} \quad \cos \beta = \frac{11}{61},$$

   find $\sin^2 \dfrac{\alpha - \beta}{2}$.

CXI.

1. If 4 per cent. be lost by selling silk at 10s. per yard, at what price should it be sold to gain 5 per cent.?

2. If

$$4x^4 + 12x^3y + px^2y^2 + 6xy^3 + y^4$$

   is a perfect square, find p.

3. The product of the sum and difference of a number and its reciprocal is $3\frac{3}{4}$; find the number.

4. The number of sergeants in a company is twice the number of officers, and equal to the number of companies in a battalion. There is an officer or sergeant to every 10 men, and the whole battalion (including 6 on the staff) numbers 600. How many companies are there?

5. Define *ratio*.

   What is meant by *duplicate* ratio, and *compound* ratio?

   Which is the greater

$$2:3 \quad \text{or} \quad 2r^2 + 2r + 1 : 3r^2 + 3r + 1?$$

6. Prove that

(i.) $\sin 45^\circ = \cdot 7071 \ldots\ldots$, (ii.) (if $x = c \tan\theta$)

$$(x^2 + c^2)^2 \sin 4\theta = 4\,cx(c^2 - x^2).$$

7. At 300 feet from the base of a tower, the elevation of the top is 15° 30′. What is its height?

(Use a book of Mathematical Tables.)

8. Prove that

(i.) $\log 6 = \log 2 + \log 3$,

(ii). $\log\left(\frac{m}{n}\right)^r = r \log m - r \log n$.

CXII.

1. What is a Simple Equation?

Prove that a simple equation cannot have two roots.

Solve

(i.) $a\frac{x-b}{a-b} + b\frac{x-a}{b-a} = 1$.

(ii.) $\frac{9}{x-4} + \frac{3}{x-8} = \frac{4}{x-9} + \frac{8}{x-3}$.

2. When is one quantity said to vary as another?

Two men start together for a race. Each runs uniformly, but their rates are different. Prove that the space between them at any instant varies as the time from the start.

3. Sum to infinity the G.P. whose first term and common ratio are the Arithmetic and Harmonic means between a and a^{-1}.

4. What is meant by a coefficient?

Find the coefficient of x in dividing

$$8x^4 + xy^3 - y^4 \quad \text{by} \quad x - \tfrac{1}{2}y.$$

5. Show that

(i.) $\sin 5A - 5\sin 3A = 16\sin^5 A - 10\sin A$,

(ii.) $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$.

6. Find the value of

$$a \cos 2\theta + b \sin 2\theta,$$

when $\tan \theta = \dfrac{b}{a}$.

7. Write down the sine, cosine, and tangent of 60°; and hence deduce $\cos 120^\circ$, $\sin 150^\circ$, $\tan 240^\circ$.

8. A circle is inscribed in a triangle ABC, and a triangle cut off at each angle by a tangent to the circle. Show that the perimeters of these three triangles are together equal to the perimeter of ABC.

CXIII.

1. If $\dfrac{x+y}{z} = \dfrac{y+z}{x} = \dfrac{z+x}{y}$, then each $= 2$, and also

$$3xyz = x^3 + y^3 + z^3.$$

2. Solve the equations :—

$$\text{(i.)} \quad \begin{cases} \dfrac{1}{x} = \dfrac{1}{a} + \dfrac{b}{ay(a+b)}, \\[2mm] \dfrac{1}{y} = \dfrac{1}{b} + \dfrac{a}{bx(a+b)}, \end{cases}$$

$$\text{(ii.)} \quad x + y = z^2, \quad (x-z)^2 = y, \quad x + z = 2y.$$

3. Eliminate x and y from

$$ax = by^2, \quad by = ax^2, \quad \text{and} \quad x^2 + y^2 = 1.$$

4. The road from A to B forms the arc of a circle whose length is 10 miles. If this arc subtend at the centre an angle of 30°, find the distance between the two places "as the crow flies."

5. In a catch of fish consisting of eels and pike, it was found that had there been one eel more and one pike less, the numbers would have been equal. The average

weight was $3\frac{3}{4}$ lb., the pike average being $\frac{3}{5}$ lb. more than the eel average. The pike weighed 28 lb. Find the number of each.

6. Simplify

$$\text{(i.)}\ \frac{\cos\theta\,\text{cosec}\,\theta - \sin\theta\sec\theta}{\cos\theta + \sin\theta},$$

$$\text{(ii.)}\ \frac{1+\tan^2\dfrac{A}{2}}{1-\tan^2\dfrac{A}{2}}.$$

7. In any triangle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \text{diameter of circumscribed circle.}$$

8. If two opposite angles of a parallelogram are equal to two right angles, a circle can be described to pass through the angular points.

CXIV.

1. Prove that the expression

$$\frac{n^2-n+1}{n^2+n+1}$$

lies between 3 and $\frac{1}{3}$.

2. A debt which might have been paid exactly with $5x$ half-sovereigns, and x half-crowns, was paid out of a £10 note, and the change was $15x$ half-crowns, and x half-sovereigns. Find x and the amount of the debt.

3. The roots of

$$x^3 - 15x^2 + 71x - 105 = 0$$

are in A.P. Find them.

4. Prove that

$$\text{(i.)}\ \cos 2A = \frac{1-(\tan A)^2}{1+(\tan A)^2},$$

$$\text{(ii.)}\ \tan 75^\circ + \cot 75^\circ = 4.$$

5. Find the least positive value of A which satisfies the equation

$$2\sqrt{3}\cos^2 A = \sin A,$$

and write down the general value of θ if $\sec^2\theta = \frac{4}{3}$.

6. Given $\log 2 = \cdot 3010300$, find $\log 125$, and $\log \cdot 0005$, and $\log \sin 30°$.

7. If $\pi = 3\cdot 1416$, find the perimeter and radius of a circle which contains $5\cdot 309304$ square feet.

8. Describe an isosceles right-angled triangle equal to a given rectilineal figure.

CXV.

1. One company pays $5\frac{1}{2}$ per cent. on shares of £100; another pays $3\frac{1}{2}$ per cent. on shares of £10. If the prices be £115 10s. and £7 15s. respectively, compare the rates of interest on an investment.

2. Find the rationalizing factor of $3^{\frac{1}{2}} - 5^{\frac{2}{3}}$.

3. Prove that

$$\frac{1+x^3}{1-\dfrac{x}{1+\dfrac{x}{1-x}}} - \frac{1-x^3}{1+\dfrac{x}{1-\dfrac{x}{1+x}}} = 2x.$$

4. Solve

$$\text{(i.)}\quad \frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}} - \frac{7+3\sqrt{5}}{2} = 0.$$

and (ii.) eliminate x, y, z from the equations,

$$y^2+z^2 = 2ayz, \quad z^2+x^2 = 2bzx, \quad x^2+y^2 = 2cxy, \quad xyz = 1.$$

5. If a, b, c are in H.P., show that

$$a+c-2b : a-c :: a-c : a+c.$$

6. Solve the equation

$$\sin\phi \sin 3\phi = \tfrac{1}{2}.$$

7. Find approximately the height of an object which subtends an angle of 1 second, at a distance of 2 miles.
8. If two circles cut, the tangents drawn to them both from any point in their common chord produced are equal.

CXVI.

1. On July 26th the sun rises at 4.15 a.m., and sets at 7.54 p.m.: when is it due south?
2. If $a : b :: c : d$, prove that
$$\frac{(a^2-b^2)^{\frac{1}{2}}+(a^2+b^2)^{\frac{1}{2}}}{(c^2-d^2)^{\frac{1}{2}}+(c^2+d^2)^{\frac{1}{2}}}=\frac{b}{d}.$$
If $15 : x :: 25 : y :: 35 : 42$, find x and y.
3. Find the value in £ s. d. of
(i.) $\frac{1}{1000}$d. + $\frac{1}{100}$d. + $\frac{1}{10}$d. + ... to 8 terms,
and of (ii.) $\frac{1}{10}$d. + $\frac{1}{100}$d. + $\frac{1}{1000}$d. + ... to infinity.
4. Find the Arithmetic, Geometric, and Harmonic means between 1 and 2; and continue the series which *the means* form, to two more terms.
5. Divide
$$(a^2-4b^2)x^3+2(a^3-ab^2)x^2+(2a^4+2a^3b-2a^2b^2-ab^3-b^4)x$$
$$+a^5-a^3b^2-a^2b^3+b^5$$
by $(a+2b)x+(a^2-b^2)$, without removing the brackets.
6. If an angle of 15° be represented by ·2, how many degrees are contained in the unit of that measure?
What number will represent a right angle in the same measure?
7. Solve the equation
$$\cot\theta-ab\tan\theta=a-b.$$
8. Prove that
$$\sec^2\alpha\operatorname{cosec}^2\alpha-2=\tan^2\alpha+\cot^2\alpha.$$

CXVII.

1. A druggist buys a certain article by Avoirdupois weight, and sells it by Troy weight. The buying price is 3d. per oz., and selling price 6d. per oz. Find his gain per cent.
2. Find the square root of
$$16x^4 - 56x^3 + 73x^2 - 42x + 9,$$
and the fourth root of
$$24\sqrt{-1} - 7.$$
3. Simplify
(i.) $(a+1)(a^2+a+1)^{-1} + (a-1)(a^2-a+1)^{-1} + 2(a^4+a^2+1)^{-1}$,
(ii.) $\left\{\dfrac{a-b}{1+ab} + \dfrac{b-c}{1+bc}\right\} \div \left\{1 - \dfrac{(a-b)(b-c)}{(1+ab)(1+bc)}\right\}$.
4. Solve the equations :—
(i.) $x - \dfrac{1}{3}\left\{x - \dfrac{1}{4}\left(x - \dfrac{x - \dfrac{x}{6}}{5}\right)\right\} = 106.$
(ii.) $\begin{cases} 4x^2 + xy = 7, \\ 3xy + y^2 = 18. \end{cases}$
5. If $x : y$ be in the duplicate ratio of $x+z : y+z$, prove that z is the Geometric Mean between x and y.
6. The sum of two angles is 1^g; the circular measure of their difference is 1 : find the circular measure of each.
7. Solve the equation
$$\tan^2\theta + \cot^2\theta = 2.$$
8. Prove that if
$$x\cos\theta + y\sin\theta = c, \quad \text{and} \quad x\sin\theta - y\cos\theta = d,$$
then $$x^2 + y^2 = c^2 + d^2.$$

CXVIII.

1. Find the value of 6 tons 7 cwt. 2 qr. 17 lb. at £3 10s. 7d. per cwt.
2. How many terms of the series 17, 15, 13, &c., amount to 56? Explain the double answer.
3. A working 6 hours a day, and B working 7 hours a day, together do a piece of work in a certain time; if A worked 7 hours, and B 10 hours a day, they would do it in 2 days less; A is twice as rapid a worker as B; how many days does the work take?
4. If $s=\frac{a+b+c}{2}$, prove that
$$4s(s-a)(s-b)(s-c)=a^2b^2-\tfrac{1}{4}(a^2+b^2-c^2)^2.$$
5. Find the square root of
$$\frac{x}{y}+\frac{y}{x}+3-2\sqrt{\frac{x}{y}}-2\sqrt{\frac{y}{x}},$$
and simplify
$$\frac{z\sqrt{xy}-xz}{yz-z\sqrt{xy}}.$$
6. The number of grades in one of the angles of an acute-angled triangle is $\frac{10}{27}$ the number of degrees in the other; find the angles in degrees.
7. Solve the equation
$$\cos x+\cos^2 x=1.$$
8. A and B are two fixed points, and CD a straight line. Find a point P in CD, such that the sum of the squares on PA, PB, is the least possible.

CXIX.

1. By selling tea at 5s. 4d. a man clears ·125 of his outlay. What does he gain per cent. if he sells at 6s. 2d.?

2. Solve

$$\text{(i.)}\ x(5x^2+9x-9)=x^3,$$
$$\text{(ii.)}\ x^4-7x^2+10x-4=0,$$
$$\text{(iii.)}\begin{cases}\dfrac{x}{b+y}=\dfrac{y}{a+x},\\ ax+by=(x+y)^2.\end{cases}$$

3. Sum

(i.) $2\cdot5+\cdot5+\cdot1+\ldots$ to infinity,

(ii.) $(x+a)+(x^2+2a)+(x^3+3a)+\ldots$ to n terms.

4. Find the infinite Geometric Series whose sum is $\frac{8}{21}$, and second term is $-\frac{1}{2}$.

5. If a straight line is divided into two parts, so that the square on one part is equal to $\frac{1}{6}$ of the rectangle contained by the whole and the other part, find the ratio of the parts.

6. Find all the Trigonometrical ratios for an angle of 585°.

7. Solve the equation

$$\sin^3\alpha+\cos^3\alpha=0.$$

8. A and B are two points on a circle ADB, whose centre is C. A circle is described through A, C, B, and a straight line APQ drawn cutting this circle at P and the former circle at Q. Prove that PB is equal to PQ.

CXX.

1. Transform 1322634 from the septenary to the denary scale; and 124·96 from the denary to the quinary.

2. If $$a+b:b+c::c+d:d+a,$$
prove that either $$a=c,$$
or $$a+b+c+d=0.$$

If x is to y in the subduplicate ratio of y to z, and x, y, z are in A.P., find the equation connecting x and z.

3. The roots of the equation

$$4x^3 - 41x^2 + 120x - 108 = 0$$

are of the form $a, \quad a+b, \quad a+\frac{1}{b}$.

Find them.

4. Prove that

(i.) $\sin^{-1}\frac{2}{2\cdot5} + \sin^{-1}\frac{1\cdot5}{2\cdot5} = 90°$,

(ii.) (if $A+B+C=180°$),

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}.$$

5. Prove that the ratio of the circumference of a circle to its diameter is constant.

Find the measure in degrees of the angle whose circular measure is $\frac{3\cdot1416}{20}$.

6. Given $\log 2 = \cdot3010300$, $\log 3 = \cdot4771213$, $\log 7 = \cdot8450980$, find the logarithms of 28000, $\cdot000036$, and $\cdot0\dot{2}8571\dot{4}$.

7. A solid sphere fits closely into the inside of a closed cylindrical box, whose height is equal to the diameter of the cylinder. Given the radius of the sphere, write down the values of the volume of the sphere, surface of the sphere, and the empty space between the sphere and the cylinder.

8. Produce a straight line AB to C, so that the rectangle $AC \cdot CB$ may be equal to the square on AB.

CXXI.

1. In Fahrenheit's thermometer, freezing point is 32°, and boiling point 212°; and Centigrade freezing point is 0°, and boiling point 100°. What does the Centigrade thermometer show when the Fahrenheit is at 65°?

2. Prove that

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}-\frac{1}{\sqrt{3}+\sqrt{2}-1}=\tfrac{1}{2}(2-\sqrt{2}).$$

3. Write down the first four terms, and the sum to $2n$ terms, of the series whose n^{th} term is

$$3-(-1)^n n.$$

4. What number of terms of the series

$$6+9+12+\ldots\ldots$$

will amount to 105?

5. The measures of an angle in 2 systems are as 5 : 2. The sum of the units is 70° ; find them.

6. Write down the general value of θ when

$$\cos\theta=-1.$$

7. Write down all the values of $\operatorname{versin}\dfrac{n\pi}{4}$ where n is any integer.

8. Divide a straight line into five equal parts.

CXXII.

1. Find the weight of a spherical iron bombshell, whose interior and exterior diameters are 8 and 10 inches respectively. (1 cub. ft. of water weighs 1000 oz. ; Sp. Gr. of iron $=7\cdot21$; $\pi=3\frac{1}{7}$.)

2. The Series $3+7+15+35+\ldots\ldots$ is made up of the sum of an Arithmetic and Geometric Series. Find them, and sum the series to n terms.

3. Prove that x^7-a^7
is divisible by $x^2+azx+a^2$,
if $z^3-z^2-2z+1=0$.

4. Find the G.C.M. of

$$(ab-xy)^2-2lc(ab-xy)-(k^2-l^2)c^2$$

and $(ab-xy)(k-l)c-c(k+l)(kc-cl+ab-xy)+(k+l)^2c^2$.

5. Find the value of

$$\cfrac{1}{\sqrt{7}+\cfrac{1}{\sqrt{7}+\cfrac{1}{\sqrt{7}+\cfrac{1}{\sqrt{7}}}}}$$

to 5 places of decimals.

6. Simplify

$$\cos 2\,\theta \cos 2\phi + \sin^2(\theta-\phi) - \sin^2(\theta+\phi).$$

7. One angle of a quadrilateral contains 45°, another 150^g, and the circular measure of a third is $\frac{\pi}{3}$. Express all four in degrees.

8. Explain *exactly* what is meant by $\cot\theta$.

CXXIII.

1. A gallon holds 10 lb. Avoir. of water. 1 cub. ft. of water weighs 997·13 oz. 1 litre = 1 cubic decimètre. 1 decimètre = 3·937 inches. Find to 2 places of decimals how many litres there are in a gallon.

2. Solve

(i.) $\dfrac{x+2}{x-1}-\dfrac{x-1}{x+2}=\dfrac{7}{12}$,

(ii.) $(x^2-3x+2)^2+2(x^2-3x)^2+(x^2-3x-2)^2=24$,

(iii.) $\begin{cases} zx-xy=84, \\ yz-xy=20, \\ x+y+z=0. \end{cases}$

3. If the cube of x vary as the square of $y+1$, and $x=4$ when $y=3$, find the equation connecting x and y.

4. If a, b, c, d are in H.P., prove that

H^c. mean between a and b : H^c. mean between c and d :: $3b-c : 3c-b$.

5. If $$\tan^2\alpha - 1 = 2\tan^2\beta,$$
prove that $$\sin^2\beta + \cos 2\alpha = 0.$$

6. Find the trigonometrical ratios for an angle of 165°.

7. Express in each system of angular measurement, the angle described by the hour hand of a watch in 20 minutes.

8. Describe a circle passing through a given point and touching a given circle at a given point.

CXXIV.

1. If 8000 mètres = 5 miles, 1 cub. fathom of water weighs 6 tons, and 1 cub. mètre of water weighs 1000 kilogrammes; find the ratio of 1 kilogramme to 1 lb. Avoir.

2. Solve the equation
$$x^2 + x + \sqrt{3x^2 - 2x + 4} = \tfrac{5}{3}x + 12.$$

3. Extract the square root of $103 - 20\sqrt{21}$,
and simplify
$$\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} + \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}.$$

4. Given three numbers a, b, c such that $a + b + c = 0$, find the number by which, if each be diminished, the sum of the fractions formed by dividing each number by its corresponding remainder, may be zero.

5. What is meant by "the sum of a Geometric series continued to infinity"?
In what case is such a sum possible?

6. Divide an angle of d degrees into two parts such that one part may contain as many English minutes as the other does French.

7. A tower subtends an angle of 30°, at a point 200 yards away from its foot, and in the same horizontal plane. Find its height.

8. Show that

$$\sin^2 24° - \sin^2 6° = \frac{\sqrt{5}-1}{8}.$$

CXXV.

1. Simplify

(i.) $\sqrt{12} + \frac{1}{10}\sqrt{75} + 6\sqrt{\frac{1}{12}}$,

(ii.) $\left(2 - \frac{3n}{m} + \frac{9n^2 - 2m^2}{m^2 + 2mn}\right) \div \left(\frac{1}{m} - \frac{1}{m - 2n - \frac{4n^2}{m+n}}\right)$.

2. Divide 183 into 3 parts in G.P., such that the sum of the first and third is $2\frac{1}{20}$ times the second.

3. Solve the equation :—

$$x^4 + 15x^3 - 270x^2 - 1080x + 5184 = 0,$$

the roots of which are in continued proportion.

4. Find $\tan \alpha$ in terms of $\tan 2\alpha$, and show *a priori* that two values will be obtained.

Prove that

$$\tan 7° \, 30' = \sqrt{6} + \sqrt{2} - \sqrt{3} - 2.$$

5. Solve the equation

$$\sin 11\,\theta + \sin 5\,\theta = \sin 8\,\theta.$$

6. A sphere just fits into a closed cylinder, whose height is equal to its diameter. If the volume of the empty space between them is 134·0416 cubic inches, find the radius of the sphere.

7. Find (by Log. Tables) the value of

$$\frac{7{\cdot}891}{{\cdot}0345} \times \sqrt[7]{89312{\cdot}7}.$$

8. If from the ends of a diameter AB of a semi-circle $AEDB$, chords ACD, BCE be drawn to meet the semi-circle in D and E; show that the square on the diameter is equal to the sum of the rectangles $AC.AD$ and $BC.BE$.

CXXVI.

1. If $a:b::c:d$ prove that
$$a^2+b^2:a^2-b^2::c^2+d^2:c^2-d^2.$$
2. In what scale of notation is 10105 equivalent to the denary number 83815 ?
3. Solve the equations :—
$$\text{(i.)}\ \sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\tfrac{10}{3},\quad x+y=10,$$
$$\text{(ii.)}\ x^2-y^2=9\ ;\ 2y-x=3,$$
and (iii.) $8x-5y=37$ in positive integers.
4. Write down the middle term of $(20-3b)^{12}$, and extract the cube root of (a^3+b^3) to 5 terms.
5. Prove that if a be the 1st term, d common difference, and s the sum of n terms of an A.P., then
$$s=\frac{n}{2}\left\{2a+\overline{n-1}d\right\}.$$
6. Solve (giving full solutions) :—
$$\text{(i.)}\ \sin\theta=1-\sqrt{3}\cos\theta,$$
$$\text{(ii.)}\ \cot\frac{A}{2}=2+\sqrt{3}.$$
7. The height of a house subtends a right angle at an opposite window, the top being 60° above a horizontal straight line. If the breadth of the street be 30 feet, find the height of the house.
8. Given the base, and vertical angle of a triangle, show that it will be greatest when isosceles.

CXXVII.

1. Extract the square root of

$$\text{(i.) } x^2 + 2\left(x + \frac{1}{x}\right) + \frac{1}{x^2} + 3,$$

$$\text{(ii.) } 79 + 30\sqrt{6}.$$

2. Solve

$$\text{(i.) } 6x^2 - 17x - 14 = 0,$$

$$\text{(ii.) } \begin{cases} x - 2y = 1, \\ x^3 - 8y^3 = 61, \end{cases}$$

$$\text{(iii.) } \frac{\sqrt{x + \sqrt{x^2 - 1}}}{\sqrt{x - \sqrt{x^2 - 1}}} = a + \sqrt{a^2 - 1}.$$

3. Sum

$$\text{(i.) } \tfrac{3}{5} + \tfrac{12}{10} + \tfrac{27}{15} + \ldots \text{ to 24 terms,}$$

$$\text{(ii.) } \frac{2}{3} + \frac{3}{3^2} + \frac{2}{3^3} + \frac{3}{3^4} + \frac{2}{3^5} + \frac{3}{3^6} + \ldots \text{ to infinity.}$$

4. Simplify

$$\frac{\left(p + \frac{1}{q}\right)^p \left(p - \frac{1}{q}\right)^q}{\left(q + \frac{1}{p}\right)^p \left(q - \frac{1}{p}\right)^q}.$$

5. Divide

$$x^{\frac{4}{3}} + y^4 \quad \text{by} \quad x^{\frac{2}{3}} - x^{\frac{1}{3}} y\sqrt{2} + y^2.$$

6. Prove that

$$\cos^2 48^\circ - \sin^2 12^\circ = \frac{\sqrt{5} + 1}{8}.$$

7. Prove that the value of

$$\sin(n+1)\beta \sin(n-1)\beta + \cos(n+1)\beta \cos(n-1)\beta$$

is independent of n.

8. The angles of a triangle are in G.P., and the greatest is double the least; express them in degrees, grades, and circular measure.

CXXVIII.

1. Simplify

$$\frac{1+ax^{-1}}{a^{-1}x-1} \times \frac{a^{-1}-x^{-1}}{a^{-1}x-ax^{-1}} \div \frac{ax^{-1}}{x-a}.$$

2. Solve

(i.) $\dfrac{x^2+a^2}{a-x} - \dfrac{x}{2} + \dfrac{3}{2}\left(x - \dfrac{1}{2}a\right) + \dfrac{15}{4}a = 0,$

(ii.) $x^2 - xy + y^2 = 1\frac{3}{4}, \quad x + y = 2.$

3. If $(a+b+c)x = (-a+b+c)y = (a-b+c)z = (a+b-c)w$, prove that

$$\frac{1}{y} + \frac{1}{z} + \frac{1}{w} = \frac{1}{x}.$$

4. Find the L.C.M. of

$$x^3 \pm 1, \quad \text{and} \quad x^3 \pm 2x^2 + 2x \pm 1$$

5. If α and β are the p^{th} and q^{th} terms of an A.P., find the $(p+q)^{\text{th}}$ and $(p-q)^{\text{th}}$ terms.

6. Find the circular measure of the angle of a regular octagon.

7. Prove *geometrically* that

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}\overline{A+B} \cos \tfrac{1}{2}\overline{A-B}.$$

8. Given $L \sin 60° = 9{\cdot}9375306$, find log 3.

$$\log 2 = {\cdot}3010300.$$

CXXIX.

1. Simplify

(1.) $\dfrac{1}{a+b} - \dfrac{b}{a^2-b^2} - \dfrac{ab^4}{a^6-b^6}.$

(ii.) $a - \cfrac{1}{b + \cfrac{1}{a + \cfrac{ab}{a-b}}}.$

2. Solve the equations :—

$$\text{(i.)}\begin{cases}\dfrac{x+y}{7}-\dfrac{2y-x}{3}=3,\\[2ex]\dfrac{3y+2x}{4}+\dfrac{9(x+1)}{8}=\dfrac{x}{2},\end{cases}$$

$$\text{(ii.)}\ \ 2x^2-3x-5=0.$$

3. Sum the series :—

(i.) $\frac{9}{4}+\frac{3}{2}+\frac{3}{4}+\ldots\ldots$ to 19 terms,

(ii.) $12+6+3+\ldots\ldots$ to infinity.

4. A merchant being asked the size of his store-room, replied that if it were 9 feet longer, and 6 feet narrower, it would be 4 square yards less; but if it were 6 feet shorter and 6 feet longer, it would be the same size as before. Find the dimensions.

5. If α and β are the roots of the equation

$$ax^2+bx+c=0$$

show that $\alpha+\beta=-\dfrac{b}{a}$, and $\alpha\beta=\dfrac{c}{a}$;

and if the roots are impossible show that

$$x^2+\frac{b}{a}x+\frac{c}{a}$$

may be put in the form

$$(x+p)^2+q^2,$$

where p and q are real.

6. Reduce 39° 56′ to French measure.

7. Prove that

$$(\sin A+\cos A)(\tan A+\cot A)=\sec A+\operatorname{cosec} A.$$

8. Find an expression for all angles with a given tangent, and hence find the solution of the equation

$$\operatorname{cosec}\theta=1+\cot\theta.$$

CXXX.

1. Find between what limits a must lie, that the roots of
$$x^2 + a^2 = 6a - 8x$$
may be real.

   Find also the condition that the roots of
$$x^2 - px + q = 0$$
may be equal and of opposite signs.

2. Two men A and B travel along a road 180 miles long in opposite directions, starting simultaneously from the ends. A travels 6 miles a day more than B, and the number of miles travelled by B each day is equal to double the number of days before they meet. Find the number of miles each travels in a day.

3. Find the condition that the roots of
$$x^3 - px^2 + qx - r = 0$$
may be in A.P.

4. In any triangle
$$\text{(i.)}\quad a = b\cos C + c\cos B.$$
$$\text{(ii.)}\quad \frac{b^2 - c^2}{\tan A} + \frac{c^2 - a^2}{\tan B} + \frac{a^2 - b^2}{\tan C} = 0.$$

5. Solve the equation
$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}.$$

6. Prove that the area of a sector of a circle is equal to $\frac{1}{2}$arc × radius.

   If the area is 2240·567 sq. ft. and the radius 33·495 ft., find the length of the arc.

7. Find by logarithms in what time a sum of money will treble itself at 3 per cent. Compound Interest.

8. The straight line joining the middle points of the diagonals of a quadrilateral with two parallel sides is parallel to those sides.

CXXXI.

1. If $a : b :: c : d$, prove that

$$a+b : a-b :: c+d : c-d,$$

and $\sqrt{a^2+b^2} : \sqrt{c^2+d^2} :: \sqrt[3]{a^3+b^3} : \sqrt[3]{c^3+d^3}$.

2. Solve the equations :—

$$\text{(i.)}\ \left(x^2+\frac{1}{x^2}\right)^2+4\left(x^2+\frac{1}{x^2}\right)=12.$$

$$\text{(ii.)}\ \sqrt{x+y}+\sqrt{x-y}=4, \quad x^2-y^2=9,$$

$$\text{(iii.)}\ \begin{cases} xy+20(x-y)=0, \\ yz+30(y-z)=0, \\ zx-12(z-x)=0, \end{cases}$$

given in (iii.) that x, y, z are positive integers and all < 7.

3. In an A.P. the first term is 81, and the fourteenth 159. In a G.P. the second term is 81, and the sixth 16. Find the Harmonic Mean between their fourth terms.

4. If one quantity be multiplied by a second, and divided by it, and the same operation be performed with the two results, what relation will the last results bear to the original quantities ?

5. The first edition of a book had 600 pages, and was divided into two parts. In the second edition, a quarter of the second part was omitted, and 30 pages added to the first. This made both parts the same length. Find what they were originally.

6. Explain what is meant by the circular measure of an angle, and find the circular measure of an angle of 240°.

7. Prove that

$$\text{(i.)}\ \cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A},$$

$$\text{(ii.)}\ \sin 2A = 2\frac{\cot A+\tan A}{\sec^2 A+\operatorname{cosec}^2 A}.$$

8. If $A+B+C=\pi$, show that

$$\sin A+\sin B+\sin C$$
$$=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right).$$

CXXXII.

1. Simplify

(i.) $\dfrac{a^3+a^2b+ab^2+b^3}{a^3-a^2b-ab^2+b^3}\times\dfrac{(a+b)^2-3ab}{(a-b)^2+3ab}\times\dfrac{(a-b)^3-(a^3-b^3)}{(a+b)^3-(a^3+b^3)}$,

(ii.) $a+\cfrac{b}{c+\cfrac{d}{e+\cfrac{f}{g+h}}}$.

2. Show that a pure circulating decimal is a G.P., and find the vulgar fraction equivalent to a circulating decimal of the form $\cdot PQQQ\ldots\ldots$

3. Multiply

$$a^2+b^2-c^2+2ab \quad \text{by} \quad a^2-b^2+c^2+2ac,$$

and divide the product by

$$a^2-b^2-c^2+2bc.$$

Divide

$$x^{\frac{2}{3}}+a^{\frac{1}{3}}x^{\frac{1}{3}}+a^{\frac{2}{3}} \quad \text{by} \quad x^{\frac{1}{3}}-a^{\frac{1}{6}}x^{\frac{1}{6}}+a^{\frac{1}{6}}.$$

4. Prove the rule for finding the square root of $a+\sqrt{b}$, and find the square roots of

(i.) $\frac{1}{2}-\frac{2}{3}\sqrt{\frac{1}{2}}$.

(ii.) $4x^4-8x^3y+4xy^3+y^4$.

5. Find the G.C.M. of

$$2x^5+2x^4-x^2-x+1 \quad \text{and} \quad 3x^5+3x^4+6x^3-3,$$

and the L.C.M. of

$$x^2-3x+2, \quad x^2-4x+3, \quad x^2-5x+6.$$

6. A farmer pays rent for his farm at a certain rate per acre. In one year he spends £2 per acre on tillage, and the total rent and tillage is £450. The next year the rent is raised 10s. an acre, and he spends £3 an acre on tillage, his total expenses being £630. Find the rent per acre, and extent of the farm.

7. Solve the equation

$$\sin 5x + \sin 3x = \sin 6x + \sin 2x.$$

8. If the moon is 240,000 miles distant from the earth, and her diameter is 2160 miles, what angle does she subtend at a point on the earth nearest to her? Give the answer in Sexagesimal measure.

CXXXIII.

1. If a mile be equivalent to 1600 mètres, find the number of square mètres in $7\frac{1}{2}$ acres.

2. Show that a quadratic equation cannot have more than two roots.

   Form the equation whose roots are 4 and 5.

   What is the condition that the roots of

$$ax^2 + bx + c = 0$$

   may be equal?

3. In a G.P. the sum of the second and fourth terms is 10, and the difference of the sixth and second is 30. Find the series.

4. When are four quantities said to be proportional?

   If a, b, c, d are proportionals, so are

$$\sqrt{a^2 + c^2},\quad \sqrt{b^2 + d^2},\quad \sqrt{ac + \frac{c^3}{a}},\quad \sqrt{bd + \frac{d^3}{b}}.$$

5. Solve the equations :—

$$\text{(i.)}\ \sqrt{x^2+x+\frac{\sqrt{x-1}}{\sqrt{x^3-x}}}=\tfrac{5}{2},$$

$$\text{(ii.)}\ x+y-xy=1, \quad x-y+x^2y^2=5.$$

6. Simplify

$$\sqrt[p]{\left\{(a+b)^{2m}\right\}}\times(a^2-b^2)^{\frac{p-2m}{p}}\times\left\{(a-b)^{\frac{m}{p}}\right\}^2.$$

7. Prove that

$$\sin 18^\circ=\frac{\sqrt{5}-1}{4}.$$

Hence find an expression for sin 9°.

8. If $\tan\theta+\tan 2\,\theta+\tan 3\,\theta=0$, find θ.

CXXXIV.

1. In any G.P. of an odd number of terms, the sum of the extremes is greater than twice the middle term.

   Show that the recurring decimal $\cdot 2\dot{3}4\dot{2}$ may be considered as a G.P., and hence evaluate it.

2. A grocer gains 20 per cent. by selling at 2s. a lb. a mixture formed of 7 lb. of a common tea, and 2 lb. of a better kind. If he had mixed 7 lb. of the latter with 2 of the former, he would have lost 20 per cent. Find the prices of the tea.

3. Find

   (i.) The sum of the squares,

   (ii.) The sum of the cubes, of the equations

$$ax^2-bx+c=0,$$

and

$$x^3-px^2+qx-r=0.$$

4. Find all the values of x which satisfy the equation

$$\tan 2x=\tan\frac{2}{x}.$$

5. The value of the divisions on the outer rim of a graduated circle is $5'$, and the distance between two successive divisions $\cdot 1$ inch. Find the radius.

   A church spire whose height is known to be 45 feet, subtends an angle of $9'$ to the eye; find its approximate distance.

6. A hollow pontoon has a cylindrical body 20 feet long, and hemispherical ends, and is made of metal $\frac{1}{8}$ inch thick. The outside diameter is 3 ft. 4 in.; and a cubic inch of the metal weighs $4\cdot 5$ oz. Find the total weight.

7. Complete the proportion

$$\sqrt[6]{9} : \sqrt[4]{7} :: \sqrt[5]{5} : x.$$

8. Describe a parallelogram whose area and perimeter shall be equal to those of a given triangle.

CXXXV.

1. What sum will amount to £$4022\cdot 3125$ in $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. Simple Interest?

2. Solve the equations :—

$$\text{(i.)}\ \frac{5x-7}{12}+\frac{5x+7}{36}=x-\tfrac{7}{2},$$

$$\text{(ii.)}\ \frac{x+1}{2}=\frac{y-1}{3},\quad \frac{y+1}{6}=\frac{x-3}{3}.$$

3. Sum the series :—

   (i.) $3, \frac{7}{3}, 1\frac{2}{3}, \ldots\ldots$ to 18 terms,

   (ii.) $2, 6, 18, 54\ldots$ to n terms,

   and show how many terms of the series

   1, 4, 7, 10, etc.,

   must be taken to make 3725.

4. Simplify

(i.) $\dfrac{4a^2}{2a-b}+b^2\div\dfrac{3a-b}{1-\dfrac{a}{2a-b}}$.

(ii.) $\dfrac{\sqrt{a-x}}{\sqrt{a}+\sqrt{a-x}}+\dfrac{\sqrt{a+x}}{\sqrt{a}-\sqrt{a-x}}-\dfrac{\sqrt{a}(\sqrt{a+x}+\sqrt{a-x})}{x}$.

5. Prove that

$$\tan 22\tfrac{1}{2}^\circ=\sqrt{2}-1.$$

6. Show geometrically that

(i.) $\sin(180^\circ-A)=\sin A$,

(ii.) $\tan(270^\circ+A)=-\cot A$.

7. Given $\log 2=\cdot3010300$, $\log 3=\cdot4771213$, find $\log 4\cdot5$.

8. If two equal circles cut each other, and through one of their common points, a straight line be drawn terminated by both circumferences, its ends are equidistant from the other common point of the circles.

CXXXVI.

1. If m and n be any numbers, prove that

(i.) $\dfrac{1}{n}+\dfrac{n}{n+1}>1$,

(ii.) $\dfrac{n}{m}+\dfrac{m}{n}>2$.

What special case is there of (ii.)?

2. Find the continued product of n terms of the G.P. of which the first term is a, and second ar.

Find also the ratio of the continued product of the odd terms, to that of the even terms, when $n=2m+1$.

3. Simplify

(i.) $\left(\dfrac{y^2}{x}+\dfrac{x^2}{y}\right)\times\dfrac{1}{y^2-x^2}+\dfrac{x}{xy-y^2}-\dfrac{y}{x^2+xy}$,

(ii.) Find the value of $\frac{1}{z}\sqrt{az^2 - a^2}$ in terms of x, when $yz = \sqrt{ay^2 - a^2}$, $xy = \sqrt{ax^2 - a^2}$.

4. A sum of £3 is divided among 50 men and women, the men receiving 1s. 6d. each and the women 1s. Find the number of each sex.

5. A boy receives a certain sum every week; and in each week spends half what he had at the beginning of it. He had no money to begin with, and at the end of the third week he had 1s. 2d. What is his weekly allowance?

6. Prove that

(i.) $\cos(A+B) = \cos A \cos B - \sin A \sin B$,

(ii.) $\dfrac{\sin A + \sin 2A}{\cos A - \cos 2A} = \cot \dfrac{A}{2}$.

7. Given $\log 2 = \cdot 3010300$, $\log 11 = 1\cdot 0413927$, find $\log 30\cdot 25$.

8. If a rhombus be circumscribed to a circle, the lines joining the points of contact of opposite sides pass through the centre.

CXXXVII.

1. If $a \propto b$, and $b \propto c$, show that $a \propto c$.

The expenses of an institution are partly constant and partly vary as the number of inmates. When the inmates are 220 and 250, the expenses are £1300 and £1450 respectively. Find the expense for 300.

2. Give a rule for finding two numbers whose sum and difference are given.

If the two numbers obtained are themselves considered as the sum and difference of two others, what relation will these last bear to the original sum and difference?

3. There are 3 numbers in G.P. whose sum is 95. If the 2 extremes are multiplied by 12, and the mean by 13, the results will be in A.P. Find the numbers.

4. Solve the equations :—

$$\text{(i.)}\quad \frac{2x}{\sqrt{x-1}}+\frac{\sqrt{x-1}}{2x}=2,$$

$$\text{(ii.)}\quad x^2-3xy+y^2=3, \quad 2x^2+y^2=6.$$

5. Prove that

$$2\sin\frac{A}{2}=\pm\sqrt{1-\sin A}\pm\sqrt{1+\sin A}.$$

Discuss the meaning of the sign $\pm$ in the above.

6. Solve

$$\sin 5B=\frac{1}{\sqrt{2}}.$$

7. Given $\log 2 = \cdot 3010300$, $\log 13 = 1\cdot 1139434$, find $\log \cdot 000208$.

8. ABC is a given triangle; construct another of equal area, having its vertex at a given point in BC, and its base lying along AB.

CXXXVIII.

1. Solve the equations :—

$$\text{(i.)}\quad \sqrt{x}-\sqrt{4-x}=\sqrt{4-3x},$$

$$\text{(ii.)}\quad x^2y^2+x=6(x-3), \quad xy=\frac{1}{y}+3y.$$

2. If $a=5$, $b=-4$, $c=3$, find the value of

$$\frac{a^3-b^3+c^3+3abc}{a^2+b^2+c^2+bc-ac+ab}.$$

3. Prove the truth of this statement :—

Take any two proper fractions whose sum is 1, subtract the square of the smaller from the square of the greater, and add 1 to the remainder : the result will be equal to twice the greater.

4. A quantity of land, partly pasture, and partly arable, is sold at £60 per acre for the pasture, and £40 for the arable, and the whole sum is £10,000. If the average price were £50 an acre, the amount would be 10 per cent. greater. Find the amount of each kind.

5. Show that

(i.) $2 \cot 2A = \cot A - \tan A$.

(ii.) $\cot(A + 15^\circ) - \tan(A - 15^\circ) = \dfrac{4 \cos 2A}{2 \sin 2A + 1}$.

6. Given

$\log 2 = \cdot 3010300$, $\log 3 = \cdot 4771213$, $\log 13 = 1\cdot 1139434$,
$\log 84962 = 4\cdot 9292247$, $\log 84963 = 4\cdot 9292298$,
find the value of $6(65)^{\frac{1}{12}}$.

7. Prove that the angle subtended at the centre of a circle by an arc of length equal to the radius, is constant.

8. The square on the side subtending an obtuse angle of a triangle is greater than the squares on the sides containing the obtuse angle. (Only Book I. to be used.)

CXXXIX.

1. Simplify

$$\text{(i.)}\ \frac{a^3 - 3a^2b + 3ab^2 - 2b^3}{a^4 + a^2b^2 + b^4},$$

$$\text{(ii.)}\ \frac{1}{x + 3y} + \frac{6y}{x^2 - 9y^2} - \frac{1}{3y - x}.$$

2. Solve the equations :—

$$\text{(i.)}\ \frac{\sqrt{x+5} - \sqrt{x+3}}{\sqrt{x+2} - \sqrt{x+1}} = 1,$$

$$\text{(ii.)}\ \sqrt[3]{x} - \sqrt[3]{y} = a, \quad \sqrt[3]{x - y} = b.$$

3. Solve the equation

$$ax^2 + bx + c = 0,$$

and if its roots are α, β, show that, for all values of x,

$$ax^2 + bx + c = a(x - \alpha)(x - \beta).$$

4. When are 4 quantities said to be proportionals?

If a, b, c, d are proportionals, prove that

$$\frac{ma + nc}{mb + nd} = \frac{pa + qc}{pb + qd}.$$

5. Find the radius of a circle in which an arc of 3 feet subtends an angle of ·25 at the centre.

6. Prove that

(i.) $\cos 2A(1 + \tan 2A \tan A) = 1$,

(ii.) $\sin 3A = 3 \sin A - 4 \sin^3 A$.

7. Find, by logs, the 12th root of 35, correct to 5 places of decimals.

8. If two consecutive sides of a hexagon inscribed in a circle be respectively parallel to their opposite sides, the two remaining sides are parallel.

CXL.

1. Solve the equations :—

(i.) $\sqrt{1+x} + (12 - x)^{\frac{1}{2}} = \dfrac{1}{(13 + 4x)^{-\frac{1}{2}}}$,

(ii.) $xy^2 - y + x = 0, \quad (1 - xy)(y^2 + 1) + \dfrac{1}{x} - y = 0$.

2. A man having 7 miles to walk, increases his pace one mile an hour after the first mile, and finds that he is half-an-hour less on the road than he would have been at his original rate. How long did he take?

3. Define variation.

If $A \propto B$, when C is constant, and $A \propto C$ when B is constant, then $A \propto BC$ when B and C both vary.

4. Find the sum of an A.P. whose first term is a and n^{th} term l.

   Show that if n be odd, the sum is n times the middle term.

5. Prove (in two ways) that
$$\sin(90^\circ + A) = \cos A.$$

6. Find the value, by Log Tables, of
$$\sqrt[5]{\frac{6300 \times \cdot 00117 \times 42 \cdot 9}{\frac{1}{2}(2197)^{\frac{1}{3}}}}.$$

7. Solve
$$\text{(i.)}\ 4\cos\theta - 3\sec\theta = 2\tan\theta,$$
$$\text{(ii.)}\ \sin 4\,\theta + \sin 2\,\theta = 0.$$

8. In the base BC of a triangle ABC, if D be taken, so that the squares on AB, BD are together equal to the squares on AC, CD, then the middle point of AD will be equidistant from B and C.

CXLI.

1. Solve the equations :—
$$\text{(i.)}\ \left(\frac{x}{x-1}\right)^2 + \left(\frac{x}{x+1}\right)^2 = n(n-1),$$
$$\text{(ii.)}\ \begin{cases} xy = a(x+y), \\ x^2y^2 = b^2(x^2+y^2). \end{cases}$$

2. If a, b, c are in H.P., and all positive, prove that
$$a : a-b :: a+c : a-c;$$
and
$$a^2 + c^2 > 2b^2.$$

3. A sum of money is divided among 12 persons in such a way that each receives £4 less than the one next older than himself. The share of the youngest is $\frac{1}{3}$ that of the oldest. Find the whole sum.

4. If $x = -\sqrt{\frac{1}{2}}$, find the value of

$$\frac{\sqrt{3-2x^2}-x}{x(1+3x)-x^2}.$$

5. Prove that

$$\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{16}{65}.$$

6. Show that

$$(\tan\alpha + \operatorname{cosec}\beta)^2 - (\cot\beta - \sec\alpha)^2$$
$$= 2\tan\alpha\cot\beta(\operatorname{cosec}\alpha + \sec\beta).$$

7. If $\tan^2\theta + 3\cot^2\theta = 4$, find $\sin\theta$.

8. Show that in a straight line divided as in II. 11, the rectangle under the sum and difference of the parts, is equal to the rectangle under the parts. (Geometrically and Analytically.)

CXLII.

1. If $a : b :: c : d$,

   then $a^2c + ac^2 : b^2d + bd^2 :: (a+c)^3 : (b+d)^3$.

2. If $a = 1$, $b = 3$, $c = 5$, find the value of

$$\frac{2a^3 + b^3 + c^3 + a^2(b-c) + b^2(2a-c) - c^2(2a+b)}{2a^3 - b^3 - c^3 + a^2(b-c) - b^2(2a-c) + c^2(2a+b)},$$

   and express

$$(a^2+b^2)(c^2+d^2)$$

   in two ways, as the sum of two squares.

3. Prove that the Arithmetic, Geometric, and Harmonic means, between two quantities, are in G.P.

   If a, b, c be the first, second, and last terms of an A.P., find the number of terms. And if the sum of n terms is zero, find the relation between a, b, c.

4. If the difference of two numbers be multiplied by the sum of their cubes; and then their sum multiplied by the difference of their cubes; and the one result

subtracted from the other, the remainder will be a multiple of the original numbers, and of their sum and difference.

5. Prove that

$$\text{(i.)}\ \cos 4\theta = \frac{\cot^2\theta - 6 + \tan^2\theta}{\cot^2\theta + 2 + \tan^2\theta},$$

$$\text{(ii.)}\ \sec A = \frac{1 + \tan^2\frac{A}{2}}{1 - \tan^2\frac{A}{2}}.$$

6. Find the value of $2^{70} \div 5^{30}$.

7. Find the value of

$$2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha$$

in terms of $\tan\alpha$.

8. The straight lines which bisect the vertical angles of all triangles on the same base, and on the same side of it, and with equal vertical angles, all pass through the same point.

CXLIII.

1. Three men A, B, C are candidates for an office. If all had demanded a poll, the number of votes for them would have been in A.P., and A would have been elected by a majority equal to C's votes. C withdraws from the contest, and his partisans distribute themselves to A and B in the ratio $1:4$; and then A is elected by a majority of 40. Find the number of electors.

2. A and B are two watches. A loses a minute a day, and B gains 2. When A is 12 o'clock one noon, B is 1 minute past 12. What will be the time by A, when B is at 12 the next noon ?

3. Show that

$$x^2+qx+1 \quad \text{and} \quad x^3+px^2+qx+1$$

have a common factor of the form $x+a$ when

$$(p-1)^2-q(p-1)+1=0.$$

4. Solve the equations :—

$$\begin{cases} x^2y^4+y^2=333, \\ xy^2+y=21\,; \end{cases}$$

and show that if

$$x^3(1-b^2)=ab, \quad \text{and} \quad y^3b=a(1-b^2)^{\frac{1}{2}},$$

then $$y^6(x^6y^6-a^4)=a^6.$$

5. Sum the series

$$1\,.\,3+2\,.\,4+3\,.\,5+\ldots \text{ to } n \text{ terms.}$$

6. Divide

$$\frac{x^3}{y^3}+\frac{5}{6}\,\frac{x^2}{y^2}+\frac{3}{2} \quad \text{by} \quad \frac{1}{3}\,\frac{x}{y^2}+\frac{1}{2}\,\frac{1}{y}.$$

7. Find θ from

$$\frac{x^2\tan^2\theta-1}{x-\tan\theta}=x+\tan\theta.$$

8. Solve the equation

$$2\tan^2\theta-a=\sec^2\theta,$$

and find what values a must have for $\theta=30°$, $60°$, and $90°$ respectively.

CXLIV.

1. A can run a mile in 4 min. 35 secs., B in 5 min. 19 secs., C in 5 min. 52 secs. What is the least exact number of yards which must be given to B and C respectively that they may beat A who starts from scratch in a 600 yards handicap?

2. If $x=a^2+1$, $y=a^{-2}+1$,

then $xy+x-y : xy-x+y :: a : a^{-1}$.

3. At a certain lecture there were three kinds of seats, and the prices of admission to them were in A.P.; also the number of persons in the first seats : number in second, as the prices charged; and the number in the third : number in second, as the prices charged. Prove that the number in the second seats $= \frac{1}{3}$ of the whole audience.

4. If a, x, y, b and x, a, b, y are in A.P. and G.P. respectively; prove that either

$$a = b, \quad \text{or} \quad 3a^2 + 2ab + 3b^2 = 0.$$

5. When may

$$\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$$

in a triangle?

6. Find θ when

$$\cot(\pi \cos \theta) = \tan(\pi \sin \theta).$$

7. Given $\tan\frac{A}{2} = \frac{a}{b}$,

find $\sin A$ and $\sin 2A$.

8. Describe a circle touching one side of a triangle, and the other two sides produced. If the diameter of the circle is equal to the perimeter of the triangle, show that the triangle is right-angled.

CXLV.

1. If

$$\frac{b}{a+b} = \frac{a+c-b}{b+c-a} = \frac{a+b+c}{2a+b+2c},$$

find the ratios of a, b, c.

2. Find the L.C.M. of

$x^4 + 1$, $x^3 + (\sqrt{2} - 1)(x^2 - x) - 1$, and $x^3 + x^2 - x - 1$; and the square root of

$$\left(xy\right)^{\frac{1}{2}}\left(x^{\frac{1}{2}} + 4y^{\frac{1}{12}}\right) + \frac{4y^{\frac{1}{3}}}{x^{\frac{1}{2}}} + \frac{1}{x} + 2y^{\frac{1}{4}}\left(1 + 2y^{\frac{5}{12}}\right).$$

3. Solve the equations :—

$$\text{(i.)}\ \sqrt{x}+\sqrt{4+x}=\frac{2}{\sqrt{x}},$$

$$\text{(ii.)}\ xyz=a^2(y+z)=b^2(z+x)=c^2(x+y).$$

4. Eliminate y and z from the equations

$$x+y+z=p,\quad xy+yz+xz=q,\quad xyz=r.$$

5. Find a number of three digits such that the sum of the first and second $=7$, the second and third $=13$, and third and first $=8$.

6. Solve the equations :—

$$\text{(i.)}\ \sec\theta\tan\theta=2\sqrt{3}.$$

$$\text{(ii.)}\ \frac{\sin\alpha}{\sin\beta}=\sqrt{2},\qquad \frac{\tan\alpha}{\tan\beta}=\sqrt{3}.$$

7. Find the value of $\cos 4A$ in terms of $\sin A$.

8. AD is parallel to BC; AC and BD meet in E; BC is produced to P so that the triangle PEB is equal to the triangle ABC. Show that PD is parallel to AC.

CXLVI.

1. If $a:b::c:d$, then

$$\frac{a^2}{m^2}+\frac{b^2}{n^2}+\frac{c^2}{p^2}+\frac{d^2}{q^2}=abcd\left\{\frac{1}{a^2q^2}+\frac{1}{b^2p^2}+\frac{1}{c^2n^2}+\frac{1}{d^2m^2}\right\}.$$

2. Solve the equations :—

$$\text{(i.)}\ x-\frac{1}{y}=3\tfrac{4}{5},\qquad y-\frac{1}{x}=4\tfrac{3}{4}.$$

$$\text{(ii.)}\ x^3+3ax^2+3bx+\frac{b^2}{a}=0.$$

3. Prove that

$$\log\left(\sqrt[n]{\frac{a}{b}}\right)^m=\frac{m}{n}\left(\log a-\log b\right).$$

Find the logarithm of $\sqrt[3]{4}$ to the base 16.

4. A hemispherical punch-bowl is 5 ft. 6 in. round the brim. If it is half-full, how many persons can be helped from it, in hemispherical glasses $1\frac{3}{4}$ in. diameter?

5. If A is the a^{th} term, and B the b^{th} term of an A.P., find the n^{th} term, and the sum to n terms.

6. Show that

$$\text{(i.)}\ \sin A = \sin(60^\circ + A) - \sin(60^\circ - A),$$

$$\text{(ii.)}\ \cot 3A = \frac{1 - 3\tan^2 A}{3\tan A - \tan^3 A}.$$

7. AC is the diameter of a circle, and a diagonal of the inscribed quadrilateral $ABCD$. Given $AB = 30$, $BC = 40$, $CD = 10$; find AD and the area of the figure.

8. The length of a road in which the ascent is 1 foot in 5 from the foot to the top of a hill is $1\frac{2}{3}$ miles. What will be the length of a zigzag from the bottom to the top, rising 1 foot in 12?

CXLVII.

1. Solve the equations :—

$$\text{(i.)}\ \sqrt{x+13} = 2 + \sqrt{x-11}.$$

$$\text{(ii.)}\ x^{\frac{1}{m}} y^{\frac{1}{n}} = a, \quad x^{\frac{1}{p}} y^{\frac{1}{q}} = b.$$

2. Show that if a^2, b^2, c^2 are in A.P., then $b+c$, $c+a$, $a+b$ are in H.P.

Also, $$a^2 + c^2 = \frac{a^2b^2}{a^2 - b^2} + \frac{b^2c^2}{c^2 - b^2}.$$

3. The sum of two numbers multiplied by the greater is 260; their difference multiplied by the less is 42. Find the numbers.

4. $ABCD$ is a field such that the straight lines joining opposite corners A, C, and B, D, meet in F at right angles. FA, FB, FC, FD are 83, 97, 125, and 228 yards long respectively. Find the area in acres, roods, etc.

5. Given
$$\tan\theta = a,$$
find $\cos 2\theta$ and $\sin 2\theta$.

6. Find x from the equation
$$\sin 3A = 4\sin A \sin(x+A)\sin(x-A).$$

7. A ladder of length l is placed against a wall so that the angle it makes with the ground is the same as the angle it makes with the wall. How far is the middle of the ladder from the wall?

8. How many sides has an equiangular polygon, four of whose interior angles are together equal to seven right angles?

CXLVIII.

1. What value of x will make
$$x^2 + 2ax + b^2$$
the square of $x + c$?
What does the result become when $a = b = c$?

2. Simplify
$$\frac{x^2 - (2y-3z)^2}{(x+2y)^2 - 9z^2} + \frac{4y^2 - (3z-x)^2}{(2y+3z)^2 - x^2} + \frac{9z^2 - (x-2y)^2}{(3z+x)^2 - 4y^2}.$$

3. Solve the equations :—
$$\text{(i.)}\quad \frac{a-bx}{a+bx} + \frac{ax-b}{ax+b} = 2\frac{a-b}{a+b},$$
$$\text{(ii.)}\quad x^2 + xy + y^2 = 49, \quad x^4 + x^2y^2 + y^4 = 931.$$

4. Cleopatra's Needle consists approximately of a frustrum of a pyramid surmounted by a smaller pyramid. If the lower base were $7\frac{1}{2}$ feet square, and the upper

$4\frac{1}{2}$ feet square, the height of the frustrum 61 feet, and of the pyramid $7\frac{1}{2}$ feet; find the cubical contents, and its weight, if 1 cubic foot weighs 170 lbs.

5. If $A+B+C=\pi$, show that

$$\sin A - \sin B + \sin C = 4 \sin\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}.$$

Express as a product

$$\sin A - \sin 2A + \sin 3A.$$

6. If two sides of a right-angled triangle are 21 and 28 feet respectively, find the perpendicular on the hypoteneuse from the right angle.

7. A person at the edge of a river observes the elevation (60°) of a tower on the other side; on retreating a feet he finds its elevation to be 45°. Find the height of the tower and the breadth of the river.

8. The area of any rectangle is half the area of the rectangle contained by the diagonals of the squares on two adjacent sides of it

CXLIX.

1. Solve the equations :—

(i.) $(7+4\sqrt{3})x^2+(2+\sqrt{3})x=2.$

(ii.) $\begin{cases} x+y+z=a+b+c, \\ bx-cy+az=ay+bz-cx=ab-(a-b)c. \end{cases}$

2. A railway train after travelling 1 hour meets with an accident which delays it an hour, after which it proceeds at $\frac{3}{5}$ths its former rate, and arrives 5 hours late. Had the accident occurred 50 miles farther on the train would have been 3 hours 20 min. late. Find the whole distance, and the rate of the train.

3. If

$$(a+b+c+d)(a+d-b-c)=(a+c-b-d)(a+b-c-d),$$

then $$a : b :: c : d.$$

4. Find the relation between a, b, c, that they may be the p^{th}, q^{th}, r^{th} terms respectively of an A.P.

The product of three integers in A.P. is the same as their sum. Find them.

5. The town C is half way between the towns D and E, and C, E, F are equidistant from each other. From D to E is 12 miles. Find the distance from D to F.

6. Whilst sailing due W., I observe two ships at anchor directly N. of me; after sailing 6 miles, the directions of the ships make angles 60° and 30° with my course. How far are they apart?

7. Given $\log 2 = \cdot 30103$, find $\log 25$ and $\log 800$.

8. Prove that

$$\tan 45^\circ = 1,$$

$$\tan 22\tfrac{1}{2}^\circ = \frac{1}{\sqrt{3+2\sqrt{2}}}.$$

CL.

1. Prove that

(i.) $$\left(\frac{x+y}{x}-\frac{x-y}{y}\right)^2-\left(\frac{x-y}{x}+\frac{x+y}{y}\right)^2=8\left(\frac{y}{x}-\frac{x}{y}\right),$$

(ii.) If $2\left(a+\frac{1}{a}\right)=ax+\frac{a}{x}$, then $a(x-1)=\pm\sqrt{2x}.$

2. Solve the equation

$$x^4-8x^3+14x^2+8x-15=0,$$

whose roots are in A.P.

3. If α and β are the roots of

$$ax^2 + bx + c = 0,$$

and $\dfrac{\alpha}{\beta}$, $\dfrac{\beta}{\alpha}$ those of

$$px^2 + qx + r = 0,$$

then $$b^2r + acq = 2apc.$$

4. If the sum to n terms of a series be

$$nx + \frac{n+1}{2} \cdot \frac{n}{x},$$

find the series.

Can a, b, c be in A.P., and also in G.P.?

5. A, B, C are three points not in a straight line; O is another point within ABC such that $OB = OC = \frac{1}{2}OA = x$; and the angle $BAO = 30°$. OC produced bisects the angle AOB. Find the angle BAC.

6. "A castle wall there was whose height was found
"To be one hundred feet from top to ground.
"Against the wall a ladder stood upright,
"Of the same length the castle was in height.
"A waggish youngster did the ladder slide,
"(The bottom of it) ten feet from the side.
"Now find how far the top did fall
"By pulling out the ladder from the wall."

7. Prove that in any triangle the sides are proportional to the sines of the opposite angles.

Show also that

$$a \cos A \cos 2B + b \cos B \cos 2A + c \cos C = 0.$$

8. Show that $$\log mn = \log m + \log n,$$

and *hence* that

$$\log 20 = \log 5 + 2 \log 2 = 1 + \log 2.$$

CLI.

1. Solve the equations :—

$$\text{(i.)} \begin{cases} a(z+y)-b(y-z)=2a, \\ a(y-z)-b(z+y)=2b, \end{cases}$$

$$\text{(ii.)} \begin{cases} x^2+y=2x, \\ y^2+x=2y, \end{cases}$$

$$\text{(iii.)} \left(x^m\right)^{\frac{1}{m}} - \left(x^{1+\frac{1}{m}}\right)^{\frac{m}{m+1}} + \sqrt[m]{x^{2m}} = a.$$

2. If
$$\frac{a+c}{b}=\frac{c}{a}=\frac{a}{c-b},$$
find the ratios of a, b, c.

3. In the Oxford and Cambridge boat race the number of minutes occupied in the race was half the average number of strokes per minute, and five times the number of miles rowed. The total number of strokes was 968. Find the length of the course, and the time.

4. A ladder of length x is placed against a wall so that the angle it makes with the ground is double that which it makes with the wall. How far is the bottom of the ladder from the wall?

5. A person travelling southwards observes two objects towards the S.E. After 8 miles' travelling, one is N.E. and the other E. Find their distance apart.

6. Solve the equations :—

$$\text{(i.)} \sin 7x - \sin x = \sin 3x,$$

$$\text{(ii.)} \tan A + \cot A = 2 \operatorname{cosec} 60^\circ.$$

7. Prove that

(i.) $\cos 10\alpha + \cos 8\alpha + 3\cos 4\alpha + 3\cos 2\alpha = 8\cos\alpha\cos^3 3\alpha.$

(ii.) $\operatorname{cosec}^2 A = 1 + \cot^2 A.$

8. What are the maximum and minimum values of the sine, tangent, and secant? Prove the truth of your answer.

CLII.

1. Prove that

$$\log \tfrac{144}{35} = 5 \log 2 + 2 \log 3 - \log 7 - 1.$$

If $7^x = 2$, find x.

2. Solve the equations :—

$$\text{(i.)} \begin{cases} x - xy + y = 0, \\ x^2 + 2x^2y^2 - y^2 = 0, \end{cases}$$

$$\text{(ii.)} \sqrt{4x^2 + 2x + 7} = 12x^2 + 6x - 119.$$

3. If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}$,

prove that $$\frac{a}{b} = \frac{(3c^2 + 4e^2)^{\frac{1}{2}}}{(3d^2 + 4f^2)^{\frac{1}{2}}}.$$

4. Prove that p is the sum, and q the product of the roots of the equation

$$x^2 - px + q = 0.$$

Form an equation whose roots shall be the square of the sum, and the square of the difference, of the roots of the equation

$$3x^2 + 3ax + a^2 = 0.$$

5. Find to the nearest hundredth of an inch the radius of a sphere,

(i.) whose volume is 1 cubic foot,

(ii.) whose area is 1 square foot.

6. Prove that the ratio of the angles denoted by a French and English second is

$$3.3^3 : 2.5^3.$$

Show that $$m' = m\grave{} + \frac{23}{27} m\grave{}.$$

7. Find the values of

$$\tan 405°, \quad \cos 780°, \quad \sin(-2100°).$$

8. At the foot of a mountain the elevation of its summit is 45°. After ascending 1 mile up a slope of 30°, the elevation is found to be 60°. Find its height.

CLIII.

1. Simplify

$$\text{(i.)}\quad \frac{1}{1-\dfrac{1}{x}}-\frac{1}{\dfrac{1}{x^3}-\dfrac{1}{x}}-\frac{1}{1-\dfrac{2x}{x^2+1}},$$

$$\text{(ii.)}\quad \frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}}-\frac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}.$$

2. Show how a debt of half-a-guinea may be paid in threepenny and fourpenny pieces.

3. What is meant by a continued fraction?

   Express 3·14159 as a continued fraction, and find the convergents.

4. Find the square root of $a-a^{\frac{1}{2}}$ to 4 terms.

5. A man invests £1,700, part in 3 per cent. stock at 90, and the rest in $3\frac{1}{4}$ per cent. stock at 104, and obtains an income of £55. How much does he invest in each?

6. An angle is the excess of $a^\circ.b'$ over $p^g.q^{`}$. Find the ratio of this angle to a right angle.

7. Eliminate θ from

$$a\cos 2\theta=b\sin\theta,$$
$$\text{and}\quad c\sin 2\theta=d\cos\theta.$$

8. Show how to find the values of $\sin 30^\circ$ and $\tan 15^\circ$.

CLIV.

1. Multiply together $e4t$ and te in the duodenary scale, and transform the result to the denary scale.

2. Find 3 numbers in G.P. such that if 1, 3, 9 be subtracted from them, they will form an A.P. whose sum is 15.

3. Solve the equations :—

   (i.) $\sqrt{ax+b^2}-\sqrt{bx+a^2}=a-b,$

   (ii.) $x^3+y^3=91,\quad x+y=7,$

   (iii.) $9x+13y=1000,$ in positive integers.

4. Find the square root of

$$\frac{x}{y}\left(2+\frac{x}{y}\right)-\frac{y}{x}\left(2-\frac{y}{x}+\frac{x}{y}\right).$$

5. When cannot a quadratic expression $(e.g.\ ax^2+bx+c)$ be resolved into two factors?

6. Given

$$L\sin 17^\circ\, 1' = 9{\cdot}4663483 \quad \text{and} \quad L\sin 17^\circ = 9{\cdot}4659353,$$

find $\log\sin 17^\circ\, 0'\, 12''$.

7. Eliminate θ from

$$x\sin\theta - y\cos\theta = \sqrt{x^2+y^2},$$
$$\frac{\sin^2\theta}{a^2}+\frac{\cos^2\theta}{b^2}=\frac{1}{x^2+y^2}.$$

8. If a quadrilateral figure be bisected by both its diagonals it is a parallelogram.

CLV.

1. Prove that

$$a(b+c-a)^2+b(c+a-b)^2+c(a+b-c)^2$$
$$+(b+c-a)(c+a-b)(a+b-c)\equiv 4abc.$$

2. Solve

(i.) $x(x+4)+\frac{1}{x}\left(\frac{1}{x}+4\right)=10,$

(ii.) $(x-9)(x-7)(x-5)(x-1)=(x-2)(x-4)(x-6)(x-10).$

3. Determine λ so that the equation in x

$$\frac{2A}{x+a}+\frac{\lambda}{x}+\frac{2B}{x-a}=0$$

may have equal roots.

4. If $$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c},$$

prove that $$\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}=\frac{1}{(a+b+c)^3},$$

and $$\frac{1}{a^{2n+1}}+\frac{1}{b^{2n+1}}+\frac{1}{c^{2n+1}}=\frac{1}{(a+b+c)^{2n+1}}$$

5. Simplify

$$\sqrt{\frac{1}{17+4\sqrt{\frac{15}{2}}}}+\sqrt{\frac{1}{17-4\sqrt{\frac{15}{2}}}}.$$

6. If $$\tan\frac{x}{2}=\frac{\tan x+a-1}{\tan x+a+1}$$

have real roots, prove that $a^2 > 1$.

7. Prove that if

$$\sin \overline{\alpha+\beta} \cos \gamma = \sin \overline{\alpha+\gamma} \cos \beta,$$

either $\beta-\gamma$ is a multiple of π, or α an odd multiple of $\frac{\pi}{2}$.

8. The figure formed by joining the middle points of the sides of any quadrilateral, is a parallelogram, and its area half that of the quadrilateral.

CLVI.

1. If $$\log\tfrac{1025}{1024}=\alpha \quad \text{and} \quad \log 2=\beta,$$
prove that $$\log 4100=\alpha+12\beta.$$

2. What integral values of y will make $\dfrac{9+y}{1+y}$ integral?

3. Solve

(i.) $x-y=3,\ \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}+\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}=\dfrac{10}{3}.$

(ii.) $x^2(a-x)=(a^2-x^2)(2x-a),$

(iii.) $x(x+y)=54-2xy, \quad y(x+y)=115-3y^2.$

4. What is the meaning of $a^{\frac{m}{n}}$? Why has it this meaning?

5. What will be the amount of £150 from Jan. 1, 1856, to May 1, 1859, at 3 per cent. Compound Interest?
(To be worked algebraically, and with use of logarithms.)
Explain why there is a different answer, when the problem is worked out by Arithmetic.

6. If $x^2+y^2 \propto x^2-y^2$,

   prove that $x+y \propto x-y$.

7. Prove that

$$\frac{\sin x}{2^n \sin \frac{x}{2^n}} = \cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}\ldots\cos\frac{x}{2^n},$$

   and hence solve the equation $\sin 16x = 0$.

8. If $\sin\theta = \frac{3}{5}$, find the other trigonometrical ratios for the angle θ.

CLVII.

1. If a, b, c are in A.P., convert

$$a^2+3ac+c^2$$

   into an expression involving a, b, c symmetrically.

   If *also* $a-\frac{x}{a}$, $b-\frac{x}{b}$, $c-\frac{x}{c}$ are in G.P., prove that

$$x = 2b^2 \pm \frac{b}{3}\sqrt{2(a+b+c)^2+9(ab+bc+ca)}.$$

2. Prove that $a^4+ma^2b^2+b^4$

   is divisible by $a^2+\sqrt{2-m}.ab+b^2$,

   and find the other factor.

3. Solve the equations :—

$$\text{(i.) } y^2+2cy-2a\sqrt{\frac{y^2+2cy+c-a}{a+c}}=0,$$

$$\text{(ii.)}\begin{cases}(x+y)^4 = 6xy(x+y)^2-27,\\(x-y)^4 = 9-4xy(x-y)^2.\end{cases}$$

4. Show how an integer greater than r can be expressed in terms of digits of numbers less than r.

   If any number be multiplied by a number which exceeds the radix by unity, show that the result may be obtained by adding each digit to the digit in front,

beginning from the right, and carrying unity to the next pair when the sum of any pair is not less than the radix.

Show also that in the product the sums of the alternate integers are equal.

5. Prove that

$$\text{(i.)}\quad \tan A - \tan\frac{A}{2} = \tan\frac{A}{2}\sec A,$$

$$\text{(ii.)}\quad 3\tan^{-1}A = \tan^{-1}\frac{3a-a^3}{1-3a^2}.$$

6. In a triangle, if

$$B = 45^\circ,\quad C = 60^\circ,\quad \text{and}\quad a = 2(\sqrt{3}+1)\text{ inches},$$

find the area.

7. Given $b = 8{\cdot}4$, $c = 12$, $B = 37^\circ\ 36'$, find A.

8. Solve the equation

$$a^x(a^x - 1) = 1,$$

and find the value of

$$10\log\tfrac{3}{2} + 7\log\tfrac{5}{18} + 4\log\tfrac{48}{25},$$

without using Log. Tables.

CLVIII.

1. There are three numbers in A.P., and if the first is increased by 1, or the third by 2, they will be in G.P. Find them.

If $s_1 = a + \frac{a}{r} + \frac{a}{r^2} + \dots$ *ad inf.*, $s_2 = b + \frac{b}{r} + \frac{b}{r^2} + \dots$ *ad inf.*,

$$s_3 = c + \frac{c}{r} + \frac{c}{r^2} + \dots \textit{ ad inf.},$$

then

$$\frac{s_1 s_2}{s_3{}^2} = \frac{ab}{c^2}.$$

2. Find the value of x to 3 places of decimals from the equation

$$3^{2x} + 3^x = 6.$$

3. Two casks, A and B, are filled with two kinds of sherry, mixed in A in the ratio $2:7$, and in B in the ratio $1:5$. What quantity must be taken from each to have 11 gallons mixed in the ratio $2:9$?

4. Prove that if
$$(a+2b)(2b+c)=(c+2a)(a+2c),$$
then $a+b+c$ either $=0$, or $=3b$.

Simplify
$$(x\sqrt{y}+z\sqrt{z})(\sqrt{x\sqrt{y}}+\sqrt{z\sqrt{z}})\left(\frac{\sqrt{xy}}{\sqrt[4]{y}}-\sqrt{\frac{z^2}{\sqrt{z}}}\right).$$

5. Prove that
$$\text{(i.)}\ \sec\left(\frac{\pi}{4}+\theta\right)\sec\left(\frac{\pi}{4}-\theta\right)=2\sec 2\theta.$$
$$\text{(ii.)}\ \tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}.$$

6. Trace the variations in sign and magnitude of $\cos\theta-\sin\theta$, as θ varies from 180° to 0°.

7. In any triangle
$$\text{(i.)}\ \sin\frac{A}{2}=\tfrac{1}{2}\sqrt{\frac{(a-b+c)(a+b-c)}{bc}},$$
$$\text{(ii.)}\ \frac{1}{r}=\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c},$$
where r, r_a, r_b, r_c, are the radii of the inscribed, and escribed circles.

8. AB is a side of an equilateral triangle inscribed in a circle whose diameter is AOC; prove that the triangle BOC is equilateral, and that the square on AB is three times the square on AO.

CLIX.

1. Simplify
$$\text{(i.)}\ \frac{a^{-1}+b^{-1}}{a^{-1}-b^{-1}}\cdot\frac{a^{-2}-b^{-2}}{a^{-2}+b^{-2}}\cdot\frac{1}{\dfrac{1}{a^{-2}-b^{-2}}+\dfrac{1}{a^{-2}+b^{-2}}},$$

(ii.) $$\frac{(x+z)\sqrt{y^2+z^2}-(y+z)\sqrt{x^2+z^2}}{z(x-y)} - \frac{2(z^2-xy)}{(x+z)\sqrt{y^2+z^2}+(y+z)\sqrt{x^2+z^2}}.$$

2. If α and β are the roots of the equation
$$x^2+px+q=0,$$
then p and q are the roots of
$$x^2+(\alpha+\beta-\alpha\beta)x-\alpha\beta(\alpha+\beta)=0.$$

3. X starts by coach from A to B, a distance of 7 miles, and Y at A catches the train for C, which starts 10 minutes after the coach. C is 7 miles from A, and $2\frac{1}{2}$ from B. If the rates of the coach and train are 12 and 42 miles per hour, at what rate must Y travel from C to B, to reach B when X does?

4. Simplify

(i.) $(12+2\sqrt{35})^{\frac{1}{2}}$,

(ii.) $\sqrt{\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}}$,

(iii.) $\dfrac{(\sqrt{5}+\sqrt{3})^5-(\sqrt{5})^5-(\sqrt{3})^5}{(\sqrt{5}+\sqrt{3})^3-(\sqrt{5})^3-(\sqrt{3})^3}$.

5. If in a triangle A be three times B,

then $$\sin B = \tfrac{1}{2}\sqrt{\frac{3b-a}{b}}.$$

6. Solve the equation
$$\log(\cdot 125)^{x-4} = \cdot 0270927.$$

7. A man observes the elevation of a mountain top to be 15°, and after walking 3 miles towards it, on level ground finds it 18° Find his original distance from it.

8. Prove that if $\log_a b = m$, and $\log_b a = n$,

then $$\left(\frac{\log_a m}{\log_b n}\right)^2 = \frac{m}{n}.$$

CLX.

1. Which is the better investment, 5 Per Cents. at 98, or 6 Per Cents. at $113\frac{1}{2}$?

2. A purse and its contents are worth £3 6s. $5\frac{1}{2}$d., and the value of the contents is to that of the purse as 9 is to 2. Find the value of the contents.

3. Solve

$$\text{(i.)}\ \frac{x^2-ab}{a}=\frac{x^2-2bx}{b},$$

$$\text{(ii.)}\ \frac{4}{3}xy=1=x-y.$$

4. Show how to find the sum, difference, and product of the roots of the equation

$$x^2+7x+10=0$$

without solving it.

5. Sum to n terms the Arithmetic Series whose first two terms are

$$\frac{1}{\sqrt{2}-1} \quad \text{and} \quad 2\sqrt{2}.$$

6. Simplify

$$\text{(i.)}\ \frac{3^{\frac{1}{2}}}{2^{-3}}-5\sqrt{12}+18(3)^{-\frac{3}{2}}.$$

$$\text{(ii.)}\ \frac{5+\sqrt{7}}{4+\sqrt{7}}+\frac{5-\sqrt{7}}{4-\sqrt{7}}.$$

7. Find the number of combinations of n things, taken r together.

8. If two circles cut, prove that their common chord is bisected at right angles by the straight line through their centres. What does this become when the circles touch each other?

CLXI.

1. Given $\log 20 = 1\cdot3010300$, $\log 30 = 1\cdot4771213$, find $\log 360$, $\log 64$, and $\log 5$.

2. Solve

(i.) $x + y = 6$, $\frac{1}{y} - \frac{1}{x} = \frac{1}{4}$,

(ii.) $27x^2 + 6x - 8 = 0$,

(iii.) $x + y - z = 4$, $\frac{x^2 + y^2 - z^2}{2} = 40 - xy$, $xz = 3(y - 1)$.

3. If $x - y = a$, $x^2 - y^2 = b^2$, $x^3 - y^3 = c^3$, prove that $a^4 + 3b^4 - 4ac^3 = 0$.

4. If $a : b :: c : d :: e : f$,

prove that (i.) $\frac{a}{b} = \frac{a + c + e}{b + d + f}$,

and (ii.) $(a^2 + c^2 + e^2)(b^2 + d^2 + f^2) = (ab + cd + ef)^2$.

5. Prove that the number of combinations of n things taken r together is

$$\frac{n(n-1)(n-2)\ldots(n-r+1)}{\underline{|r}}.$$

How many selections of 3 can be made from a class of 12 boys? In how many would the same boy appear?

6. Divide

$$x^{\frac{7}{8}} + 3xy^{\frac{4}{3}} - 6y^{\frac{7}{3}} \text{ by } x - 2y \text{ to 3 terms.}$$

7. Prove that the circumferences of circles vary as their radii. The circular measure of an angle is equal to the ratio of the number of degrees in it to the number of grades. Find the measure of it in degrees.

8. Prove that if

$x = a\cos\theta + b\cos 2\theta$, and $y = a\sin\theta + b\sin 2\theta$;

then $a^2\{y^2 + (x + b)^2\} = (y^2 + x^2 - b^2)^2$.

CLXII.

1. Show, without using Log. Tables, that

$$\frac{\log\sqrt{27}+\log 8-\log\sqrt{1000}}{\log 1\cdot 2}=1\tfrac{1}{2}.$$

2. Solve

$$\text{(i.)}\ y^2z=2\sqrt{3},\quad z^2x=3,\quad x^2y=\sqrt{2}.$$

$$\text{(ii.)}\ \frac{1}{x}+y=\frac{1}{y}+x,\quad 4(x^2+y^2)=17.$$

3. A man invests £p in the m per cent. stock at £a, and £q in the n per cents at £b. What percentage does he get on his whole capital?

4. Sum to n terms the series whose r^{th} terms are

$$\text{(i.)}\ r^3,\qquad \text{(ii.)}\ ra^r.$$

5. If x_r represent the number of combinations of x things, r together, prove that

$$x_r=x_{x-r}.$$

6. In any triangle if $b-a=3c$, show that

$$A+\frac{C}{2}=\cos^{-1}\left(3\cos\frac{C}{2}\right),$$

and $$3\sin B\cot\frac{B-A}{2}=1+3\cos B.$$

7. The number of grades in an angle of a regular rectilineal figure is to the number of degrees in an angle of another as $5:3$. Find the number of sides in each.

8. Show that

$$\cos 50^\circ+\cos 70^\circ+\cos 170^\circ=0.$$

CLXIII.

1. Solve the equations:—

$$\text{(i.)}\ 2y^2=2x^2+1,\quad 2(y^2-1)=xy,$$

$$\text{(ii.)}\ 3x^n\sqrt[3]{x^n}+2\frac{x^n}{\sqrt[3]{x^n}}=16.$$

2. If $x^2 - yz = a^2, \quad y^2 - xz = b^2, \quad z^2 - xy = c^2,$
prove that
$$a^2x + b^2y + c^2z - (a^2 + b^2 + c^2)(x + y + z) = 0.$$

3. Find the square root of
$$\frac{x^2}{y^2} + \frac{y^2}{x^2} - \frac{1}{\sqrt{-1}}\left(\frac{x}{y} - \frac{y}{x}\right) - \frac{9}{4}.$$

4. If $x + y : 3a - b :: y + z : 3b - c :: z + x : 3c - a$
then $x + y + z : ax + by + cz :: a + b + c : a^2 + b^2 + c^2.$

5. Show that
$$\frac{a}{c} + \frac{c}{a} > 2.$$

6. The sides of a triangle are 141, 100, 53; find the length of the perpendicular on the longest side from the opposite angle.

7. Solve the equations :—
$$\text{(i.)}\ \tan^2\theta - a\tan\theta = b^2,$$
$$\text{(ii.)}\ \sin^{-1}\frac{3}{x} - \sin^{-1}\frac{x}{13} = \sin^{-1}\frac{16}{65}.$$

8. Through a given point A, draw a straight line meeting two parallel lines at B and C, so that BC may be of given length.

CLXIV.

1. Find the square root of
(i.) $7 - 4\sqrt{3}$,
(ii.) $x^3(x^3 + 2) + 2x^2(x^2 + 1) - x(x - 2) + 1.$

2. Find expressions for the sum to n terms, and the sum to infinity, of a G.P.

Find the fourth term in
(i.) $2 + 2\frac{1}{2} + 3 + \ldots$
(ii.) $2 + 2\frac{1}{2} + 3\frac{1}{8} + \ldots$
(iii.) $2 + 2\frac{1}{2} + 3\frac{1}{3} + \ldots$

3. Find the number of combinations of n things, taken r together, assuming the corresponding number of Permutations.

   Deduce the expansion of $(a+x)^n$,

   n being a whole number.

4. Two rectangular lawns have the same area (a^2), but the perimeter of the one is $\frac{1}{4}$ greater than that of the other, which is a square. Find the dimensions.

5. Find an expression for all angles with a given cotangent. Solve the equation

$$\tan\left(\frac{\pi}{3}-\theta\right)=\sqrt{3}.$$

6. Find the values of $\sin 2A$, $\tan 2A$, and $\sin\frac{A}{2}$, in terms of the ratios of A.

7. Prove that

$$1-\tan^2 A\tan^2 B=\frac{\cos^2 B-\sin^2 A}{\cos^2 A\cos^2 B}.$$

8. Perpendiculars are drawn from any point within an equilateral triangle, on the three sides; prove that their sum is constant.

CLXV.

1. Given $7^x=823543$, find x by logarithms.

2. Simplify

$$\text{(i.)}\quad \frac{x^2-bc}{(a-b)(a-c)}+\frac{x^2-ca}{(b-c)(b-a)}+\frac{x^2-ab}{(c-a)(c-b)}.$$

$$\text{(ii.)}\quad \left\{\left(\frac{x^2}{y}+y-x\right)\div\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}+1\right)\right\}\times\left(\frac{1}{x}+\frac{1}{y}+\frac{y}{x^2}\right).$$

3. If a, b, c are in H.P., prove that
$$2a(a-b), \quad b(a-c), \quad 2c(b-c)$$
are in G.P.

4. If α and β are the roots of
$$x^2 - px + q = 0,$$
find the values of
$$\alpha^2 + \beta^2, \quad \text{and} \quad \alpha\beta^{-1} - \beta\alpha^{-1}.$$
Verify the results for $x^2 - x + 1 = 0$.

5. If the number of combinations of n things, r together, be denoted by c_r, find the value of
$$c_1 + c_2 + c_3 + \ldots + c_{n-1}.$$

6. Solve
$$\text{(i.)} \begin{cases} xy + ab = 2ax, \\ x^2y^2 + a^2b^2 = 2b^2y^2, \end{cases}$$
$$\text{(ii.)} \; 3x^2 + 2x - 4 = 3x(x-1)^{\frac{1}{2}}(x+2)^{\frac{1}{2}}.$$

7. Trace the changes in the value of $\operatorname{cosec}\theta$ as θ goes from 0 to $-180°$.

8. Solve
$$\sin 2x + \cos 2x + \sin x - \cos x = 0$$

CLXVI.

1. Solve the equation $5x + 2y = 58$ in positive integers.

2. Express 17161 in the duodenary scale, and divide it by te in that scale.

3. Distinguish between permutations and combinations. If 60 people are waiting for a train which can only take 56, in how many different ways may they be taken? In how many of these groups would the same man get a seat?

4. Solve the equations :—
$$\text{(i.)} \; 2x(x+b)(a+c) = (a+b)\{(2a+c)x + ac\},$$
$$\text{(ii.)} \; 3x^2y - 2xy^2 = 5 = 10x^3 - 15xy^2,$$

$$\text{(iii.)}\begin{cases} 2\left(y+\frac{1}{z}\right)\left(z+\frac{1}{y}\right)=9, \\ 3\left(z+\frac{1}{x}\right)\left(x+\frac{1}{z}\right)=16, \\ 6\left(x+\frac{1}{y}\right)\left(y+\frac{1}{x}\right)=49. \end{cases}$$

5. Write down the general value of θ when

$$\sin\theta = \tfrac{1}{2}.$$

6. If $\sin\alpha = \tan\beta$,

prove that $\sin(\alpha-\beta)\cos\beta = \sin 2\beta \sin^2\frac{\alpha}{2}$.

7. Eliminate θ from the equations :—

$$a\sin\alpha - b\cos\alpha = 2b\sin\theta,$$
$$a\sin 2\alpha - b\cos 2\theta = a.$$

8. Inscribe a square of given size in a given square.

CLXVII.

1. If a, b, c, d, are in continued proportion then $a : b+d :: c^3 : c^2d+d^3$.

2. Simplify

$$\frac{(4+\sqrt{15})^{\frac{3}{2}}+(4-\sqrt{15})^{\frac{3}{2}}}{(6+\sqrt{35})^{\frac{3}{2}}-(6-\sqrt{35})^{\frac{3}{2}}}.$$

3. What is meant by a scale of notation?
Find the area of a room 12 ft. 8 in. by 11 ft. 10 in. by the use of the duodecimal scale.

4. Solve the equations :—

(i.) $(x^2-9)^2 = 3+11(x^2-2)$,

(ii.) $\dfrac{x+1}{2}+\dfrac{y-1}{3}=8, \quad \dfrac{x-1}{3}+\dfrac{y+1}{2}=9,$

(iii.) $x^4-1=0$.

5. Prove the Binomial Theorem for a positive integral index. Expand

$$(3x+2a)^{10}, \quad \text{and} \quad (2x-5a)^{-5}$$

to 5 terms.

6. In any triangle prove that

(i.) $\tan\dfrac{B-C}{2}=\dfrac{b-c}{b+c}\cot\dfrac{A}{2}$,

(ii.) the area $=\dfrac{a^2}{2}\sin B\sin C\operatorname{cosec}(B+C)$.

7. Express in the centesimal method

(i.) the sum of the angles of a quindecagon,

(ii.) one of the angles, when the figure is regular.

8. Construct a triangle, having given the base, the vertical angle, and the point in the base on which the perpendicular falls.

CLXVIII.

1. Find the square root of

$$(a^2+b^2+c^2)(x^2+y^2+z^2)-(cy-bz)^2-(az-cx)^2-(bx-ay)^2.$$

2. Solve the equations :—

(i.) $\begin{cases}(x+5)(y+7)=(x+1)(y-9)+112, \\ 2x+5=3y-4,\end{cases}$

(ii.) $x-\dfrac{x^3-8}{x^2+5}=2$.

3. If α, β are the roots of

$$ax^2+bx+c=0,$$

find the equations whose roots are

(i.) α^2, β^2,

(ii.) $\alpha(1+\beta)$, $\beta(1+\alpha)$.

4. Assuming the law of the expansion of a binomial to hold for $(1+x)^n$, prove that it holds for $(1+x)^{n+1}$, n being a positive integer.

Apply the Binomial Theorem to find the value of 99^5.
What is the sum of the coefficients in the expansion of $(a+b+c)^8$?

5. In consequence of a defect in a clock, the minute hand points 2·5 minutes to the left of 12, when the hour hand points to 12. What will be the position of the hour hand, when the minute hand points to 3?

6. Show, by the "ratio definitions," which of the trigonometrical ratios are never < 1, and which can be $>$ or < 1.

7. A and B are two points in a plane, 32 feet apart. The angles of elevation of a tower at C, in a line with AB, are $\cot^{-1}\frac{2}{5}$ at A, and $\cot^{-1}\frac{2}{3}$ at B. Find the height of the tower.

8. Describe a circle to touch a given straight line, and pass through two given points.

CLXIX.

1. Solve the equation

$$2^{2x} \,.\, 7^{3x} = 1882384$$

by logarithms.

2. Find the sum of the coefficients of $(1+x)^n$.

3. Solve

$$\text{(i.)} \quad \begin{cases} 2xy + 2x + 5y = 73, \\ 3xy + 7x + 3y = 103. \end{cases}$$

$$\text{(ii.)} \quad x + y = 7xy, \quad x^2 + y^2 = 25x^2y^2.$$

4. If a, b, c are in A.P. and b, c, d in H.P., then a, $\frac{c^2}{d}$, c are in H.P., and $\frac{1}{b}$, $\frac{b}{ad}$, $\frac{1}{d}$ in A.P.

5. Divide

$$x + y + z - 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} \quad \text{by} \quad x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}}.$$

6. Simplify

$$\text{(i.)}\ \frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c}+1.$$

$$\text{(ii.)}\ \frac{x^4-5x^2+4}{x^2+1}\times\frac{\frac{1}{x}-\frac{1}{x+2}}{1-\frac{1}{x^2}}\div\frac{2-\frac{4}{x}}{1+\frac{1}{x^2}}.$$

7. Find the angle, whose cosine is to its tangent as 3 is to 2.

8. Find the trigonometrical ratios for an angle of $22\frac{1}{2}°$.

CLXX.

1. Simplify

$$\frac{x^6-y^6}{x^4+x^2y^2+y^4}\times\frac{x^2+y^2}{x^2+2xy+y^2}\times\frac{x+y}{x^3-y^3},$$

and find the L.C.M. of

$$x^3+x^2-4x-4, \quad \text{and} \quad x^3+6x^2+11x+6.$$

2. Solve the equation

$$ax^2+bx+c=0,$$

and find the forms which the roots assume

(i.) when $a=0$,

(ii.) when $a=0$ and $b=0$.

Solve the equations :—

(α) $x^2+\sqrt{4x^2+24x}=24-6x$,

(β) $x^3+y^3=a^3, \quad x+y=b.$

3. Find the number of Combinations of n things, r together.

A crew of 8 rowers and 1 coxswain has to be formed out of 12 men, 9 of whom can row but not steer, and 3 can steer but not row; in how many ways can it be done? If one of the 3 could both row and steer, in how many ways could it be done?

4. Find the value of

$$\frac{x^3 + 3x^2 - 20}{x^4 - x^2 - 12}$$

when $$x = 2,$$

and multiply

$$1 + x^{\frac{1}{4}} + x^{\frac{1}{2}} \quad \text{by} \quad 1 - x^{\frac{1}{4}}.$$

5. Prove that in any triangle

(i.) $b^2 = a^2 + c^2 - 2ac \cos B$,

(ii.) $a \sin (B - C) + b \sin (C - A) + c \sin (A - B) = 0$.

6. Solve the equation

$$2 \tan^2\theta = \sec^2\theta.$$

7. Given $\log 20{\cdot}867 = 1{\cdot}3194600$, $\log 20{\cdot}866 = 1{\cdot}3194392$, find the number whose logarithm is $-1{\cdot}6805452$.

8. Divide a given arc of a circle into two parts, so that the chords shall be in a given ratio.

CLXXI.

1. Multiply

$$x^{\frac{1}{2}} + 2x^{\frac{1}{4}} + 2 \quad \text{by} \quad x^{\frac{1}{2}} - 2x^{\frac{1}{4}} + 2,$$

and divide

$$a^2b^{-2} + b^2a^{-2} + 1 \quad \text{by} \quad ab^{-1} + ba^{-1} - 1.$$

2. Solve

(i.) $\dfrac{a}{x} + \dfrac{x}{a} = \dfrac{b}{x} + \dfrac{x}{b}$,

(ii.) $x^2 + y^2 = 169 = 5x + 12y$.

3. Enunciate the Binomial Theorem.
Show that the coefficient of the middle term of $(1 + x)^{2n}$ is the sum of the coefficients of the two middle terms in the expansion of $(1 + x)^{2n-1}$.

4. A man invests £500, part at 5 per cent. and part at 3 per cent.; he thus gets $4\frac{1}{2}$ per cent. on the whole. Find the amounts invested.

5. Prove that a ratio of greater inequality is diminished, and one of less inequality increased, by adding the same number to each of its terms.

If $$a : a-b :: c : c-d$$
then $$a+b : b :: c+d : d.$$

6. Find the value of

(i.) $\log_{100} \cdot 01$;

(ii.) $\sqrt[5]{\dfrac{(5\cdot 42580)^2}{(\cdot 8300204)^3}}$.

7. Solve the equations :—

(i.) $\sin^4\theta - \cos^4\theta = 1$,

(ii.) $\tan(\cot x) = \cot(\tan x)$.

8. Find the value of

$$\cot^{-1}\frac{ab+1}{a-b} + \cot^{-1}\frac{bc+1}{b-c} + \cot^{-1}\frac{ca+1}{c-a}.$$

CLXXII.

1. If a, b, c, d are in continued proportion, prove that

(i.) $a : d :: a^3 : b^3$,

(ii.) $\dfrac{2a+3d}{3a-4d} = \dfrac{2a^3+3b^3}{3a^3-4b^3}$.

2. Prove that any number in the denary scale, diminished by the sum of its digits, is divisible by 9.

State the corresponding theorem for *any* scale.

3. What is the ninth term in the expansion of

$$\left(3x - \frac{y}{2}\right)^{-\frac{3}{4}}?$$

4. Find the G.C.M. of

$$3x^3 - 4x^2 - x - 14 \quad \text{and} \quad 6x^3 - 11x^2 - 10x + 7;$$

and the L.C.M. of

$$x^2 - 4, \quad x^2 - 5x + 6, \quad x^3 - 8.$$

5. Prove that

(i.) $\sin(A-B)=\sin A\cos B-\cos A\sin B$,

(ii.) $\sin^{-1}\sqrt{\dfrac{x}{a+x}}=\tan^{-1}\sqrt{\dfrac{x}{a}}=\dfrac{1}{2}\cos^{-1}\dfrac{a-x}{a+x}$.

6. If $\sec^2\alpha=\frac{4}{3}$,
find an expression for all angles which have the same tangent as α.

7. In any triangle, prove that

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c},$$

and hence that

$$\frac{\sin B\sin C+\sin C\sin A+\sin A\sin B}{bc+ca+ab}=\frac{\sin^2 A}{a^2}.$$

8. In any quadrilateral, the squares on the diagonals are together equal to twice the sum of the squares on the straight lines joining the middle points of opposite sides.

CLXXIII.

1. Sum to n terms the Series whose n^{th} terms are

(i.) $2n-1+2^n$,

(ii.) $(n-1)x^n$,

(iii.) $n(n+2)$.

2. Find the greatest coefficient in the expansion of $(1+x)^{2n}$, and the sum of all the coefficients preceding it.

3. Find the number of permutations of n things, taken all together, of which p, q, r are alike.

Five times the number of permutations of n things all together, of which $n-2$ are alike, $=6$ times the number of combinations of n different things, four together. Find n.

4. If
$$a : \frac{b^2}{c} = c : \frac{d^2}{e} = e : \frac{f^2}{a},$$
show that
$$(a+c+e)(b+d+f) = (b^2c^{-1} + d^2e^{-1} + f^2a^{-1}) \times (acb^{-1} + ced^{-1} + aef^{-1}).$$

5. Solve the equations :—
$$\text{(i.)}\ \sin^2 n\,\theta - \sin \overline{n-1}\,\theta = \sin^2\theta,$$
$$\text{(ii.)}\ \tan\theta + \cot\theta = \frac{4}{\sqrt{3}}.$$

6. Find the circular measure of $(x^{\circ}.y' - z^{g})$.

7. Show that
$$\text{(i.)}\ \sin(A + 45^{\circ}) + \sin(A + 135^{\circ}) = \sqrt{2}\cos A,$$
$$\text{(ii.)}\ 2\operatorname{cosec} 4\alpha + 2\cot 4\alpha = \cot\alpha - \tan\alpha.$$

8. ABC, DEF are triangles, having the angle A equal to the angle D, and AB equal to DF. Show that the areas of the triangles are as $AC : DE$.

CLXXIV.

1. Water is poured into a cylindrical reservoir, 20 feet in diameter, at the rate of 400 gallons a minute. If a gallon of water measure $277\frac{1}{4}$ cubic inches, how many inches does the water rise per minute? $(\pi = \frac{22}{7}.)$

2. Given $\log 2 = \cdot 3010300$, $\log 3 = \cdot 4771213$, find logs. of 72, 135, and $\sqrt[3]{\frac{1}{6}}$.

3. What is the coefficient of x^5 in the expansion of
$$(1-x)^{-\frac{3}{2}}?$$

4. The sum of a number formed by 2 digits, and the number formed by reversing the digits is 121; and the product of the digits is 28. Find the number.

5. Simplify
$$\frac{3-\sqrt{-2}}{2-\sqrt{-3}} + \frac{3+\sqrt{-2}}{2+\sqrt{-3}}.$$

6. Sum

(i.) $1\frac{1}{4}+2\frac{1}{2}+3\frac{3}{4}+\ldots\ldots$ to n terms,

(ii.) $12+4+1\frac{1}{3}+\ldots\ldots\ldots$ to 8 terms, and to infinity.

7. Solve the equations :—

$$\text{(i.)}\ \sin 2\theta=\cos\theta,$$

$$\text{(ii.)}\ \operatorname{cosec}\phi=\operatorname{cosec}\frac{\phi}{2}.$$

8. Construct a triangle, having given the base, vertical angle, and altitude.

CLXXV.

1. Expand

$$(1-x)^{-\frac{p}{q}}$$

to 4 terms, and give the r^{th} term.

2. Find the number of combinations of n things r at a time.

How many things are contained in each combination of the set which gives the greatest possible number of combinations of m things?

The greatest possible number of m things is double the greatest possible number of $m-1$ things, if m be even.

3. Solve the equations :—

(i.) $x^2-8x+15=0,$

(ii.) $x+\dfrac{6}{y}=4,\quad y+\dfrac{4}{x}=5,$

(iii.) $$\frac{(x-yz)^2}{(1-y^2)(1-z^2)}=a^2,\quad \frac{(y-zx)^2}{(1-z^2)(1-x^2)}=b^2,$$
$$\frac{z-xy}{(1-x^2)(1-y^2)}=c^2,$$

(iv.) $$x+2y-xy^2+\sqrt{3}(1-2xy-y^2)$$
$$=y+2x-x^2y+(2+\sqrt{3})(1-2xy-x^2)=0.$$

4. Prove that

(i.) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$,

(ii.) $\cos(36° + a)\cos(36° - a) + \cos(54° + a)\cos(54° - a)$
$= \cos 2a$.

5. Find the radius of a circle described about a triangle, and if twice the square on its diameter is equal to the sum of the squares on the three sides, show that the triangle is right angled.

6. In the triangle ABC, $b = 5$, $c = 4$, $A = 60°$. Find B and C.

7. Two circles, whose radii are a and b, touch externally. Show that the angle between their common tangents is

$$2\tan^{-1}\tfrac{1}{2}\left(\sqrt{\frac{a}{b}} \sim \sqrt{\frac{b}{a}}\right).$$

8. Solve the equation

$$5^x + 5^{-x} = 125\cdot008.$$

CLXXVI.

1. (i.) Find the sum of the first n odd numbers,

(ii.) Sum $\frac{2}{3} - 1 + \frac{3}{2} - \ldots$ to n terms,

(iii.) Find two Harmonic Means between 2 and ∞.

2. Solve the equations :—

$$\text{(i.)} \begin{cases} \dfrac{x+2y}{7} = \dfrac{3y+4z}{8} = \dfrac{5x+6z}{9}, \\ x + y - z = 126, \end{cases}$$

$$\text{(ii.)} \begin{cases} (a+b)x + (a-b)y = 4ab, \\ (a-b)x + (a+b)y = 2a^2 - 2b^2, \end{cases}$$

$$\text{(iii.)} \sqrt{x^2 - 8x + 31} + (x-4)^2 = 5.$$

3. Prove the Binomial Theorem for positive integral indices.

Find the 6th term of $(1 - 2x)^{\frac{3}{2}}$.

4. Twelve men are to be divided into three parties containing 3, 4, 5 men respectively. In how many ways can it be done?

5. Prove that if $\cos 3A$ be given, there are three, and only three, values of $\cos A$;
and find $\cos A$ if

$$\cos 3A = \cos 135^\circ.$$

6. Explain what is meant by circular measure.
If a right angle be reckoned at 100, find the value on the same scale of the angle whose circular measure is $\frac{1}{2}$.

7. Find the values of $\sin 15^\circ$, and $\cos 15^\circ$.

8. Given the middle points of the sides of a triangle, construct the triangle.

CLXXVII.

1. Expand $(a-x)^6$; and $(1-2x)^{\frac{5}{8}}$ to 6 terms.

2. Simplify

$$\text{(i.)}\ \frac{\sqrt{5}+\sqrt{3}}{\sqrt{4+\sqrt{15}}},$$

$$\text{(ii.)}\ \frac{a-b}{a^{\frac{1}{3}}-b^{\frac{1}{3}}}-\frac{a+b}{a^{\frac{1}{3}}+b^{\frac{1}{3}}}.$$

3. Solve

$$\text{(i.)}\ \frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}-\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}=8x\sqrt{x^2-3x+2},$$

$$\text{(ii.)}\ x^2y^2=c^3z,\quad x^2z^2=b^3y,\quad y^2z^2=a^3x.$$

4. Prove that any number is divisible by 11 if the sums of the alternate digits are the same.

5. Prove that the coefficient of x^n in the expansion of

$$\frac{4x-6}{x^3-6x^2+11x-6}$$

is

$$1+\frac{1}{2^n}-\frac{1}{3^n}.$$

(Break it up into its partial fractions, and then expand by Binomial Theorem.)

6. Determine the values of the Trigonometrical Ratios for an angle of 5700°.

7. Prove that

$$\sin 4\alpha \tan^4\alpha + 4\tan^3\alpha + 2\sin 4\alpha \tan^2\alpha - 4\tan\alpha + \sin 4\alpha = 0.$$

8. If the angle C of a triangle ABC be 60°, the line joining the orthocentre and circumcentre of the triangle being produced, cuts BC at an angle of 60°.

(Orthocentre, intersection of perpendiculars from the angles on the opposite sides. Circumcentre, centre of circumscribed circle.)

CLXXVIII.

1. Sum the Series :—

(i.) $16\frac{1}{2} + 14 + 11\frac{1}{2} + \ldots\ldots$ to 14 terms,

(ii.) $1 + 2r + 3r^2 + 4r^3 + \ldots$ to n terms.

If the Arithmetic Mean between two numbers $= 1$, the Harmonic Mean is the square of the Geometric Mean.

2. Prove that a quadratic equation has two, and only two, roots.

Find the equation whose roots are the cubes of the roots of

$$2x(x - a) = a^2.$$

3. Find the fifth power of

$$a + \sqrt{-x},$$

and show that it will be real if

$$x = (5 \pm 2\sqrt{5})a^2.$$

4. Prove that

$$\sin 18° = \frac{\sqrt{5} - 1}{4},$$

and $$\sin^2 30° = \sin 18° \sin 54°.$$

Show that in any circle the chord of an arc of $108°$ is equal to the sum of the chords of arcs of $36°$ and $60°$.

5. Find an expression for $\tan 3A$ in terms of $\tan A$.
Show that
$$\tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A.$$

6. It is proposed to add to a square lawn of 58 feet side two circular ends, the centre of each circle being the intersection of the diagonals of the square. How much turf will be required?

7. Find the values of $\log_{10}\cdot 004$, and $\log_4 100$.
How many terms of the Series
$$\cdot 04, \quad \cdot 08, \quad \cdot 16, \quad \cdot 32, \quad \text{etc.}$$
will amount to 41943?

8. If a quadrilateral have one vertex at the centre, and the others on the circumference of a circle, one of the three angles on the circumference will be equal to the sum of the other two.

CLXXIX.

1. Show that 123 is a perfect cube in the quaternary scale.

2. Simplify
$$\text{(i.)}\ \sqrt{17 + 12\sqrt{2}},$$
$$\text{(ii.)}\ \sqrt{a - c + 2\sqrt{ab + bc - ca - b^2}}.$$

3. Solve
$$\text{(i.)}\ 2^{x^2} : 2^{2x} :: 8 : 1,$$
$$\text{(ii.)}\ \sqrt{x+y} + \sqrt{x-y} = \sqrt{c}, \quad b(x-a) + a(b-y) = 0,$$
$$\text{(iii.)}\ \sqrt{2x^2 + 3x + 9} - \sqrt{2x^2 + 3x - 2} = 1.$$

4. Write down the general term in the expansion of $(x+a)^n$, and find the coefficient of x^5 in the expansion of
$$(1 + 2x + 3x^2)^7.$$

5. A and B run a race. A runs 1 mile an hour slower

than B, and starts first by 14 minutes. They both arrive at the 7-mile stone together. Find their rates.

6. Find the circular measure of the angle of a regular n-sided figure.

7. Solve

$$\text{(i.)}\ \cot\theta = 2 - \tan\theta,$$
$$\text{(ii.)}\ \cos 3\theta + \cos\theta = 0.$$

8. Prove that

$$\frac{\sin\beta\cos\alpha(\tan\alpha + \tan\beta)}{1 - \cos(\alpha+\beta)} + \frac{\sin\frac{1}{2}(\alpha-\beta)}{\cos\beta\sin\frac{1}{2}(\alpha+\beta)} = 1.$$

CLXXX.

1. Evaluate $\sqrt{13\sqrt{5} \div \sqrt{7}}$, and $\sqrt{13 + \sqrt{5} - \sqrt{7}}$, by logarithms.

2. Sum to 10 terms

$$1 + \sqrt{10} + 10 + \ldots\ldots,$$

and to n terms

$$a + 3a^2 + 5a^3 + \ldots\ldots$$

3. The p^{th} term of a series is $p(p+1)$. Write down the first 5 terms, and find the sum of n terms.

4. Define the *convergents* of a continued fraction, and state their principal properties.

5. How can the ratio of two straight lines be determined readily in the form of a continued fraction by the use of a pair of compasses?

6. What is the meaning of a^0? Give reasons for your answer.

7. Bring $48°\ 6'\ 30''$ to circular measure, and $\dfrac{\pi}{10}$ to grades.

8. If θ be the circular measure of an angle less than a right angle, prove

$$\sin\theta < \theta < \tan\theta.$$

CLXXXI.

1. Stock to the amount of £2700 was sold at 90, and re-invested in the 5 per cents. at 125. What will be the annual income? Work by logarithms.

2. Sum

   (i.) $3\frac{1}{3} + 4\frac{1}{2} + 5\frac{2}{3} + \ldots\ldots$ to 19 terms,

   (ii.) $18 + 12 + 8 + \ldots\ldots$ to 5 terms.

3. If x, y, z are three positive numbers in continued proportion, and if $x + y : x - y :: 2y : 3z$, show that $$y = 3z.$$

4. Simplify

   (i.) $$\frac{\sqrt{6 + 2\sqrt{5}}}{\sqrt{5} - 1} + \frac{\sqrt{6 - 2\sqrt{5}}}{\sqrt{5} + 1},$$

   (ii.) $a^{\frac{2}{3}}(b^{\frac{1}{2}}c^{-2})^{\frac{1}{2}} \div a^{\frac{1}{3}}b^{\frac{1}{4}}c.$

5. Solve

   (i.) $15x + 19y = 132$ in positive integers,

   (ii.) $\sqrt{x - 2y} - \sqrt{x - 3y} = 2; \quad 2x - 5y = 34.$

6. Reduce
$$\sin\alpha + \sin\beta - \cos\alpha\sin(\alpha + \beta)$$
   to the form of a product.

7. Prove that

   (i.) $$(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2\frac{A - B}{2},$$

   (ii.) $$\sin 22\tfrac{1}{2}^\circ = \frac{\sqrt{2 - \sqrt{2}}}{2} \quad \text{and} \quad \cos 22\tfrac{1}{2}^\circ = \frac{\sqrt{2 + \sqrt{2}}}{2}.$$

8. What is the locus of a point whose distance from one given point is double its distance from another given point? To be done geometrically and analytically if possible.

CLXXXII.

1. Find the square root of $55 - 7\sqrt{24}$.
2. Solve

$$\text{(i.)} \quad \sqrt{x + \sqrt{x^2 - 4}} = \sqrt{\frac{x+2}{2}} + \sqrt{2(x-2)},$$

$$\text{(ii.)} \quad y + \sqrt{x^2 - 1} = 2, \quad \sqrt{x+1} - \sqrt{x-1} = \sqrt{y},$$

$$\text{(iii.)} \quad \frac{x-1}{x+2} + \frac{3x}{x-2} = \frac{3x^2+2}{x^2-4}.$$

3. If $a : b :: c : d :: e : f$,
   prove that $a : b :: a + c + e : b + d + f$.
4. In an A.P., difference $= 2$, and number of terms $=$ second term. Find the first term, so that the sum $= 35$.
5. Expand

$$(a - 3b)^{-\frac{10}{3}} \text{ to 5 terms,}$$

   and show that if $a = 5$, and $b = 1$, the fourth term is greater than either the third or fifth.
6. Is

$$\sec^2\theta = \frac{4ab}{(a+b)^2}$$

   a possible equation, if a and b are unequal?
7. Find the values of

$$\sin(270° - A), \quad \cos(540° + A), \quad \tan(810° - A).$$

8. Show how to find the centre of gravity of a number of heavy particles.

CLXXXIII.

1. Sum

$$\sqrt{1 + \sqrt{2}} + 1 + \sqrt{\sqrt{2} - 1} + \sqrt{2} - 1 \ldots \text{ to } n \text{ terms.}$$

   Test your answer by putting $n = 1$.
2. Prove the Binomial Theorem for positive integral indices.

   Expand $(2 - 3x)^{-\frac{2}{3}}$ to 4 terms.

3. Transform $112t3$ from the duodenary to the septenary scale.

4. Find the value of $\sqrt[15]{(86)^4}$ to 6 places of decimals.

5. Solve

$$\text{(i.)}\ x+\sqrt{x^2-3x+4}=\frac{x^2}{3}+2.$$

$$\text{(ii.)}\ \sin^3\theta+\cos^3\theta=0.$$

6. Form the equations :—

 (i.) Of a line perpendicular to $3x-y=7$, and passing through the origin,

 (ii.) A line through the points $(-1, 2)$ and $(3, -2)$; and

 (iii.) Find the point of intersection of (i.) and (ii.).

7. Find the values of

$$\sin 810^\circ, \quad \cos 1560^\circ, \quad \text{and} \quad \tan 1125^\circ.$$

8. The resultant of two forces at right angles is 51 lb.; one of the components is 24. Find the other, and the angles which the direction of the resultant makes with the lines of action of the forces.

CLXXXIV.

1. Find, by logarithms, the value of

$$\frac{1}{73^{-\frac{1}{7}}}$$

to 6 places of decimals.

2. Multiply

$$x^{\frac{1}{2}}+y^{\frac{1}{2}}-x^{-\frac{1}{2}}y \quad \text{by} \quad xy^{-\frac{1}{2}}-x^{\frac{1}{2}}+y^{\frac{1}{2}},$$

and divide

$$a-bx \quad \text{by} \quad c-dx$$

to 4 terms of the quotient.

What would be the coefficient of x^n in the quotient?

3. Simplify

$$\frac{\left(1+\frac{x}{y}\right)^m\left(1-\frac{y}{x}\right)^n}{\left(1+\frac{y}{x}\right)^n\left(1-\frac{x}{y}\right)^m}.$$

4. Rationalise the denominator of

$$\frac{x+2+\sqrt{x^2-4}}{x+2-\sqrt{x^2-4}}.$$

5. How much tea at 3s. 9d. per lb. must be mixed with 45 lbs. at 3s. 4d., that the mixture may be worth 3s. 6d. per lb.?

6. If the altitude of an equilateral triangle be 12 inches, find the length of a side, and the area of the triangle.

7. Solve

$$\begin{aligned} &\text{(i.)}\ 2x^2-3x-5=0. \\ &\text{(ii.)}\ x-y=5, \quad x^3-y^3=35, \\ &\text{(iii.)}\ \cos^2\theta=\sin^2\theta, \\ &\text{(iv.)}\ \sin\theta+\cos\theta=\sqrt{2}. \end{aligned}$$

8. A stone let fall down a well, strikes the water after 3 secs. Find the depth of the surface of the water.

CLXXXV.

1. If ${}_nC_r$ denote the number of combinations of n things r together, show by general reasoning that

$${}_nC_r={}_nC_{n-r}.$$

Also show that

$${}_{n+2}C_{r+1}={}_nC_{r+1}+2{}_nC_r+{}_nC_{r-1}.$$

2. Find the first five terms of the expansion of

$$(1-3x)^{-\frac{2}{3}}.$$

Show that the coefficient of x^8 in the expansion of

$$(1+x+x^2+x^3)^{-4}$$

is 14.

3. Solve the equation
$$x^3 - x^2 - 8x + 12 = 0$$
which has two equal roots.

4. The angular altitude of a lighthouse seen from a point on shore is 12° 31′ 43″. From a point 500 feet nearer to it, it is 26° 33′ 55″. Find its height.

5. Solve the equations
$$\text{(i.)}\ \cos n\theta - \cos(n-2)\theta = \sin\theta,$$
$$\text{(ii.)}\ \cos^{-1}\frac{1-x^2}{1+x^2} + \tan^{-1}\frac{2x}{1-x^2} = \frac{4\pi}{3}.$$

6. Express the volume of a cone in terms of the radius of the base, and the vertical height.

If the diameters of the circular ends of a frustrum of a cone be 4 inches and 6 inches, and the volume of the frustrum 209 cubic inches, find the height (i.) of the frustrum, (ii.) of the cone.

7. Calculate the integral part of $(190815)^{\frac{11}{10}}$; and find the number of integers in $(2{\cdot}1)^{200}$.

8. If a straight line meet the sides BC, CA, AB of a triangle in X, Y, Z, respectively, then
$$\frac{BX \,.\, CY \,.\, AZ}{CX \,.\, AY \,.\, BZ} = 1.$$

CLXXXVI.

1. Solve the equations :—
$$\text{(i.)}\ x^3 + \frac{1}{x^3} = \frac{65}{8},$$
$$\text{(ii.)}\ x + y = 7, \quad x^3 + y^3 = 91,$$
$$\text{(iii.)}\ 7x - \frac{\sqrt{3x^2 - 8x + 1}}{x} = \left(\frac{8}{\sqrt{x}} + \sqrt{x}\right)^2.$$

2. In laying a submarine cable it is found that the whole length employed is 10 per cent. more than the distance

between the places; and that in the first third of the distance, two miles more than one-third of the whole cable had been used, and the slack was 11 per cent. of the distance run. Find the distance between the two places.

3. Show how to transfer a number from the common scale to the scale whose radix is r.

   Express 1866 in the binary and duodenary scales.

4. If p be the difference between any fraction and its reciprocal, q the difference between their squares, find the relation between p and q.

5. Prove that

   (i.) $\tan^2\frac{A}{2}=\frac{\sec A-1}{\sec A+1}$,

   (ii.) $(2\cos A+1)(2\cos A-1)(2\cos 2A-1)$
   $$=2\cos 4A+1.$$

   How may (ii.) be extended?

6. Given
$$\cos 3x=-\frac{3\sqrt{3}}{4\sqrt{2}},$$
   prove that the three values of $\cos x$ are
$$\sqrt{\frac{3}{2}}\sin\frac{\pi}{10},\ \sqrt{\frac{3}{2}}\sin\frac{\pi}{6}-\sqrt{\frac{3}{2}}\sin\frac{3\pi}{10}.$$

7. The sides of a triangle are 20, 21, 29; find the largest angle and the area.

8. Prove, analytically, that the bisectors of an angle and its adjacent supplement are at right angles.

CLXXXVII.

1. Find the value of
$$\frac{1089\times\cdot01881\times\cdot405}{\sqrt[3]{(729)^4}}.$$

2. Simplify
$$\frac{\sqrt{12+6\sqrt{3}}}{\sqrt{3}+1}.$$

3. Solve the equations :—

(i.) $\dfrac{x}{x+1}+\dfrac{x+1}{x+2}=\dfrac{x-2}{x-1}+\dfrac{x-1}{x}$,

(ii.) $2x^2+6x=5-\sqrt{x^2+3x-1}$,

(iii.) $\begin{cases} x^2+y^2=20, \\ x+y=\sqrt{2xy}+2. \end{cases}$

4. A circular disc of cardboard one foot in diameter is divided into six equal sectors. A circle is inscribed in each sector. If the circles are cut out, find the area of cardboard remaining.

5. Prove that
$$m(a \pm b) = ma \pm mb.$$
To what proposition in Euclid does this correspond?

6. Find the square root of
$$x^{\frac{5}{3}} - 4x^{\frac{4}{3}} + 2x^{\frac{7}{6}} + 4x - 4x^{\frac{5}{6}} + x^{\frac{2}{3}}.$$

7. Find the number of degrees in the angle subtended at the centre of a circle whose radius is 20 feet, by an arc 5 in. long.

8. Forces P, $P\sqrt{3}$, and $2P$, act on a particle. Find the angle between their directions if they maintain equilibrium.

CLXXXVIII.

1. Prove the Binomial Theorem for a positive integral index.

2. Find the first four terms of
$$\left(1-\frac{x}{3}\right)^{-3}$$
and the 15th term.

3. Show that, if the last 2 digits of a number are divisible by 4, the whole number must be so divisible.
4. Solve

$$\text{(i.)} \quad \sqrt{x} + \frac{1}{\sqrt{y}} = 4 \cdot 2,$$
$$\sqrt{y} + \frac{1}{\sqrt{x}} = 5 \cdot 25,$$
$$\text{(ii.)} \quad x^2 + 2ax = a^3,$$
$$\text{(iii.)} \quad x^3 - 1 = 0.$$

Verify all your solutions.
5. Express in French measure $70° \; 20' \; 6''$.
6. Solve

$$3 \cot\left(\frac{\pi}{4} + a\right) = \cot\left(\frac{\pi}{4} - a\right).$$

7. If R be the resultant of two forces P and Q acting on a particle, and S the resultant of P and R, show that the resultant of S and $Q = 2R$.
8. Find the equation of the straight line joining the origin to the point of intersection of

$$x - 3y = 4 \quad \text{and} \quad 5x - y + 2 = 0.$$

CLXXXIX.

1. Find the square root of $\frac{35}{36} - \sqrt{\frac{2}{3}}$.
2. If $\frac{p}{q}, \frac{p'}{q'}$ are two consecutive convergents, prove that

$$p'q - pq' = \pm 1.$$

3. Find the square root of 24 to 5 places of decimals by the Binomial Theorem.
4. Solve

$$\text{(i.)} \quad \frac{x^2}{6} + \frac{27}{2} = \frac{1}{3}\sqrt{13(x^2 - 30x + 81)} + 5x,$$

$$\text{(ii.)} \quad x^{\frac{1}{2}} + y^{\frac{1}{2}} = 5, \quad x^{-\frac{1}{2}} + y^{-\frac{1}{2}} = \frac{5}{6},$$

(iii.) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0, \quad \frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4, \quad \frac{5}{x}+\frac{1}{y}+\frac{1}{z}=20.$

5. What is meant by the Arithmetic and Geometric Means between a and b? Find them, and prove the first is the greater.

6. If the space described in falling for 11 secs. be 556·6 feet, find the acceleration.

7. Find the log. of $81\sqrt{27}$ to the base $\sqrt{3}$.

8. If the angle subtended in a circle by an arc of 1 inch be 15°, what is the radius?

CXC.

1. Find the sum of n terms of an A.P.

Sum 9 terms of an A.P. of which the middle term is 9.

2. If a, b, c, d are in G.P., show that

$$a^2+d^2 > b^2+c^2;$$

and if x be greater than 9, then

$$\sqrt{x} > \sqrt[3]{x+18}.$$

3. Solve the equations :—

(i.) $\begin{cases} x-y=5, \\ x^3-y^3=35, \end{cases}$

(ii.) $\sqrt{x+\sqrt{x^2-4}}=\sqrt{\frac{x+2}{2}}+\sqrt{2(x-2)}.$

4. A farmer sells a certain number of bushels of wheat at 7s. 6d. per bushel, and 200 bushels of barley at 4s. 6d. per bushel. The average price is 5s. 6d. per bushel. How much wheat did he sell?

5. If

$$\frac{\sin^4\theta}{\sin^2\alpha}+\frac{\cos^4\theta}{\cos^2\alpha}=1,$$

prove that $\theta=n\pi\pm\alpha$

where n is any integer.

6. Show that

$$\frac{2\cos 2^n\theta+1}{2\cos\theta+1}$$

$$=(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 2^2\theta-1)\ldots(2\cos 2^{n-1}\theta-1).$$

7. If θ be the circular measure of a small angle, prove that $\sin\theta$ nearly $=\theta-\dfrac{\theta^3}{6}$.

Given

$$\frac{\sin\theta}{\theta}=\frac{1013}{1014},$$

prove that $\theta=4°\ 24'$ nearly.

8. A, B, C are three points in a straight line, and D a point at which AB and BC subtend equal angles. Show that the locus of D is a circle. (Geometrically and Analytically.)

CXCI.

1. Find the number of permutations of n things taken r together.

If the number of permutations of n things, two together, is thrice the number of combinations of n things, three together, find n.

2. Solve the equation

$$x^2+px+q=0,$$

and show how to approximate to its roots by the Binomial Theorem.

3. Solve the equations :—

$$\text{(i.)}\ \begin{cases}(a-b)x+(a+b)y=a^2+b^2,\\ bx=ay,\end{cases}$$

$$\text{(ii.)}\ \begin{cases}x+2y+3z=14,\\ 2x+3y+z=3x+y+2z=11,\end{cases}$$

$$\text{(iii.)}\ \begin{cases}a_1x+b_1y+c_1z=d_1,\\ a_2x+b_2y+c_2z=d_2,\\ a_3x+b_3y+c_3z=d_3\end{cases}$$ by determinants.

4. Show how to find n Arithmetic Means between a and b.

Find the relation between a and b if the r^{th} mean between a and $2b = $ the r^{th} mean between $2a$ and b; n means being inserted in each case.

5. Prove that

$$\text{(i.)}\ 1 + \tan 2A \tan A = \sec 2A,$$

$$\text{(ii.)}\ 2\cot^{-1}x = \operatorname{cosec}^{-1}\frac{1+x^2}{2x},$$

the latter geometrically.

6. Solve the equation

$$\operatorname{cosec}\theta + \cot\theta = \sqrt{3}.$$

7. Find what value of θ between $0°$ and $180°$ will make $\cos(\alpha - \theta)\cos\theta$ have the greatest value. ($\alpha > 0°$ and $< 90°$.)

8. Describe a circle to pass through two given points, A and B, and to touch a given straight line.

Show that if it touches the given line at P, then P is the point in the line at which AB subtends the greatest angle.

CXCII.

1. Evaluate, using only the logs of 2 and 3,

$$\sqrt{\frac{3\log 1728}{1 + \frac{1}{2}\log \cdot 36 + \frac{1}{3}\log 8} \times \frac{1}{6}},$$

and, if possible, do it without using these logarithms.

2. Express $\sqrt{47}$ as a continued fraction, and find its first four convergents.

3. Solve (i.) $\sqrt{x^2 - 2x + 9} - \dfrac{x^2}{2} = 3 - x.$

$$\text{(ii.)}\ x + \sqrt{a^2 + x^2} = \frac{a^2 + a}{2\sqrt{a^2 + x^2}}.$$

4. Find the cube root of

$$1728x^6 + 1728x^4y^3 + 576x^2y^6 + 64y^9.$$

5. Multiply

$$a^{-\frac{2}{3}} + a^{-\frac{1}{3}} + 1 \quad \text{by} \quad \frac{1}{\sqrt[3]{a}} - 1,$$

and prove your answer.

6. Solve

$$\sin 9\theta + \sin 5\theta + 2\sin^2\theta = 1.$$

7. If two forces acting at right angles to each other be in the proportion of 1 to $\sqrt{3}$, and their resultant be 10 lbs., find the forces.

8. Prove that the bisectors of the angles of a triangle are concurrent (all meet in a point.) Analytically and geometrically.

CXCIII.

1. The ratio of the fifth term of the expansion of $(1+3x)^n$ to the seventh, is $3 : 7x^2$. Find the least value of n.

2. Prove that

$$\log_a b \times \log_b a = 1,$$

and expand $\log_e(1+x)$.

3. A bag contains n sovereigns and n shillings; show that the number of different ways in which they can be drawn out in succession, one at a time, is

$$\frac{1 \,.\, 3 \,.\, 5 \ldots (2n-1)}{|\!\underline{n}}2^n.$$

4. If a, b, c, d be in A.P.; a, e, f, d in G.P.; a, g, h, d in H.P. respectively; prove that

$$ad = ef = bh = cg.$$

5. Find the acute angles of a right-angled triangle, whose hypoteneuse is four times as long as the perpendicular on it from the right angle.

6. Given $a=2b$, $C=120°$, $\log 3 = \cdot 4771213$,
$L \tan 10° \, 53' = 9\cdot 2839070$, $diff. = 6808$,
find A and B.

7. A right prism on a triangular base, each of whose sides is 21 inches, is touched by a sphere on each of its five faces. Find the volume of the sphere, and the space between it and the prism.

8. The diagonal BD of a parallelogram $ABCD$ is divided in E, so that BE is one-third of ED, and AE, DC produced meet in F. Show that FC is double AB

CXCIV.

1. Solve the equations :—

$$\text{(i.)} \quad x^{\frac{1}{2}} + (4a+x)^{\frac{1}{2}} = 2(b+x)^{\frac{1}{2}},$$

$$\text{(ii.)} \quad \begin{cases} x - ay + a^2 z = a^3, \\ x - by + b^2 z = b^3, \\ x - cy + c^2 z = c^3. \end{cases}$$

2. Five persons are chosen by lot out of 10. In how many ways may this be done, and what is the chance that any particular person will be chosen ?

3. Write down the middle term in the expansion of

$$\left(\frac{x}{2} - 3y\right)^8.$$

4. What are the different ways in which a sum of £11 5s. may be paid in half-crowns and napoleons (16s.) ?

5. Solve the equations :—

$$\text{(i.)} \quad \cos x + \cos 7x = \cos 4x,$$

$$\text{(ii.)} \quad \cot \theta - \tan \theta = 2.$$

6. If $\alpha + \beta$ is constant, find the limits of

$$\sin \alpha \sin \beta.$$

7. If $A + B + C = \pi$, prove that

$$\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C.$$

8. Prove analytically that the sum of the distances of any point within a triangle from the angular points, is less than the sum of the sides of the triangle.

CXCV.

1. Solve the equations :—

$$\text{(i.)}\ x\sqrt{x^2+1}+\sqrt{x^2-1}=x,$$

$$\text{(ii.)}\begin{cases} x-y=24 \\ x^2+y^2=296. \end{cases}$$

$$\text{(iii.)}\begin{cases} \dfrac{x^3}{y}=108-x^4 \\ \dfrac{y^3}{x}=\dfrac{4}{3}-y^4. \end{cases}$$

2. In what scales of notation are 69 and 1253 represented by 234 and 2345 respectively ?

3. Prove that

$$\log_a b \times \log_b a = 1.$$

Given $\log_8 3 = m$, and $\log_{25} 24 = n$, find $\log_{10} 45$.

4. Find the first four convergents to $\sqrt{7}$.

5. Solve the equations :—

$$\text{(i.)}\ \tan^{-1}\frac{x-1}{x+1}+\tan^{-1}\frac{2x-1}{2x+1}=\tan^{-1}\frac{23}{36},$$

$$\text{(ii.)}\ 2\tan^{-1}(\cos\theta)=\tan^{-1}(2\operatorname{cosec}\theta).$$

6. Prove that in every triangle :—

(i.) $c = a\cos B + b\cos A$,

(ii.) $\cos 2A + \cos 2B - \cos 2C = 1 - 4\cos C\sin A\sin B$.

7. A man walking towards a tower on the edge of which is a flagstaff, observes that when he is c feet from the tower, the flagstaff subtends the greatest angle at his eye, and that this angle is α; prove that the height of the flagstaff is $2c\tan\alpha$. (Compare Ex. CXCI. 8).

8. Find the equation of the straight line joining $(a, 2a)$, and $(3a, 4a)$;

and find where this straight line meets the circle

$$x^2 + y^2 - 2ax = a^2.$$

CXCVI.

1. Find by logarithms the amount of £55,000 in fifteen years at 5 per cent. compound interest.
2. A bag contained £5 in shillings and half-crowns; after taking out 18 shillings and 4 half-crowns there were twice as many shillings as half-crowns left. How many were there of each?
3. If the first 2 terms of an A.P. and G.P. are the same, and the first term positive, then the third term of the G.P. is greater than the third term of the A.P.
4. Find the number of combinations of n things, r at a time. If the number of combinations of 16 things, r together : the number of combinations of 15 things, $\overline{r-2}$ together : : 52 : 3, find r.
5. In any triangle, prove that:—

$$\text{(i.)}\quad \cos A = \frac{b^2 + c^2 - a^2}{2bc},$$

$$\text{(ii.)}\quad \frac{\cos\frac{A}{2}}{\sqrt{a(s-a)}} = \frac{\cos\frac{B}{2}}{\sqrt{b(s-b)}} = \frac{\cos\frac{C}{2}}{\sqrt{c(s-c)}}.$$

6. What is the meaning of $\sin^{-1} m$?

Prove that

$$\sin^{-1}\frac{2mn}{m^2+n^2} + \sin^{-1}\frac{m^2-n^2}{m^2+n^2} = \frac{\pi}{2},$$

for all values of m and n.

7. Solve the equation

$$2\sin\theta\sin 3\theta - \sin^2 2\theta = 0.$$

8. In the triangle ABC, BC is bisected at E, and AB at G; AE and CG are produced to F and H, so that $EF = AE$, and $GH = CG$. Prove (analytically) that F, B, H, are in the same straight line.

CXCVII.

1. If $$(1+x)^n = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n,$$ prove that
$$c_0c_2 + c_1c_3 + \ldots + c_{n-2}c_n = \frac{\lfloor 2n}{\lfloor n+2 \; \lfloor n-2}.$$

Find the coefficient of $x^3y^2z^4$ in the expansion of
$$(x-y-z)^9.$$

2. Show that
$$\sqrt{a^2+4a} = a + \frac{2}{1+}\,\frac{1}{a+}\,\frac{1}{1+}\,\frac{1}{a+}\,\frac{1}{1+}\ldots\ldots$$

3. Find the coefficient of x^r in the expansion of
$$\frac{1-9x+32x^2}{(1-5x)^2(1+7x)}.$$

4. Solve the equations :—

(i.) $1 - \cos 2\theta = 2(\cos\alpha \cos\theta - \cos 2\alpha)$.

(ii.) $\sin(\cot^{-1}\frac{1}{2}) = \tan(\cos^{-1}\sqrt{x})$.

5. If the sines of the angles of a triangle be in the ratio $13 : 14 : 15$, find the ratio of the cosines.

6. Given $a = 2{\cdot}7402$, $b = {\cdot}7401$, $C = 59° \, 27' \, 5''$, solve the triangle.

7. The extremity of the shadow of a flagstaff, 6 feet high, standing on the top of a regular pyramid with a square base, just reaches a side of the base, and is distant 56 feet and 8 feet from the ends of that side. Find the sun's altitude, if the height of the pyramid be 34 feet.

8. The area of an equilateral triangle is 17320·5 square feet. About each angular point as centre, a circle is described with radius equal to half a side of the triangle. Find the area of the space included within the three circles.

CXCVIII.

1. Divide $x+x^{-1}+1$ by $x^{\frac{1}{2}}+x^{-\frac{1}{2}}-1$, and simplify

$$\left[(ab)^{\frac{1}{2}}+(ac)^{\frac{1}{2}}+(bd)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}\right]\times$$
$$\left[(ab)^{\frac{1}{2}}-(ac)^{\frac{1}{2}}-(bd)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}\right].$$

2. Solve the equations :—

(i.) $\sqrt{x+7}-\sqrt{x-7}=\sqrt{2x}$,

(ii.) $\begin{cases} x^2(x+y)=300, \\ y^2(x+y)=588, \end{cases}$

(iii.) $8x+23y=141$ in positive integers.

3. Find the $(r+1)^{\text{th}}$ term in the expansion of

$$(1-3x)^{-\frac{1}{3}},$$

and the greatest in the expansion of

$$(1-\tfrac{1}{6})^{-17}.$$

4. A man puts by 2d. at the end of the second week of the year, 4d. at the end of the fourth, 8d. at the end of the sixth, and so on every fortnight. Find the sum he would put by for the last fortnight. (Use logarithms.)

5. Find the value of

$$\frac{165\times(30)^9\times\sqrt[13]{24}}{(121)^7}.$$

6. Solve the equations :—

(i.) $\sin 4\theta-\sin\theta=0$,

(ii.) $\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{4} + \tan^{-1}\dfrac{1}{5} + \tan^{-1}x = \dfrac{\pi}{4}$.

7. A man, 6 feet high, standing due S. of a lighthouse, observes that his shadow is 24 feet long; walking 100 yards due E. he finds it 30 feet. Find the height of the lighthouse.

8. Transform to polar coordinates :—

(i.) $x^2 + y^2 = a^2$,

(ii.) $x \cos a + y \sin a = a$.

Interpret the equations.

CXCIX.

1. Solve the equations :—

(i.) $\begin{cases} ax + by = c \\ bx - ay = d, \end{cases}$

(ii.) $2(2 - x^{-1}) - 20x^{-2} = 3(1 - x^{-1})(1 + 2x^{-1})$,

(iii.) $5x^2 - 2xy + 10 = y^2, \quad 3x - 2y + 1 = 0$.

2. A horse is sold for £24, and the number expressing the profit per cent. is the same as the cost price in £. Find the cost.

3. The difference between two numbers is 72, and the Arithmetic mean between them exceeds the Geometric mean by 8. Find them.

4. Express $\sqrt{17}$ as a continued fraction, and find the first four convergents.

5. If $$\frac{\tan(\theta - 15^\circ)}{\tan(\theta + 15^\circ)} = \tfrac{1}{3}, \text{ find } \theta.$$

6. Eliminate θ between the equations :—

$$x = 2a \cos\theta \cos 2\theta - a \cos\theta$$
$$y = 2a \cos\theta \sin 2\theta - a \sin\theta.$$

7. The angles of elevation of a balloon are observed from two stations a mile apart, and a point midway between

them, to be 60°, 30°, and 45° respectively. Find the height of the balloon in yards.

8. In a right-angled triangle a perpendicular is let fall from the right angle on to the hypoteneuse; prove, analytically, that the square on this perpendicular is equal to the rectangle contained by the segments of the base.

CC.

1. If
$$x^2+y^2=a^2,$$
expand y in a series of ascending powers of x.

2. Solve the equations:—

(i.) $x^4-4x^3+8x-165=0$,

(ii.) $x-y=2, \quad x^4+y^4=706$,

(iii.) $7x+15y=225$ in positive integers.

3. It is desired to cover a piece of ground 80 feet square by a pyramidal tent, 30 feet in perpendicular height. Find the cost of the canvas at $4\frac{1}{2}$d. per square yard.

4. Prove that any trinomial is a complete square, if the square of the middle term is equal to four times the product of the other two.

Find the square root of
$$x^4+x^2yz+\frac{y^2z^2}{4}-2x^2z^2-yz^3+z^4.$$

5. Solve the equations:—

(i.) $\sin\frac{\theta}{2}=\operatorname{cosec}\theta-\cot\theta$,

(ii.) $\begin{cases}\cos(x+y)+\cos(x-y)=2,\\ \sin\frac{x}{2}+\sin\frac{y}{2}=0.\end{cases}$

6. If
$$\sec\alpha\sec\beta+\tan\alpha\tan\beta=\tan\gamma,$$
prove that $\cos 2\gamma$ cannot be positive.

7. If $A = 52^\circ\ 19'$, $b = 166{\cdot}5$, $a = 162{\cdot}5$,
prove that there are two values of B, and find them.

8. Prove, analytically, that the straight line bisecting two sides of a triangle is parallel to the base.

CCI.

1. A man bought 80 lb. of tea, some at 2s. a lb., and some at 3s.; selling the whole at 3s. a lb. he will gain 10s. more than if he added 6d. a lb. to the price of each. How much of each did he buy?

2. Prove the Binomial Theorem for a negative index.
Expand $(3x + 2a)^{10}$
and $(2x - 5a)^{-5}$
to 5 terms.

3. Find the values of

$$\text{(i.)}\quad \sqrt{10 + 4\sqrt{6}},$$
$$\text{(ii.)}\quad \sqrt[3]{44 + \sqrt{1944}}.$$

4. In a triangle, if $\dfrac{\tan A}{\tan B} = \dfrac{\sin^2 A}{\sin^2 B}$,
the triangle is either isosceles or right-angled.

5. In what scale is the denary number 59055 written 40407?

6. Solve the equations:—

$$\text{(i.)}\quad (3x^2 + 1)(3a^2 + 1) = ax(x^2 + 3)(a^2 + 3).$$

$$\text{(ii.)}\quad \begin{cases} x + y + z = 6, \\ x^3 + y^3 + z^3 = 216, \\ x^2 + y^2 + z^2 = 38. \end{cases}$$

A farmer spent £39 on cows at £7 each, and sheep at £3 each. How many did he buy?

7. Evaluate

$$\tfrac{3}{4} \times \sqrt{24{\cdot}41} \times \frac{1}{\sqrt[3]{187}} \div {\cdot}00395.$$

8. The straight lines AD, BE, bisecting the sides BC, AC of a triangle, meet in G; prove (analytically) that $AG = 2GD$.

CCII.

1. Find the greatest coefficient in the expansion of $(1+x)^n$. Write down (i.) the r^{th}, (ii.) the greatest, coefficient in the expansion of

$$\left(1-\frac{5}{6}x\right)^{-\frac{3}{5}}.$$

2. Find a third proportional to
the Harmonic mean between 3 and $\frac{3}{7}$, and the Geometric mean between 2 and 18.

3. Find the sum of all the numbers that can be expressed by the digits 1, 2, 3, 4, 5 in the senary scale; giving the result in the denary scale.

4. Show how to sum a Geometrical Progression, given the first and last terms, and the number of terms.

Sum m terms of the series whose n^{th} term is $2.3^n + 3.2^n$.

5. Solve the equations :—

(i.) $\cos x \cos 3x = \cos 2x \cos 6x$,

$$\text{(ii.)}\begin{cases}\tan^{-1}x + \tan^{-1}y = \dfrac{\pi}{4},\\ 3x = 2y.\end{cases}$$

6. Prove that if the roots of

$$\tan\frac{x}{2} = \frac{\tan x + a - 1}{\tan x + a + 1}$$

be real, then $a^2 > 1$.

7. If r, r_a, r_b, r_c, be the radii of the inscribed and escribed circles of the triangle ABC, then

$$r_a \cot\frac{A}{2} = r_b \cot\frac{B}{2} = r_c \cot\frac{C}{2} = r \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}.$$

8. Prove that

$$(a, a), \quad \left(-\frac{a}{3}, \frac{a}{3}\right), \quad (-a, -a), \quad \left(\frac{a}{3}, -\frac{a}{3}\right)$$

are the angular points of a parallelogram, and that the sum of the squares on its sides is equal to the sum of squares on the diagonals.

CCIII.

1. Find the greatest term in the expansion of

$$(1-x)^{-\frac{4}{3}},$$

when $x = \frac{12}{13}$.

Show that

$$1 + \frac{1}{6} + \frac{1 \cdot 3}{1 \cdot 2} \cdot \frac{1}{6^2} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \cdot \frac{1}{6^3} + \text{ etc.} = \sqrt{\frac{3}{2}}.$$

2. One root of the equation

$$x^3 - 13x^2 + 15x + 189 = 0$$

exceeds another by 2. Solve the equation.

3. Expand

$$\frac{4x-1}{(1-x)(1-2x)}$$

in a series of ascending powers of x, and write down the coefficient of x^n.

4. Prove that

(i.) $$\frac{\sin 3\theta + 2\sin 5\theta + \sin 7\theta}{\sin 5\theta + 2\sin 7\theta + \sin 9\theta} = \frac{\sin 5\theta}{\sin 7\theta},$$

(ii.) $\sin^3\alpha + \sin^3(120° + \alpha) + \sin^3(240° + \alpha) = -\frac{3}{4}\sin 3\alpha.$

5. At each end of a base of length $2a$ it is found that the angular altitude of a certain peak is θ, and at the middle point of the base the altitude is ϕ. Prove that the height of the peak is

$$\frac{a \sin\theta \sin\phi}{\sqrt{\sin(\theta+\phi)\sin(\phi-\theta)}}.$$

6. A man borrowed £11,000 for two months at 5 per cent. per annum. At the end of the time the interest was added on, and the debt renewed for another two months. This was repeated till the end of two years. Find the amount of the debt and interest at that time.
7. A halfpenny is 1 inch in diameter. Six halfpennies are placed so that each touches two others, and their centres all lie on the circumference of a circle. Find (i.) the area of this circle, (ii.) the area enclosed by the coins.
8. The circumference of each of two circles passes through the centre of the other. Inscribe a square in the figure bounding the space common to both circles.

CCIV.

1. Find the square root of

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} - x^{\frac{1}{6}}y^{\frac{1}{6}}\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)\sqrt{2} + \frac{5}{2}x^{\frac{1}{3}}y^{\frac{1}{3}}.$$

2. Find the area of the triangular field ABC from the following measurements on the scale of 25 inches to the mile. AC 4·1 inches, perpendicular from B on AC 1·59 inches.

   Calculate the same area from the three sides, AB measuring 3·3 inches, and BC 2 inches.

   Express the mean of the two in acres.
3. Solve the equations :—

$$\text{(i.)}\quad \sqrt{3x-3} + \sqrt{5x-19} = \sqrt{3x+4},$$

$$\text{(ii.)}\begin{cases} 3\sqrt{x^2+xy+x+y} = 4, \\ 3x+y=2, \end{cases}$$

$$\text{(iii.)}\quad x^4+1=0.$$

   Verify the solution of (iii.).

4. Assuming the law of the expansion of a Binomial to hold for $(1+x)^n$, where n is a positive integer, prove that it is true for $(1+x)^{n+1}$.

(i.) Find the coefficient of x^r in the expansion of

$$(6-7x+2x^2)^{-1}.$$

(ii.) Find by the Binomial Theorem the value of $(128)^{\frac{1}{3}}$ to 7 places of decimals.

5. Given $\log 1\frac{1}{5} = \cdot 0791812$, and $\log 2\frac{2}{5} = \cdot 3802112$, find the value of

$$\sqrt{(3\cdot 6)^3} \times \sqrt[4]{\tfrac{1}{25}} \div \sqrt[3]{8\tfrac{49}{72}},$$

the mantissae for 14863 and 14864 being 1721065 and 1721357.

6. In a triangle ABC, $b=14$, $c=11$, $A=60°$, find the other angles.

7. If the cosecant of an angle between 90° and 180° is $\dfrac{2}{\sqrt{3}}$, what is the secant?

Prove that $2 \operatorname{cosec} 4\alpha + 2 \cot 4\alpha = \cot \alpha - \tan \alpha$.

8. Prove, by Analytical Geometry, that the angle-bisectors of a triangle meet in a point.

CCV.

1. A sum of £500 is allowed to accumulate at 5 per cent., Compound Interest. What will it amount to in 21 years?

2. Find the sum of a G.P. to n terms, and to infinity.

What ought to be paid for the purchase of a leasehold of which the rental is £p per annum, and of which the lease has n years to run, Compound Interest at r per cent. per annum?

3. Show how to express a given number in any proposed scale.

Show also that if the radix be odd, the difference between the number and the sum of its digits must be even.

4. Show that if $3s = a + b + c$,

$$(s-a)^3 + (s-b)^3 + (s-c)^3 = 3(s-a)(s-b)(s-c).$$

5. Prove the Binomial Theorem for a positive integral exponent.

Write down the general term in the expansion of

$$(1-x)^{-\frac{3}{2}},$$

and prove that

$$\sqrt{8} = 1 + \frac{3}{4} + \frac{3 \, . \, 5}{4 \, . \, 8} + \frac{3 \, . \, 5 \, . \, 7}{4 \, . \, 8 \, . \, 12} + \ldots\ldots$$

6. What is "acceleration," and how is it measured?

If 72 is the measure of an acceleration when 4 feet and 3 seconds are the units, what will it be when 8 feet and 12 seconds are the units?

7. Prove that

$$\text{(i.)} \quad \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{2}{9} = \frac{\pi}{4},$$

$$\text{(ii.)} \quad \sec(A \pm B) = \frac{\sec A \sec B}{1 \mp \tan A \tan B}.$$

8. If each of three equal circles of radius r touch the other two, the area included between the circles is nearly equal to the square on $\frac{2r}{5}$.

CCVI.

1. Find the value of

$$\sqrt{\left(\frac{\sqrt{13}+3}{\sqrt{13}-3}\right)}$$

to five places of decimals.

2. A certain number of men, working together, could do a piece of work in x days. It is however done in y days, one of the men working only half the time, and another one-third of the time, and the rest the whole time. How many men were there?

3. Find the value of

$$(x^2+y^2)^{\frac{1}{3}}$$

when $x=1+2\sqrt{-1}$, and $y=2+\sqrt{-1}$.

(ii.) Express $\sqrt{2n\sqrt{-1}}$ in the form $\sqrt[4]{\alpha}+\sqrt[4]{\beta}$.

4. Solve the equations :—

$$\text{(i.)}\begin{cases} x-y+2z=11, \\ 2x-y+z=9, \\ x-2y+z=0, \end{cases}$$

$$\text{(ii.)}\ 2\sqrt{x}-\sqrt{4x-\sqrt{7x+2}}=1,$$

$$\text{(iii.)}\begin{cases} x+\sqrt[3]{xy^2}=\dfrac{10}{x}, \\ \\ y+\sqrt[3]{x^2y}=\dfrac{810}{y}. \end{cases}$$

5. If

$$\sin x=\tfrac{4}{5},\quad \sin y=\tfrac{7}{25},\quad \sin z=\tfrac{5}{13},$$

find the value of

$$\cos(x+y+z).$$

6. If

$$x=r\sin\tfrac{1}{2}(\theta-\alpha)\qquad y=r\sin\tfrac{1}{2}(\theta+\alpha),$$

prove that $x^2-2xy\cos\alpha+y^2=r^2\sin^2\alpha$.

7. Trace the loci

$$y^2=4x,\quad \text{and}\quad 2x+y=6,$$

by use of a piece of paper ruled in squares.

8. Three forces acting at a point are in equilibrium; the greatest force is 5 lb., and the least 3 lb.; the angle between two of the forces is a right angle. Find the other force.

CCVII.

1. Solve the equations :—

$$\text{(i.)}\begin{cases}\dfrac{x}{2}+\dfrac{y}{5}=5,\\[2mm]\dfrac{2}{x}+\dfrac{5}{y}=\dfrac{5}{6},\end{cases}$$

$$\text{(ii.)}\quad \frac{3x-2}{2}+\sqrt{2x^2-5x+3}=\frac{(x+1)^2}{3}.$$

2. Three brothers, whose ages are in A.P., contribute towards a charity, each giving as many shillings as he is years old. They do this again a few years later, and find that the youngest gives 20 per cent., and the eldest 12 per cent., more than the former time. Altogether they have given 129 shillings. Find their ages at the second contribution.

3. If the number of combinations of n things, five at a time, be the same as when taken ten at a time, what will be the number when taken two at a time?

4. Sum the following series :—

(i.) $\frac{1}{9}$, $\frac{1}{3}$, $\frac{5}{9}$, $\frac{7}{9}$,............ to 21 terms,
(ii.) $\frac{1}{9}$, $\frac{1}{3}$, 1, 3,............ to n terms,
(iii.) 3, 6, 11, 27,............ to n terms,
(iv.) $4a$, $5a^2$, $6a^3$, $7a^4$,.... to infinity,

a being a proper fraction.

5. If

$$\sin(y+z-x),\quad \sin(z+x-y),\quad \sin(x+y-z)$$

are in A.P., so are

$$\tan x,\quad \tan y,\quad \tan z.$$

6. If α and β are the roots of the equation

$$1-\cos 2\theta = 2(\cos A\cos\theta-\cos 2A),$$

prove that $\cos\alpha+\cos\beta = -\cos A.$

✓✓7. Eliminate θ from

$$p = \sec\theta - \cos\theta,$$
$$q = \operatorname{cosec}\theta - \sin\theta.$$

8. Find the area of the triangle formed by the lines

$$x + y = 6, \quad 2x + y = 4, \quad \text{and} \quad x + 2y = 5,$$

and the equation of the circumscribed circle.

CCVIII.

1. Find the value of

$$\frac{\frac{3}{2}\sqrt{5\cdot 2}}{5\sqrt{11\cdot 31}} \times \left(\frac{3}{7}\right)^{-\frac{1}{2}}.$$

2. In what scale will 5261 be expressed by 40205? What are the greatest and least numbers that can be expressed with 5 digits in that scale?

3. Express $\sqrt{50}$ as a continued fraction.

4. If $\cos\alpha = \cos\beta\cos\theta$, prove that

$$\tan\frac{\alpha+\beta}{2}\tan\frac{\alpha-\beta}{2} = \tan^2\frac{\theta}{2}.$$

5. Solve the equations :—

$$\text{(i.)}\ \frac{a}{x} + \frac{b}{y} = p, \quad \frac{b}{x} + \frac{a}{y} = q,$$

$$\text{(ii.)}\ \sqrt{3}\sin\theta - \cos\theta = \sqrt{2},$$

$$\text{(iii.)}\ \sin^2 2\theta - \sin^2\theta = \sin^2\frac{\pi}{6}.$$

6. Express $\cos 5\theta$ in terms of $\cos\theta$.

7. Trace the loci :—

$$y^2 = 5x + 3, \qquad \frac{x^2}{4} + \frac{y^2}{16} = 1.$$

8. A man supports two weights slung on ends of a pole 40 inches long, placed across his shoulder. The one weight is $\frac{2}{3}$ the other, and the weight of the pole may be disregarded. Find the point of support.

CCIX.

1. Find the n^{th} term of the series whose terms are
$$1, \quad 3+5, \quad 7+9+11, \quad \text{etc.}$$
2. A quantity y varies as the sum of two numbers, one constant, and the other a multiple of x^3; when $x=0$, $y=a$, and when $x=a$, $y=2a$. Find y when $x=ab$.
3. Eliminate θ between
$$a+b\cos\alpha=c\cos\theta, \quad b\sin\alpha-a=c\sin\theta.$$
4. Solve
$$\text{(i.)}\ (x^8+1)a^4=(a^8+1)x^4,$$
$$\text{(ii.)}\ \begin{cases} x^2+y^2+z^2=a^2+b^2+c^2, \\ ax+by+cz=a^2+b^2+c^2. \end{cases}$$
5. Expand $\left(1-2x+x^2\right)^{-\frac{1}{4}}$ to 5 terms.
6. Reduce 5678 and ·5678 from the scale of 8 to that of 10.

Prove your answers by reducing them back.
7. If $1-\cos A$, $1-\cos B$, $1-\cos C$ are in A.P., and
$$\sqrt{1-\cos A}, \quad \sqrt{1-\cos B}, \quad \sqrt{1-\cos C}$$
in G.P., then $\sin\frac{A}{2}$, $\sin\frac{B}{2}$, $\sin\frac{C}{2}$ are equal.
8. If B and C move on two parallel lines, and AB, AC be inclined to these lines at angles α, β respectively, and $AB=mAC$, find the locus of A.

CCX.

1. Prove that the sum of the squares of the roots is the same for the two equations
$$ax^2+bx+c=0 \quad \text{and} \quad ax^2+3bx+c+\frac{4b^2}{a}=0.$$

If the two equations

$$x^2 + 7x + 12 = 0 \quad \text{and} \quad x^2 + 21x + a = 0$$

are such that the sums of the squares of their roots are equal, prove that $a = 208$.

2. Find the sum of the cubes of the first n natural numbers, and the sum of n terms of the series whose r^{th} term is

$$1 + 3r + 3r^2.$$

3. Prove that the total number of combinations of n things, 1, 2, 3, ... n, at a time is

$$2^n - 1.$$

Show that the number of ways of making a party of four or more out of 10 persons is 848.

4. Assuming the truth of the Binomial Theorem for a positive integral index, prove its truth for positive fractional indices.

Write down the coefficient of x^n in the expansions of

$$\frac{1}{1-x}, \quad \frac{1}{\sqrt{1-x}}, \quad \sqrt{a-x},$$

and find the expression of which the n^{th} term, in expanding by the Binomial Theorem, is

$$\frac{5 \,.\, 7 \,.\, 9 \ldots\ldots (2n-7)}{3 \,.\, 6 \,.\, 9 \ldots\ldots (3n-3)} . 3^{\frac{7}{2}}.$$

5. In a triangle

$$4 \times \text{area} = (b^2 + c^2 - a^2) \tan A.$$

6. Prove that

$$\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ.$$

7. Prove that

$$\cos 2A = \cos^2 A - \sin^2 A$$

by a Geometrical method of proof.

8. Draw the locus represented by $r = a \cos \theta$ by giving θ the successive values 0°, 30°, 45°, etc.

CCXI.

1. A series $\qquad 1, \quad 3, \quad 6, \quad 10, \quad$ etc.,
is formed by omitting after each term as many of the successive natural numbers as the number of the term. Find the n^{th} term, and prove that the sum to n terms is
$$\tfrac{1}{6}(n^3 + 3n^2 + 2n).$$

2. Show that the middle term of $(1+x)^{2n}$ is the $\overline{n+1}^{\text{th}}$ term of $\left(1-4x\right)^{-\frac{1}{2}}$.

3. Find the present value of an annuity of £A, to commence at the end of n years, and then go on for n years. Compound Interest at r per cent.

An usurious money-lender offers to lend a £5 note for six monthly repayments of £1, the first to be paid one month after the loan. Show that the rate per cent. charged is approximately 80 per cent. per annum.

4. Write down the r^{th} term in the expansion of
$$\left(a-x\right)^{-\frac{1}{n}}.$$
If a_0, a_1, a_2, etc., be the successive coefficients of powers of x in the expansion of $(1+x)^n$, prove that
$$a_0 - a_1 + a_2 - \ldots\ldots + (-1)^r a_r = (-1)^r \frac{\underline{|n-1}}{\underline{|r}\ \ \underline{|n-r-1}}.$$

5. Find the tangent of the angle between
$$y = mx + c \quad \text{and} \quad y = m_1 x + c_1.$$

6. Prove that in any triangle
$$\frac{\sin^2 \frac{A}{2}}{s-c} + \frac{\sin^2 \frac{B}{2}}{b} + \frac{\sin^2 \frac{C}{2}}{c} = \frac{1}{b}.$$

7. If $\qquad a = 9, \quad b = 12, \quad A = 30^\circ, \quad$ find $\quad c.$

8. Find the centre of gravity (i.) of a triangle, (ii.) of a quadrilateral.

CCXII.

1. Distinguish between permutations and combinations.
   Find the number (i.) of permutations (all together), (ii.) of combinations (3 together) that can be formed of the letters of the word *examination.*
2. Solve the equations :—
$$\text{(i.)}\quad \sqrt{x+y}+\sqrt{x-y}=4,\quad x^2+y^2=41,$$
$$\text{(ii.)}\quad \begin{cases} x^2+y^2+z^2=50, \\ yz+xy-zx=7, \\ xy-yz-zx=47. \end{cases}$$
3. Write down the first four terms in the expansion of $(x+y)^n$.
   Find the expression whose second, third, and fourth terms are 240, 720, 1080. $(n > 4.)$
4. A and B start from the same place, B five days after A. A goes 1 mile the first day, 2 miles the second, 3 miles the third, and so on; B goes 12 miles a day. When will they be together?
   Explain the double answer.
5. Prove that
$$\text{(i.)}\quad \sin^{-1}\left(\frac{x-a+b}{2b}\right)^{\frac{1}{2}}=\tfrac{1}{2}\cos^{-1}\frac{a-x}{b},$$
$$\text{(ii.)}\quad \tan 7^\circ\ 30'=(\cot 30^\circ-\operatorname{cosec} 45^\circ)(\sec 45^\circ-1).$$
6. If
$$(a-b)\tan\theta=2\sqrt{ab}\sin\frac{C}{2},$$
   find θ, when $a=5$, $b=2$, $C=120^\circ$.
7. The elevation of a steeple at a place due S. of it is 45°, and at another place due W. it is 15°. If the distance between the stations is a, show that the height of the steeple is
$$\frac{a}{2}\left(3^{\frac{1}{4}}-3^{-\frac{1}{4}}\right).$$

8. Enunciate and prove De Moivre's Theorem.
Hence find the fifth roots of unity.

CCXIII.

1. The natural numbers are divided into groups,

$$1; \quad 2, 3, 4; \quad 5, 6, 7, 8, 9;$$

and so on. Find the numbers in the n^{th} group, and their sum.

2. Find the value of

$$\frac{1}{a+}\,\frac{1}{b+}\,\frac{1}{c+}\,\frac{1}{a+}\ldots\ldots - \frac{1}{b+}\,\frac{1}{a+}\,\frac{1}{c+}\,\frac{1}{b+}\ldots\ldots$$

3. If y vary as the sum of three quantities, of which the first is constant, the second varies as x, and the third as x^2; and if $(a, 0)$, $(2a, a)$, $(3a, 4a)$ are pairs of corresponding values of x and y, prove that when $x = na$, $y = (n-1)^2 a$.

4. Given $\log \frac{1}{2} = \bar{1}\cdot 69897$, solve the equation

$$20^x = 100.$$

Find $\log_{2\sqrt{2}} 256$.

5. Prove that

$$\sin(A + B) = \sin A \cos B + \cos A \sin B;$$

and deduce from it the formula for $\cos(A + B)$.

6. Solve the equations :—

$$\text{(i.)}\quad 3 \tan 3x - \tan 2x = \tan x,$$

$$\text{(ii.)}\quad \begin{cases} \cos x + \cos y = a, \\ \sin x + \sin y = b. \end{cases}$$

7. Prove that in any triangle, if $\cos\theta(\sin B + \sin C) = \sin A$, then $\tan^2\frac{\theta}{2} = \tan\frac{B}{2}\tan\frac{C}{2}$.

8. Prove, analytically, that if a point be taken within a parallelogram, the sum of the triangles formed by joining it to the ends of two opposite sides, is half the area of the parallelogram.

CCXIV.

1. Simplify

$$\frac{1}{\sqrt{2+\sqrt{3}}+\sqrt{2}}-\frac{1}{\sqrt{2-\sqrt{3}}-\sqrt{2}}.$$

2. If α and β are the roots of

$$x^2-px+q=0,$$

find the equation whose roots are α^2, and β^2.

3. Prove that

$$\text{(i.)}\ \left(1+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}+\ldots\ldots\right)^x=1+\frac{x}{\underline{|1}}+\frac{x^2}{\underline{|2}}+\ldots\ldots$$

$$\text{(ii.)}\ \log_e\sqrt{\frac{1+x}{1-x}}=\frac{x}{1}+\frac{x^3}{3}+\frac{x^5}{5}+\ldots\ldots$$

4. Find the sum of n terms of a G.P., of which the first term and common ratio are given.

If S_n represent this sum, find the sum of

$$S_1+S_3+S_5+\ldots+S_{2n-1}.$$

5. Solve the equations :—

$$\text{(i.)}\ \operatorname{cosec}^2\theta=\frac{2\sec\theta}{\tan^2\theta},$$

$$\text{(ii.)}\ \sin^2 2\theta-\sin^2\theta=\frac{1}{4}.$$

6. Prove that

$$\text{(i.)}\ \sin A+\sin(72^\circ+A)-\sin(72^\circ-A)$$
$$=\sin(36^\circ+A)-\sin(36^\circ-A),$$

$$\text{(ii.)}\ \tan A+2\tan 2A+4\tan 4A+8\cot 8A=\cot A.$$

7. In any triangle, prove that

$$\text{(i.)}\ \frac{a^2+b^2-ab\cos C}{a\sin A+b\sin B+c\sin C}=\frac{a}{2\sin A},$$

$$\text{(ii.)}\ 8r\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=R(\sin 2A+\sin 2B+\sin 2C).$$

8. Find the area between the lines

$$x+y=a,\quad 2x=y+a,\quad 2y=x+a.$$

CCXV.

1. If n Harmonic means be inserted between a and b, find the first two and the last two.

   If a, b, c are in H.P., so are
$$\frac{1}{a}+\frac{1}{b+c},\ \frac{1}{b}+\frac{1}{c+a},\ \frac{1}{c}+\frac{1}{a+b}.$$

2. If the square of the sum of an A.P. of n terms be subtracted from n times the sum of the squares, the remainder will be equal to
$$\frac{(n-1)n^2(n+1)}{12}\text{ (common difference)}^2.$$

3. The number of combinations of $2n$ things, three at a time, is equal to twelve times the number of combinations of n things, two at a time. Find n.

4. Prove the Binomial Theorem for a positive integral index.

   If $$(1+x+x^2)^n = 1 + A_1x + A_2x^2 + \ldots\ldots$$
   prove that $$1 + A_3 + A_6 + \ldots\ldots = A_1 + A_4 + A_7 + \ldots\ldots$$
$$= A_2 + A_5 + A_8 + \ldots\ldots = 3^{n-1}.$$

5. Eliminate α and β from
$$x = (a\sin^2\alpha + b\cos^2\alpha)\cos^2\beta + e\sin^2\beta,$$
$$y = a\cos^2\alpha + b\sin^2\alpha,$$
$$z = (b-a)\sin\alpha\cos\alpha\cos\beta.$$

6. If the sides a, b, c of a triangle are in A.P.,

   prove that $$2\sin\frac{A}{2}\sin\frac{C}{2} = \sin\frac{B}{2}.$$

7. Find $\cos 3\alpha$ in terms of $\cos\alpha$.

   Prove that $2\sqrt{2}\cos\alpha$ is one of the roots of the equation
$$x^3 - 6x - 4\sqrt{2}\cos 3\alpha = 0,$$
   and find the others.

8. A point moves so that the square of its distance from a fixed point is four times its distance from a fixed line. Find the locus.

CCXVI.

1. If α, β, and γ are the roots of the equation

$$x^3 - px^2 + qx - r = 0,$$

show that

$$\alpha + \beta + \gamma = p, \quad \alpha\beta + \beta\gamma + \gamma\alpha = q, \quad \alpha\beta\gamma = r.$$

Show that the roots of the equation

$$x^2 - 186x + 25 = 0$$

are the squares of the roots of

$$x^2 - 14x + 5 = 0.$$

2. Find the number of Permutations of n things r at a time.

In the Morse telegraph the marks are of two kinds, show that the number of different signals that can be made when n marks are used for a signal, is 2^n.

3. Show that no square number when divided by 3 can leave a remainder 2.

If the sum of the squares of 2 numbers be a square, the product of the numbers will be divisible by 3 and 4.

4. Prove that each successive convergent approaches more nearly to the value of the continued fraction

$$a + \frac{1}{b+}\;\frac{1}{c+}\;\frac{1}{d+}\ldots\ldots$$

If of three successive convergents the first and third are known, find the second.

5. Prove that

(i.) $\sec\left(\dfrac{\pi}{4} + \theta\right)\sec\left(\dfrac{\pi}{4} - \theta\right) = 2\sec 2\theta,$

(ii.) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \dfrac{\pi}{4} = \sin^{-1}\dfrac{1}{\sqrt{5}} + \cot^{-1}3.$

6. In a triangle $a = 540$, $b = 420$, $C = 52°\ 6'$. Find the other angles.

7. Find the equation of the lines through $(3, 4)$ making an angle of $45°$ with $x - y = 2$.

8. Two circles intersect in P, Q. A straight line MPN is drawn terminated by the circles. Tangents at M and N intersect at T. Prove that a circle can go through M, N, Q, T.

CCXVII.

1. Prove that the difference between two consecutive convergents to a continued fraction has unity for its numerator, and the product of the denominators of the convergents for its denominator.

   Find the first four convergents to the greater root of
$$x^2 - 11x - 2 = 0.$$

2. Form the equation whose roots are the squares of the reciprocals of the roots of the equation
$$ax^2 + bx + c = 0.$$

3. Prove that the Arithmetic, Geometric, and Harmonic means between two positive quantities are in descending order of magnitude.

   The Arithmetic mean between two quantities exceeds the Harmonic by a, and the square of the Arithmetic mean exceeds the sum of the squares of the Geometric and Harmonic means by a^2. Find the quantities.

4. Show that $$\cos\frac{p\theta}{q} + \sqrt{-1}\sin\frac{p\theta}{q}$$
   is one of the values of
$$(\cos\theta + \sqrt{-1}\sin\theta)^{\frac{p}{q}}.$$
   Hence find the values of $(-1)^{\frac{1}{3}}$.

5. Given $$\cos(\alpha - \beta)\cos\alpha = a\sin\beta\cos\beta,$$
   and $$\sin(\alpha - \beta)\cos\alpha = \frac{a}{2}\cos^2\beta,$$
   prove that $$a = \frac{2\sin\beta}{1 + 3\sin^2\beta}.$$

6. Having given $\log_{10}2 = \cdot3010300$, $\log_{10}7 = 8450980$, find $\log_{10}98$, and $\log_{1000}\sqrt{\frac{8}{343}}$.

7. Two men A and B start to walk at the same time along two straight roads inclined at an angle of 37°. Both walk uniformly, A at 5 miles an hour. After 3 hours they are $9\frac{1}{2}$ miles apart. Show that there are two rates at which B may walk to fulfil the conditions, and find them.

8. Find the equation of the straight line from the origin to the intersection of

$$3x + 4y - 5 = 0 \quad \text{and} \quad 5x - 7y + 2 = 0.$$

CCXVIII.

1. In a church where the numbers of the hymns are given out by putting up tablets, each marked with a separate figure, what is the least number of tablets necessary for a collection containing 554 hymns, if there are 3 hymns at each service?

2. Solve the equation

$$\frac{2x-1}{x+1} + \frac{3x-1}{x+2} = 4 + \frac{x-7}{x-1},$$

and show that if α and β are the roots of the equation $ax^2 + bx + c = 0$,

then $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ are the roots of

$$acx^2 + (2ac - b^2)x + ac = 0.$$

3. Find the sum of a given number of terms in G.P.

Sum the series

$1 + 2x + 3x^2 + 4x^3 + \ldots\ldots$ to n terms, and to infinity.

What is the condition that the latter summation may be possible?

4. Find the amount of £P put out at Compound Interest for n years, at $100r$ per cent.

   If £K be withdrawn each year, what will be the amount at the end of n years, and when will the sum be exhausted?

5. A body is observed three times at intervals of a second, and found to have velocities 36, 60, 84. Find the spaces passed over.

6. Prove that the area of a triangle $=\frac{1}{4}abcR^{-1}$.

7. Show that the connector of the points $(a+f,\ b+g)$ and $(a-f,\ b-g)$ passes through, and is bisected at, $(a,\ b)$.

8. Find the locus of a point, the ratio of whose distances from two fixed points is given. (Geometrically or Analytically.)

CCXIX.

1. In the scale whose radix is r, show that the sum of the digits divided by $r-1$, will have the same remainder as the number divided by $r-1$.

   What practical use is made of this fact?

2. Two men start at the same time to meet each other from towns which are 25 miles apart. One takes 18 minutes longer than the other to walk a mile, and they meet in 5 hours. How fast does each walk?

3. Simplify

   (i.) $\dfrac{6a^2b^2}{x+y} \div \left(\dfrac{3(x-y)a}{7(c+d)} \div \left\{\dfrac{4(c-d)}{21ab^2} \div \dfrac{c^2-d^2}{4(x^2-y^2)}\right\}\right)$,

   (ii.) $\left(\sqrt{\dfrac{a+x}{x}} - \sqrt{\dfrac{x}{a+x}}\right)^2 - \left(\sqrt{\dfrac{x}{a}} - \sqrt{\dfrac{a}{x}}\right)^2 + \dfrac{x^2}{a(a+x)}$.

4. Prove that

$$e^x = 1 + \frac{x}{\underline{|1}} + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \ldots\ldots \textit{ ad inf.}$$

If $$x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots\ldots,$$

find y in terms of x.

5. Prove De Moivre's Theorem for a negative quantity.

6. If $\tan^2\theta - \sec^2\alpha = 1$, prove that

$$\sec\theta + \tan^3\theta \operatorname{cosec}\theta = (3 + \tan^2\alpha)^{\frac{3}{2}}.$$

7. ABC is a triangle, D any point in BC. Construct (graphically) a straight line to represent in magnitude and direction the resultant of three forces represented by AB, AC, AD.

8. Find the equation of the straight line through the origin perpendicular to

$$Ax + By + C = 0.$$

Transform both to polar co-ordinates.

CCXX.

1. Show that $$\frac{a+b+c+d}{p+q+r+s}$$ is greater than the least and less than the greatest of the fractions $$\frac{a}{p}, \frac{b}{q}, \frac{c}{r}, \frac{d}{s},$$ each letter representing a positive quantity.

2. Divide 20 into four parts in A.P., and such that the product of the first and fourth shall be to the product of the second and third as $2:3$.

3. Prove that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots$$

If $$y = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots,$$ find x in terms of y.

4. Find the number of balls in a triangular pile, each side of the base containing 25 balls.

5. If a sum of £1000 is lent on condition that it shall bear compound interest at the rate of 5 per cent. for the first 5 years, 10 per cent. for the next 5 years, and 15 per cent. afterwards; what will the debt amount to in 20 years?

6. Draw the circle $$x^2 = 2ay - y^2,$$ and transform it to polar co-ordinates.

7 If $A + B + C = 90°$, prove that
$$\tan B \tan C + \tan C \tan A + \tan A \tan B = 1.$$

8. Given the base, vertical angle, and ratio of the sides, construct the triangle.

CCXXI.

1. Find the condition that the equation
$$ax^2 + bx + c = 0$$
should have (i.) equal roots, (ii.) equal roots with opposite signs.

2. Show that the number of permutations of *aabc* all together, in which the *a*'s come together, is half the whole number of permutations.

3. If $$a : b = c : d = e : f,$$
show that $a : b = a^2 + c^2 + e^2 : ab + cd + ef.$

4. Show how to find a series of fractions convergent to a given fraction; and prove that one of two consecutive convergents is less, and the other greater than the given fraction. Find four convergents to 3·1416.

5. A man standing at A, due S. of a tower on a horizontal plane, finds the altitude of the tower to be 60°. At B, due W. of A, it is 45°, and at C, in AB produced, it is 30°. Prove that $AB = BC$.

6. Solve the equations :—

$$\text{(i.)}\quad \cos 4\theta = \pm\frac{1}{\sqrt{2}},$$

$$\text{(ii.)}\quad \sin 6\theta - \sin\theta = \sin 2\tfrac{1}{2}\theta,$$

giving all the values of θ between $0°$ and $180°$.

7. Prove that $\cos 18° \cos 54° = \frac{\sqrt{5}}{4}$.

8. Two triangles are on the same base, and between the same parallels; prove (analytically and geometrically) that their sides intercept equal lengths on any straight line parallel to their bases.

CCXXII.

1. Find the value of $\frac{(2\cdot4806)^8}{(1\cdot2701)^{10}}$ by log. tables.

2. Solve the equations :—

$$\text{(i.)}\quad \begin{cases} x + y + z = 5, \\ 3x - 5y + 7z = 75, \\ 9x - 11y + 14 = 0, \end{cases}$$

$$\text{(ii.)}\quad \begin{cases} x^2 + y^2 : x^2 - y^2 :: 25 : 7, \\ xy = 48, \end{cases}$$

(iii.) $409x - 77y = 1$ in positive integers.

3. Express

$$\frac{1 + x}{1 - 2x + 2x^2 - 2x^3 + x^4}$$

in the form of the sum of three simple fractions.

4. Transform 15526 from the septenary to the nonary scale.

5. If $2\tan A = 3\tan B$, prove that

$$\tan(A - B) = \frac{\tan B}{2 + 3\tan^2 B} = \frac{\sin 2B}{5 - \cos 2B}.$$

6. Solve the equation

$$\sin\frac{n+1}{2}\theta + \sin\frac{n-1}{2}\theta = \sin\theta.$$

7. Find the length of the perpendicular from (2, 3) on
$$2x+y-4=0.$$

8. The base of a triangle is a length c on the x-axis, and the sum of the squares of the 2 sides $=m^2$. Find the locus of the vertex.

CCXXIII.

1. Simplify

(i.) $(-\sqrt{-1})^5$, (ii.) $\dfrac{1}{\sqrt{7-4\sqrt{3}}}$, (iii.) $\sqrt{61-28\sqrt{3}}$.

2. If £8600 be invested at $4\frac{1}{2}$ per cent. Compound Interest, and £200 added to it annually, what will the interest amount to in seventeen years?

3. Expand $\dfrac{1}{\sqrt{1-x}}$ to 5 terms, and write down the r^{th} term.

Indicate how the first few terms might be obtained without the use of the Binomial Theorem.

4. In how many ways can a debt of £4 11s. 6d. be paid in half guineas and half crowns; and what is the smallest number of coins with which it can be done?

5. Eliminate ϕ from
$$\frac{\cos\theta}{a+\cos\phi}=\frac{\sin\theta}{\sin\phi}=b.$$

6. Prove that $$1+\sec 2\theta=\frac{\tan 2\theta}{\tan\theta},$$

and hence that
$$(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)\ldots(1+\sec 2^n\theta)=\frac{\tan 2^n\theta}{\tan\theta}.$$

7. Straight lines are drawn through a fixed point. Find the locus of the mid points of the parts intercepted between two fixed lines.

8. Find the co-ordinates of the in-centre of a triangle, whose base lies on the x-axis, and one of its angles is at the origin; the other angular points being given.

(ii.) If only the base BC, and the value of $\tan\frac{B}{2}\tan\frac{C}{2}$ be given, find the locus of the in-centre

CCXXIV.

1. A spends £a in buying a number of articles all at the same price; B spends £b in buying n more than A at £c less for each; write down the equation to find how many A bought.

2. Solve the equations :—

(i.) $\dfrac{4^x+1}{5}=2^{x-1}$,

(ii.) $x^4-3x^3-2x^2+3x+1=0$,

(iii.) $3x+4y=23$ in positive integers.

3. Prove that the Geometric Mean between two numbers is also the Geometric Mean between their Arithmetic and Harmonic Means.

Sum the series :—

(i.) $25\frac{1}{2}+24+22\frac{1}{2}+\ldots\ldots$ to 15 terms,

(ii.) $4\frac{4}{15}+2\frac{2}{3}+1\frac{2}{3}+\ldots\ldots$ to n terms and to infinity.

4. Expand

$$(a-x)^{11}, \quad \text{and} \quad (a+x)^{\frac{3}{2}} \quad \text{to 5 terms.}$$

5. Prove that if $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are in H.P., so are

$$\tan(\beta-\alpha), \quad \tan\beta, \quad \tan(\beta-\gamma).$$

6. If

$$\frac{x\cos\phi}{a}+\frac{y\sin\phi}{b}=1,$$

and $x\sin\phi-y\cos\phi=\sqrt{a^2\sin^2\phi+b^2\cos^2\phi}$,

prove that

$$\frac{x^2}{a}+\frac{y^2}{b}=a+b.$$

7. Prove that the equation

$$ax^2 + 2hxy + by^2 = 0,$$

represents two straight lines.

8. Show that the straight lines

$$bx^2 - 2hxy + ay^2 = 0$$

are respectively perpendicular to the two above.

CCXXV.

1. Write down the n^{th} terms of the expansion of

$$(1-x)^{-3} \quad \text{and} \quad (1-x)^m.$$

Show that the sum of the coefficients of the first $(r+1)$ terms of the latter series is

$$\frac{\lfloor m-1}{\lfloor r \, \lfloor m-1-r}.$$

2. Find the number of permutations of n things r together.

A party of 6 ladies and 6 gentlemen sit down at a round table. How many different arrangements can be made so that they may be alternately lady and gentlemen?

3. Change 32·042 into the scale whose radix is 5.

4. If p be the difference between any given fraction and unity, q the difference between its reciprocal and unity, find the relation between p and q.

5. Prove that the value of

$$\sin^2\alpha + \sin^2(\alpha+\beta) - 2\cos\beta \sin\alpha \sin(\alpha+\beta)$$

is always the same, whatever the value of α.

Also, give a Geometrical proof of the above.

6. Prove that if θ be the circular measure of an angle less than a right angle

$$\sin\theta \quad \text{lies between} \quad \theta \quad \text{and} \quad \theta - \frac{\theta^3}{4}.$$

Hence calculate $\sin 5''$ approximately.

7. If ABC be a triangle, right-angled at C, then

$$2\tan^{-1}\frac{a}{b+c} = \cos^{-1}\frac{b}{c}.$$

8. Prove that the equation of the straight lines through the origin which each make an angle α with $x-y=0$ is

$$x^2 - 2xy\sec 2\alpha + y^2 = 0.$$

CCXXVI.

1. Solve the equation :—

(i.) $x^4 + \dfrac{1}{x^4} = a^4 + \dfrac{1}{a^4}$,

(ii.) $7yz = 10(y+z)$, $3zx = 4(z+x)$, $9xy = 20(x+y)$,

(iii.) $x^2+y^2+z^2+a^2+b^2+c^2 = 2ax+2by+2cz$.

2. Find the number of permutations of n things all together, three of which are alike.

How many sets of lawn tennis might be formed from a party of ten ladies and six gentlemen, each set composed of two ladies and two gentlemen?

3. Enunciate the Binomial Theorem, and write down the coefficient of x^r in the expansion of $(1-x)^{-r-1}$.

Expand

$$(1-x+x^2)^{-\frac{1}{2}}$$

to 4 terms in ascending powers of x.

4. If the 6th, 7th, and 8th terms of the expansion of $(x+y)^n$ be 112, 7, and $\frac{1}{4}$, find x, y, n.

5. Express 45792 and ·0045792 in the duodenary scale.

Find the sum of all the numbers which can be expressed by the first six natural numbers in the scale of 7.

6. Solve the equation

$$\sin^{-1}1 + \sin^{-1}\sqrt{1-x^2} = \frac{\pi}{2}.$$

7. Of what angle is 1·5708 the circular measure? 90°

If θ be the circular measure of an angle, show that it contains $\dfrac{\theta}{\sin 1''}$ seconds nearly.

8. Prove, geometrically and analytically, that the straight line joining the right angle of a triangle to the centre of the square on the hypoteneuse, bisects the right angle.

CCXXVII.

1. Find the number of permutations $({}_nP_r)$ of n things r together.

If $${}_nP_4 : {}_{n-1}P_5 :: 3 : 22, \quad \text{find } n.$$

2. Show that the number of terms in the expansion of $(a+x)^n$ is $n+1$, if n is a positive integer.

Apply the Binomial Theorem to find $(10{\cdot}001)^7$ to 5 places.

3. If α and β are the roots of
$$ax^2 + bx + c = 0$$
find the value of $\alpha^4 + \beta^4$.

4. Solve the equations :—
$$\text{(i.)}\quad 144x^2 - 1 + 6\sqrt{9x^2 - x} = 16x,$$
$$\text{(ii.)}\begin{cases} x^2(x^2 - y^2) = 25, \\ y^2(x^2 + y^2) = 19\tfrac{1}{8}, \end{cases}$$
$$\text{(iii.)}\quad \sqrt[3]{14 - x} + \sqrt[3]{14 + x} = 4.$$

5. Find $\sec(A+B)$ in terms of $\sec A$ and $\sec B$; and prove that
$$\sec 105^\circ = -\sqrt{2}(1 + \sqrt{3}).$$

6. Prove that
$$\text{(i.)}\quad \sin 3A = 3\sin A - 4\sin^3 A,$$
$$\text{(ii.)}\quad \sin 54^\circ = \frac{\sqrt{5}+1}{4},$$
$$\text{(iii.)}\quad \frac{\sin(A-C)}{\cos A \cos C} + \frac{\sin(B-A)}{\cos B \cos A} + \frac{\sin(C-B)}{\cos C \cos B} = 0.$$

7. In any triangle show that

$$\text{(i.)}\quad \frac{\sin A}{\sin(B+A)} = \frac{a}{c},$$

$$\text{(ii.)}\quad \text{area} = \frac{a^2}{4}\sin 2B + \frac{b^2}{4}\sin 2A.$$

8. Prove that the equation of the circle whose diameter is the line joining the points (x_1, y_1), (x_2, y_2), is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.

CCXXVIII.

1. The sun seems to revolve round the earth once a year, and the moon thirteen times in a year. If they were all in the same plane, find at what intervals, and how many times in a century, the lines joining earth and sun, and earth and moon, will be at right angles.

2. Show, without using any formula, that the number of permutations of n things all together is equal to n times that of $n-1$ things all together.

3. Explain the meanings of the symbols a^{-p}, a^0.

Multiply $x^n + x^{\frac{n}{2}} + 1$ by $x^{-n} - x^{-\frac{n}{2}} + 1$.

4. Solve the equations :—

$$\text{(i.)}\quad \sqrt{x-1} + \sqrt{x} = \frac{2}{\sqrt{x}},$$

$$\text{(ii.)}\quad x^2 + \frac{y^2}{4} = a^2, \quad x^2 + xy = m;$$

And find the limiting values of m^2, that real and positive values of x and y may be obtained from (ii.)

5. Prove that to turn seconds into circular measure we must multiply by ·000004848.

Prove the formula $\theta = \dfrac{\text{arc}}{\text{radius}}$.

6. In any triangle

(i.) $xyz = \dfrac{(a+b+c)^3}{abc} r^3,$

where x, y, z are the perpendiculars of the triangle;

(ii.) $\sin \dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}} = \dfrac{r}{\sqrt{(r_b - r)(r_c - r)}}.$

7. A measured line (l) is drawn from a point on a horizontal plane at right angles to the line joining that point to the base of a tower on the plane. The angles of elevation of the tower from the ends are 30° and 18°. Find the height of the tower.

8. Interpret the equations:—

(i.) $xy = 0,$

(ii.) $x^2 - y^2 = 0.$

CCXXIX.

1. Find four proportionals, such that the sum of the extremes is 21, the sum of the means 19, and the sum of the squares of all four is 442.

2. Prove the Binomial Theorem for a negative exponent.
Find the first negative coefficient in the expansion of

$$(1+3x)^{\frac{13}{3}}.$$

3. Solve the equations:—

(i.) $(x-5) - 4\sqrt{x-5} = 0,$

(ii.) $\begin{cases} x+y+z = xyz = 6, \\ xy + yz + zx = 11. \end{cases}$

4. Find a formula for the number of permutations of n things, all together, which are not all different.
How many different numbers can be formed from the figures of 111223, taken 1, 2, ...6, at a time.

5. Prove that

$$\frac{\sin(p+q) - 2\sin p + \sin(p-q)}{\cos(p+q) - 2\cos p + \cos(p-q)} = \tan p.$$

6. Evaluate

$$\{165 \times (30)^9 \times \sqrt[3]{24}\} \div (121)^7,$$

and solve the equation

$$5^x = 3125.$$

7. Prove that the straight line $3x + 4y = 60$ touches the circle

$$x^2 + y^2 - 10(x+y) + 25 = 0,$$

and find the point of contact.

8. Find the equation of the tangents from (x', y') to

$$(x-\alpha)^2 + (y-\beta)^2 = r^2.$$

CCXXX.

1. The roots of the equation

$$x^2 - ax + b = 0$$

are α and β; find the equations whose roots are

$$\alpha \text{ and } -\beta, \quad \beta \text{ and } -\alpha, \quad \text{respectively;}$$

and verify by comparing them with the biquadratic whose roots are $\pm\alpha$, $\pm\beta$.

2. Enunciate and prove the Binomial Theorem for a fractional index.

Expand $\sqrt{1 + 2x + 4x^2}$ as far as x^4.

3. Find the value of

$$\frac{a(b-c)}{a-b}$$

when a, b, c are (i.) in A.P., (ii.) in G.P., (iii.) in H.P.

If a, b, c be in A.P., b, c, d in G.P., and c, d, e in H.P., prove that a, c, e are in G.P.

4. A crew can row a certain course up stream in $8\frac{4}{7}$ minutes, and if there were no stream they could row it in 7 minutes less than it takes them to drift with the stream. How long would they take to row with the stream?

5. Prove that

$$\sin 3\alpha + \sin 2\alpha - \sin \alpha = 4 \sin \alpha \cos \frac{\alpha}{2} \cos \frac{3\alpha}{2}.$$

6. If $$4 \cot 2\theta = \cot^2\theta - \tan^2\theta$$
all possible values of θ are given by

$$\theta = n\pi \pm \frac{\pi}{4}.$$

7. Find the area of a triangle in terms of its sides, and prove that it is equal to

$$\frac{a^2}{2} \frac{\sin B \sin C}{\sin (B + C)}.$$

8. Find the equation of the straight line through (4, 5), which makes an angle of 45° with the x-axis.

Find the equation of a series of circles which cut this line at points equidistant from the point (4, 5), and pass through the origin.

CCXXXI.

1. Prove the Binomial Theorem for a fractional index.

(i.) Find the first four terms in the expansion of

$$\left(1 - \frac{x}{3}\right)^{-3}$$

and write down the 15th term.

(ii.) If $1, \alpha, \beta, \gamma, \ldots\ldots$ be the coefficients in order of the expansion of $(1 + x)^n$, and n be a positive integer, show that

$$1 - 2\alpha + 3\beta - 4\gamma + \ldots\ldots = 0.$$

2. If one solution of the equation

$$ax + by = c$$

be given in positive integers, give formulæ for finding the rest.

In how many different ways is it possible to pay £2 3s. 6d. in half-crowns and shillings?

3. Show that the two values of x given by the equation

$$\frac{x^2 + c^2 - a^2}{2cx} = \frac{\sqrt{3}}{2}$$

will be positive if $c > a < 2a$.

Explain the Geometrical meaning of this when applied to Trigonometry.

4. Solve the equations:—

(i.) $\dfrac{x - \sqrt{2x} + 1}{x + \sqrt{2x} + 1} = \dfrac{a}{b}$,

(ii.) $2x^2 - xy + y^2 = 2y, \quad 2x^2 + 4xy = 5y$,

(iii.) $\begin{cases} x + y + z = 1, \\ x^2 + y^2 + z^2 + 6xy = 0, \\ \dfrac{x}{y+z} + \dfrac{y}{z+x} + \dfrac{z}{x+y} = 0. \end{cases}$

5. On a circle of 10 ft. radius, an angle of 22° 30′ was found to be subtended by an arc 3 ft. $11\frac{1}{8}$ in. long. Hence calculate π to four places of decimals.

6. In any triangle prove that

(i.) $a = (b - c)\sec\theta$, where $\tan\theta = \dfrac{2\sqrt{bc}}{b-c}\sin\dfrac{A}{2}$;

(ii.) $\tan^{-1}\dfrac{a}{b+c} + \tan^{-1}\dfrac{b}{a+c} = 45°$, if $C = 90°$.

7. Given $C = 90°$, $B = 30°$, $BC = 100$ ft.; solve the triangle.

8. Find the equation of the lines which cut off a length b from the y-axis, and make angles β with

$$x \cos \alpha + y \sin \alpha = p.$$

CCXXXII.

1. Evaluate

$$\frac{\sqrt[5]{25} \times 11^7 \times \sqrt[11]{7}}{11^{10}} \text{ by logarithms.}$$

2. Expand $(ax - x^2)^{-\frac{1}{5}}$ to 5 terms.

3. On how many nights may a different patrol of 6 men be draughted from a garrison of 60? On how many nights would one man be off duty?

4. Resolve into partial fractions:—

$$\frac{a - bx}{ec - (ed + fc)x + fdx^2}.$$

5. Prove that

$$\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A},$$

and hence show that if $A + B + C = m\pi$, where m is any integer,

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C.$$

6. A flagstaff, leaning due E., subtends an angle α at a point in the plane in which it stands, a yards W. of the place. At a point b yards E. it subtends an angle β. Find at what angle it leans.

7. Find the locus of the vertex (A) of a triangle, given the base, and that

$$\tan C = m \tan B.$$

8. Prove that the following equation represents two straight lines, and find the lines:

$$x^2 - 5xy + 4y^2 + x + 2y - 2 = 0.$$

CCXXXIII.

1. Evaluate

$$\tfrac{3}{4} \times \sqrt{24{\cdot}41} \times \frac{1}{\sqrt[3]{187}} \div {\cdot}00395.$$

2. Find the number of combinations of n things, taken r at a time.

If this number be denoted by C_r, find the value of

$$1 + C_1 + C_2 + \ldots\ldots + C_n,$$

and of $nC_1 + (n-1)C_2 + \ldots\ldots + 2C_{n-1} + C_n$.

3. Find the value of

$$\text{(i.)}\ \sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}},$$

$$\text{(ii.)}\ \frac{1}{\sqrt{2+\sqrt{3}} + \sqrt{3}}.$$

4. Sum the series :—

$$\text{(i.)}\ \frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} \quad \text{to } n \text{ terms,}$$

$$\text{(ii.)}\ \frac{a}{x} - \frac{a^2}{x^{\frac{3}{2}}} + \frac{a^3}{x^2} - \ldots\ldots \quad \text{to 8 terms.}$$

In what case can (ii.) be summed to ∞?

5. Solve

$$\text{(i.)}\ \sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta = 0,$$

$$\text{(ii.)}\ 4\sin^2\theta + \sin^2 2\theta = 3.$$

6. Eliminate α and β from the equations

$$a = \sin\alpha\cos\beta\sin\theta + \cos\alpha\cos\theta,$$

$$b = \sin\alpha\cos\beta\cos\theta - \cos\alpha\sin\theta,$$

$$c = \sin\alpha\sin\beta\sin\theta.$$

7. Reduce $$r = 2a\sec\left(\theta + \frac{\pi}{6}\right)$$

to rectangular coordinates.

8. A straight line moves so that the sum of the perpendiculars, AP, BQ, on it from two fixed points, A and B, is constant. Find the locus of the middle point of PQ.

CCXXXIV.

1. Resolve
$$\frac{2x^2}{(x+1)(x+2)^2}$$
into its partial fractions.

2. Sum to n terms :—

(i.) $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots\ldots + n(n+1)$,

(ii.) $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots$,

(iii.) $12 + 6 + 3 + \ldots\ldots$ to infinity.

3. Reduce $\frac{19}{12}$ to a continued fraction, and find the convergents.

4. Solve

(i.) $(x-3)^2 + (3x-22) = (x^2 - 3x + 7)^{\frac{1}{2}}$,

(ii.) $x^4 - x^2(2x-3) = 2x + 3$,

(iii.) $\sqrt{2x-1} + \sqrt{3x+10} = \sqrt{11x+9}$.

5. Prove

(i.) $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$,

(ii.) $\dfrac{\cos\alpha + \cos\beta}{\cos\alpha - \cos\beta} = -\dfrac{\cot\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$,

(iii.) $\dfrac{\cos 3^\circ - \cos 33^\circ}{\sin 3^\circ + \sin 33^\circ} = \tan 15^\circ$.

6. Prove that the area of a triangle
$$= \frac{a^2 \sin B \sin C}{2\sin(B+C)} = \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\left(\frac{a^2}{\sin A} + \frac{b^2}{\sin B} + \frac{c^2}{\sin C}\right).$$

7. Find the locus of a point P such that if perpendiculars PM, PN be drawn to two fixed lines, MN is parallel to a fixed line.

8. Find the locus of the vertex of a triangle, given the base and the ratio of the sides. (Geometrically and Analytically.)

CCXXXV.

1. Expand
$$\frac{x^2}{(x-1)(x-2)(x-3)}$$
in ascending powers of x, by means of Partial Fractions;
and
$$\frac{1}{1+x+x^2},$$
by assuming it $= a_0 + a_1x + \ldots\ldots,$ and then multiplying across, and equating coefficients of powers of x.

2. Show that the solutions of
$$29x - 17y = 46,$$
and of
$$ax - by = c$$
in positive integers, each give rise to two series in A.P.

3. Prove that

(i.) $\cos\overline{\alpha+\beta}\sin\beta - \cos\overline{\alpha+\gamma}\sin\gamma$
$= \sin\overline{\alpha+\beta}\cos\beta - \sin\overline{\alpha+\gamma}\cos\gamma,$

(ii.) $\sin^2 A + \sin^2 B + \sin^2 C - 2\cos A\cos B\cos C = 2$
if $A + B + C = \pi$.

4. Solve

(i.) $\cos 2x + 2\cos x\cos\alpha - 2\cos 2\alpha = 1,$

(ii.) $\sin mx = \sin nx \cos\overline{m+p}\,x\cos\overline{n+p}\,x,$

(iii.) $2\sin x = \sin 3x1.$

5. If
$$x\cos\phi + y\sin\phi = x\cos\theta + y\sin\theta = 2a$$
and
$$2\cos\frac{\phi}{2}\cos\frac{\theta}{2} = 1,$$
prove that
$$y^2 = 4a(a+x).$$

6. If r_a, r_b, r_c be the radii of the escribed circles, prove that
$$r_a\cot\frac{A}{2} = r_b\cot\frac{B}{2} = r_c\cot\frac{C}{2}.$$

7. Straight lines are drawn through a fixed point. Find the locus of the mid points of the parts intercepted between two fixed straight lines.

8. Prove that the straight line $y = mx + \frac{a}{m}$
touches the parabola $y^2 = 4ax$,
and find the point of contact.

CCXXXVI.

1. Expand a^x in ascending powers of x, and find the sum to infinity of the series, whose n^{th} term is

$$\frac{n+1}{\lfloor n}.$$

2. If $a \propto b$, when c is constant, and $a \propto c$, when b is constant, then $a \propto bc$, when b and c both vary.

If the work per hour varies as $12 - x$, x being the number of working hours per day, find x in order that the work of 27 days may be equivalent to the work of 20 days of 9 hours each.

3. If l, m, n are three numbers in G.P., prove that the first term of an A.P., whose l^{th}, m^{th}, and n^{th} terms are in H.P., is to the common difference as $m + 1 : 1$.

4. If α, β are the roots of

$$x^2 - px + q = 0,$$

and if α is a mean proportional between β and q, prove that $$\alpha = \beta^2.$$

5. Two straight lines OA, OB, of lengths a and b, include an angle α, and straight lines AP, BP, meeting in P, are drawn at right angles to OA, OB: prove that if P fall within the angle α, the area of $OAPB$ is

$$\frac{2ab - (a^2 + b^2)\cos\alpha}{2\sin\alpha}.$$

6. Eliminate c between

$$a^2 = b^2 + c^2 - 2bc\cos A$$

and $$b^2 = a^2 + c^2 - 2ac\cos B.$$

7. If
$$\tan(\pi \cot\theta) = \cot(\pi \tan\theta),$$
show that
$$\tan\theta = \frac{2n+1}{4} \pm \frac{\sqrt{(4n^2+4n-15)}}{4},$$
where n is an integer.

8. Through M, the mid point of the base BC of a triangle, a straight line DME is drawn to cut off equal parts from the sides AB, AC. Show that BD is equal to CE. (Take A as origin.)

CCXXXVII.

1. Transform 4053 from the senary to the septenary scale.

2. Express $\sqrt{180}$ as a continued fraction, and find the first four convergents.

3. Sum to n terms the series whose n^{th} terms are

(i.) $n + a^{n-1}$,

(ii.) $\dfrac{1}{n(n+1)(n+2)}$,

(iii.) n^3.

4. Prove that the number of combinations of n things r together, is the same as that of n things $n-r$ together, and greater than that of $n-1$ things $r-1$ together.

How many different numbers can be made with all or any of the figures of the number 1881, and what is their sum?

5. Prove that

(i.) $\cos 2A + \cos 2B = 2\cos\overline{A+B}\,.\cos\overline{A-B}$,

(ii.) $$\tan A \tan B = \frac{\left(\cos\frac{A-B}{2}\right)^2 - \left(\cos\frac{A+B}{2}\right)^2}{\left(\cos\frac{A-B}{2}\right)^2 - \left(\sin\frac{A+B}{2}\right)^2}$$

6. Two sides of a triangle are 76 and 57 yards, and the angle opposite the former is $53° \ 7' \ 49''$. Solve the triangle.

7. If θ is the circular measure of an angle which is $\frac{1}{3}$ of a right angle, find the value of $\dfrac{\sin \theta}{\theta}$.

8. AB is drawn parallel to the y-axis, intersecting two given straight lines at B and B', and the x-axis at A. P is taken on AB so that $AP^2 = PB \,.\, PB'$: find the locus of P.

CCXXXVIII.

1. Show that if m and n be positive integers, the coefficients of x^m and x^n in the expansion of $(1+x)^{m+n}$ are the same.

2. Find the Present Value of an annuity to continue for n years, allowing Compound Interest.

   I lend £100, to be repaid in 12 years by instalments of £10 per annum. Find an equation for determining the rate of interest reckoned, and show that it is less than 10 per cent.

3. Solve the equations :—

$$\text{(i.)} \quad 2x^2 - \sqrt{x^2 - 2x - 3} = 4x + 9,$$

$$\text{(ii.)} \begin{cases} (x^2 - y^2)(x - y) = 16xy, \\ (x^4 - y^4)(x^2 - y^2) = 640x^2y^2. \end{cases}$$

4. Given the equations

$$ax^2 + bx + c = 0$$

and
$$acx^2 + (2ac - b^2)x + ac = 0,$$

   find the roots of the latter in terms of those of the former.

5. When is a quantity said to vary directly, and when inversely as another?

A circular target consists of $n-1$ concentric rings, and a central circle, the areas of all which are equal. The area of a circle varies as the square of its radius; if the cost of the paint for any ring varies as the square of the radius of its outer edge, show that the cost of the whole varies as $n(n+1)$.

6. Prove that

(i.) $\sin 4A = 4\cos A(\sin A - 2\sin^3 A)$,

(ii.) $(\tan 4A + \tan 2A)(1 - \tan^2 3A \tan^2 A)$
$$= 2\tan 3A \sec^2 A.$$

7. If in a triangle $$R^2 = 2r,$$ the triangle is equilateral.

8. Prove that the straight line $x - y = 0$ bisects the angle between the lines
$$4x^2 - 17xy + 4y^2 = 0.$$

CCXXXIX.

1. Given $$p = x + x^{-1},$$ show by summing a series to infinity that
$$p^{-1} = x^{-1} - x^{-3} + x^{-5} - \ldots\ldots$$

2. Solve the equations:—

(i.) $\dfrac{x}{x+y-1} = \dfrac{y}{x+z} = \dfrac{z}{1-x} = x + y - z$,

(ii.) $x^4 + 4x^3 - 18x^2 + 20x - 7 = 0$, which has two equal roots,

(iii.) $x^3 - 12x - 16 = 0$.

3. Find the volume and surface of a cylinder, whose radius is 4 feet, and height 100 feet.

If a wedge is cut out of this cylinder by two planes through its axis inclined at an angle of 45°, what will be the volume left?

4. If
$$\sqrt{\frac{a+x}{a-x}}$$
be expanded in ascending powers of $\frac{x}{a}$, the coefficients of the $(2r-1)^{\text{th}}$ and $(2r)^{\text{th}}$ terms are equal.

Find the value of the infinite series
$$1+\frac{3}{2^3}+\frac{1.3.3^2}{1.2.2^6}+\frac{1.3.5.3^3}{1.2.3.2^9}+\text{etc.}$$

5. In any triangle show that

(i.) $R+r=\frac{1}{2}(a\cot A+b\cot B+c\cot C)$,

(ii.) $\tan A+\tan B+\tan C=\tan A\tan B\tan C$.

6. In a plane triangle $AB=1114{\cdot}5$ ft., the angle $CAB=38°\ 41'$, the angle $CBA=45°\ 39'$, find BC.

7. Two forces are represented in magnitude and direction by OA and $2OB$; show that the resultant is represented by $3OC$, where C is one of the points of the trisection of AB.

Generalize this for OA and $n\,.\,OB$.

8. What force must act on a mass of 65 lb. to increase its velocity from 32 to 33 feet per second in passing over 50 feet?

CCXL.

1. Prove that if
$$e^x=y+\sqrt{1+y^2},$$
then
$$y=x+\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 5}+\ldots\ldots\ ad\ inf.$$

2. Find the logarithm of 9 to the base $3\sqrt{3}$, and of 125 to the base $\sqrt{5}\times\sqrt[3]{5}$.

If $a^x=e^z$ find the value of $\frac{x}{z}$.

3. Prove that the number of combinations of $p+q$ things taken p together, or q together, is the same.

A cricket club consists of 13 members, of whom only 4 can bowl. In how many ways can an eleven be chosen so as to include *at least* 2 bowlers?

4. Expand $\dfrac{1}{(1-x)^4}$ as far as 6 terms.

In the expansion of

$$x^m\left(1-\frac{1}{x}\right)^{-n}$$

write down the term which does not contain x, and prove that it will be a positive integer whenever n is so.

5. Solve the triangle in which $a=180$, $b=140$, $C=64°\ 12'$.

6. Solve the equations:—

$$\text{(i.)}\ \sin\theta\cos\theta=\tfrac{2}{5},$$
$$\text{(ii.)}\ 6\cot^2\theta-4\cos^2\theta=1,$$

and show that if

$$\cos\phi=n\sin\alpha,\quad\text{and}\quad\cot\phi=\sin\alpha\cot\beta,$$

then $$\cos\beta=\frac{n}{\sqrt{1+n^2\cos^2\alpha}}.$$

7. At three positions in a horizontal plane, distant 60, 80, and 100 feet from each other, the elevation of a tower is found to be 45°. Find its height.

8. Straight lines through a fixed point cut two fixed straight lines which are at right angles. Find the locus of the mid points of the intercepts between the lines.

CCXLI.

1. Solve the equations:—

$$\text{(i.)}\begin{cases}x^2-2xy+y^2+2x+2y-3=0,\\ y(x-y+1)+x(x-y-1)=0,\end{cases}$$
$$\text{(ii.)}\ x^5+(1-x)^5=11.$$

Interpret the result of (i.) geometrically.

2. Simplify $(9 + 4\sqrt{5})^{\frac{1}{3}} + (9 - 4\sqrt{5})^{\frac{1}{3}}$.

3. If $(1 + x)^n = c_0 + c_1 x + c_2 x^2 + \ldots\ldots + c_r x^r + \ldots\ldots$ show that

$$2c_0 + 2^2\frac{c_1}{2} + 2^3\frac{c_2}{3} + 2^4\frac{c_3}{4} + \ldots\ldots = \frac{3^{n+1} - 1}{n+1}.$$

4. Eliminate θ between the equations

$$m = \operatorname{cosec}\theta - \sin\theta, \quad n = \sec\theta - \cos\theta.$$

5. In a plane triangle given

$$b = 2 \text{ ft. } 6 \text{ in.}, \quad c = 2 \text{ ft.}, \quad A = 22^\circ\ 20';$$

find the other angles, and show that $a = 1$ foot nearly.

6. Show that the sum of the moments of any three forces represented in magnitude and direction by the sides of a triangle taken in order, is the same about every point in the plane of the triangle.

7. A pair of compasses, each of whose legs is a uniform bar of weight W, is supported, hinge downwards, at the middle points of the legs in a horizontal line, the legs being kept apart at an angle A with one another, by a weightless rod joining their ends. Find the thrust in this rod, and the force acting on the hinge.

8. Draw the following loci :—

(i.) $x^2 + y^2 - 3x - 4y = 0$,

(ii.) $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

CCXLII.

1. Sum the series

(i.) $2 . 5 . 8 + 5 . 8 . 11 + 8 . 11 . 14 + \ldots\ldots$,

(ii.) $1^3 + 3^3 + 5^3 + \ldots\ldots$,

(iii.) Prove that Arithmetic and Geometric Progressions are Recurring Series, and find the Scales of Relation.

2. Prove that

$$(b-c)(1+a^2b)(1+a^2c)+2 \text{ similar terms}$$
$$\equiv abc(a+b+c)(c-b)(b-a)(a-c).$$

3. Show that

$$(x^2+y^2)^2 > 4xy(x^2-xy+y^2).$$

4. Solve the triangle

$$a=1{\cdot}56234, \quad b={\cdot}43766, \quad C=58^\circ\ 42'\ 6''.$$

5. Find x from the equation

$$3\tan^{-1}(x+1)=2\tan^{-1}(x-1)+\tan^{-1}\frac{x}{2-x}.$$

6. A smooth sphere is kept at rest on a plane inclined at 30° to the horizon by means of a string of length equal to the radius, attached to the plane.

   Find the direction and tension of the string, and the pressure on the plane.

7. Two equal beams AC, BC, freely jointed at C, stand with their ends A, B on a rough horizontal plane, with ACB vertical. If the coefficient of friction be ·5, show that the angle ACB cannot exceed a right angle, and find the thrust at C, when the angle ACB is equal to θ.

8. If a train ascends a gradient of 1 in 40 by its own momentum for a distance of 1 mile, the resistance from friction, etc. being 10 lbs. per ton, find its initial velocity.

CCXLIII.

1. The interior of a building is in the form of a cylinder of 15 feet radius, and 12 feet altitude, surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and the cubical content of the building.

2. Find the total number of combinations of n things.

If ${}_nC_r$ denote the number of combinations of n things r together, prove that

$$\frac{1}{n}{}_nC_1+\frac{1}{n-1}\cdot\ {}_nC_2+\frac{1}{n-2}\cdot\ {}_nC_3+\ldots+\frac{1}{1}{}_nC_n=\frac{2}{n+1}(2^n-1).$$

3. Prove that the integral part of

$$\frac{1}{\sqrt{3}}(\sqrt{3}+\sqrt{5})^{2n-1}$$

is divisible by 2^n.

4. State De Moivre's Theorem, and prove that when the index is fractional, of the form $\dfrac{p}{q}$, q and no more than q different values of the expansion can be obtained.

5. In a given circle an equilateral triangle is inscribed, in that another circle, and so on *ad infinitum;* show that the sum of the circumferences of all the inner circles, is equal to that of the given circle, and the sum of their areas one-third of its area.

6. Having given the four sides of a quadrilateral inscribed in a circle, find its area and angles.

7. Prove that the equation of the tangent at the point (h, k) of the circle

$$x^2+y^2=c^2$$

is

$$hx+ky=c^2,$$

and that, if α and β are the intercepts of the axes by this tangent,

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{1}{c^2}.$$

8. If a heavy body is suspended from a fixed point about which it can move freely, prove that, when it is at rest, its centre of gravity is in the vertical line through its point of suspension.

A heavy triangular lamina is at rest inside a smooth hemispherical bowl; show that the pressures at the three angular points are equal.

CCXLIV.

1. If $(1+x)^n = c_0 + c_1 x + c_2 x^2 + \ldots\ldots + c_n x^n$, find the value of $c_0 c_1 + c_1 c_2 + \ldots\ldots + c_{n-1} c_n$.

2. If
$$(5\sqrt{2}+7)^{2m+1} = n + \alpha,$$
where m and n are positive integers and $\alpha < 1$; then
$$\alpha(n+\alpha) = 1.$$

3. Transform
$$(1+x)(1+x^2)(1+x^4)(1+x^8)\ldots\ldots \textit{ ad inf.}$$
into a series of the form
$$1 + a_1 x + a_2 x^2 + \ldots\ldots.$$

4. Show that in any triangle
$$\text{(i.)}\ \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c} = \frac{1}{r},$$
$$\text{(ii.)}\ r_a + r_b + r_c - r = 4R.$$

5. Discuss the triangle in which
$$a = 75, \quad b = 40, \quad B = 69^\circ\ 18'.$$

6. Solve the equations :—
$$\text{(i.)}\begin{cases} \sin(2x+3y) = \cos\alpha, \\ \quad 3x + 2y = \pi, \end{cases}$$
$$\text{(ii.)}\ \sin^2\theta + \cos^2 2\theta = \tfrac{3}{4}, \text{ in which } \theta > 0 < \frac{\pi}{2}.$$

7. Interpret the equations :—
$$\text{(i.)}\ r\cos\theta = a,$$
$$\text{(ii.)}\ r = a\cos\theta,$$
$$\text{(iii.)}\ r\sin^2\theta = 4a\cos\theta,$$
$$\text{(iv.)}\ 2a = r(1+\cos\theta),$$
$$\text{(v.)}\ \frac{1}{r^2} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}.$$

8. Prove the formulæ

$$\text{(i.)}\ s = ut + \tfrac{1}{2}at^2,$$
$$\text{(ii.)}\ v^2 = u^2 + 2as.$$

Two particles start from rest at the same instant, with velocities u and v respectively; the motion of the first is uniformly retarded, that of the second uniform. Prove that by the time the first comes to rest the distances traversed will be as

$$u : 2v.$$

CCXLV.

1. If $$(1+x)^n = c_0 + c_1x + c_2x^2 + \ldots\ldots + c_nx^n,$$
find the value of
$$c_0{}^2 - c_1{}^2 + c_2{}^2 - c_3{}^2 \ldots\ldots + (-1)^n c_n{}^2.$$

2. Prove that the coefficient of x^n in the expansion of
$$\frac{x}{(1-x)^2 - cx}$$
is $$n\left\{1 + \frac{n^2-1^2}{\lfloor 3}c + \frac{(n^2-1^2)(n^2-2^2)}{\lfloor 5}c^2 + \ldots\ldots\right\}$$

3. Assuming De Moivre's Theorem, deduce the expansion of $\sin n\theta$ and $\cos n\theta$, in powers of $\sin\theta$ and $\cos\theta$, and write down the last term in each series.

4. If in a triangle $b = 5$, $c = 3$, $A = 120°$, find the other angles.

5. A cubical box, 5 feet deep, is filled with layers of spherical balls, whose diameters, where they touch, are in vertical and horizontal lines; the diameter of each ball is half-an-inch; find the space left unfilled.

6. If the tangents at the extremities of a focal chord Pp of a parabola meet at T, prove that ST is a mean proportional between AS and Pp.

7. Find the equation of the tangent to the parabola

$$y^2 = 4ax,$$

in terms of the tangent of its inclination to the axis.

Hence show that two tangents can be drawn to a parabola from a point (h, k).

What if the point be within the parabola?

8. A body whose mass is m lbs. is attached to a weightless string over a smooth pulley and fastened at the other end to another body of m' lbs. lying on a table, the line joining the position of m' and the pulley being inclined at an angle α to the vertical. If the string be at first slack and become stretched after m has fallen through 1 foot, find the impulsive tension of the string and the velocity of m'.

Show that the acceleration of m, directly after the string is stretched, is

$$\frac{m - m' \cos \alpha}{m + m'} g.$$

CCXLVI.

1. Show that

$$\frac{n(n+1)\ldots\ldots(n+p-1)}{\lfloor p} - \frac{n}{1} \cdot \frac{(n-1)n\ldots\ldots(n+p-2)}{\lfloor p}$$
$$+ \frac{n(n-1)}{\lfloor 2} \cdot \frac{(n-2)(n-1)\ldots\ldots(n+p-3)}{\lfloor p} + \ldots\ldots = 0,$$

if $n > p$.

2. Show that $(x^2 + xy + y^2)^n$

can be put in the form

$$X^2 + XY + Y^2,$$

where X and Y are real and rational.

3. Find the sum of the products, two and two, of the terms of an A.P., whose first term is a, and last l, and number of terms n.

4. If x, y, z are the perpendiculars let fall on the sides of a triangle from the opposite angles,

show that $$\frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}=\frac{a^2+b^2+c^2}{2R}.$$

5. Find the coefficient of x^n in the expansion of
$$e^x \cos x.$$

6. Find the equation of the normal to the parabola
$$y^2=4ax,$$
at the point $$\left(\frac{a}{m^2}, \frac{2a}{m}\right).$$

If two tangents be drawn from any point in the latus rectum produced, the sum of their inclinations to the axis will be $\frac{\pi}{2}$.

7. Six forces are represented by the straight lines drawn from the angular points of the regular hexagon $ABCDEF$ to the point where BD, CF intersect. Find the direction of the straight line representing their resultant, and show that it is three times as long as a side of the figure.

8. Define the terms *poundal* and *foot-poundal*. Show that a foot-poundal is nearly equal to the work done in lifting an ounce through six inches.

CCXLVII.

1. Sum the series :—

(i.) $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\ldots\ldots$ to n terms,

(ii.) $\frac{0}{\underline{|1}}+\frac{1}{\underline{|2}}+\frac{2}{\underline{|3}}+\ldots\ldots+\frac{n-1}{\underline{|n}}$,

(iii.) $3+11x+31x^2+95x^3+283x^4+\ldots\ldots$ to n terms.

2. If $$\frac{x}{y+z}=a, \quad \frac{y}{z+x}=b, \quad \frac{z}{x+y}=c,$$
prove that

(i.) $ab+ac+bc=1-2abc$,

(ii.) $$\frac{x^2}{a(1-bc)}=\frac{y^2}{b(1-ac)}=\frac{z^2}{c(1-ab)}.$$

3. The expression
$$\frac{ax^2+2bx+c}{a'x^2+2b'x+c'}$$
will be capable of all real values if
$$(ac'-a'c)^2<4(a'b-ab')(b'c-bc').$$

4. If n be a large number, and d any small increase or decrease to it, prove that
$$\log_a(n \pm d)=\log_a n \pm \frac{1}{\log_e a} \cdot \frac{d}{n} \quad \text{approximately.}$$

5. If $$(a+b\sqrt{-1})^{\frac{1}{2}}=x+y\sqrt{-1}$$
find the values of x and y in terms of a and b.

6. A stone is thrown horizontally from the top of a tower of height h, with velocity $\sqrt{2gh}$. At what distance from the foot of the tower, and at what angle, will it strike the ground?

7. A man, whose weight is $1\frac{1}{2}$ cwt., climbs a ladder, 30 feet long, and $\frac{1}{2}$ cwt. in weight, which is supported at an angle $\tan^{-1}\frac{3}{7}$ to the vertical, by resting at a point 3 feet from its upper end against a smooth horizontal scaffolding pole. Write down the equations from which may be found the pressure on the ground, and the friction at the lower end of the ladder, at any point of his ascent.

If the coefficient of friction be $\frac{1}{3}$, how far up the ladder can he go?

8. A sphere of one foot radius rests on a table. Find the volume of the right hollow cone which can just cover it, the section of the cone through the axis being an equilateral triangle.

CCXLVIII.

1. If $$(1+x)^n = c_0 + c_1 x + c_2 x^2 + \ldots\ldots + c_n x^n,$$
   (i.) prove that
   $$c_0^2 + c_1^2 + \ldots\ldots + c_n^2 = \frac{2n(2n-1)\ldots\ldots(n+1)}{\lfloor n},$$
   (ii.) find the value of
   $$\frac{c_0}{1} + \frac{c_1}{2} + \frac{c_2}{3} + \ldots\ldots + \frac{c_n}{n+1}.$$
2. Prove that the integral part of
   $$(3+\sqrt{5})^n + 1$$
   is divisible by 2^n. (By Mathematical Induction.)
3. If $$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c},$$ show that
   $$a^{(2n+1)} + b^{(2n+1)} + c^{(2n+1)} = (a+b+c)^{(2n+1)}.$$
4. Given in a triangle the angle C, and the sides a and b, show how to find the angles A and B.
   Express the length of the perpendicular from the vertex C on the opposite side.
5. P is a point in the base AB of a triangle ABC, dividing it so that
   $$AP : PB :: m : n,$$
   and the angle $CPB = \theta$.
   Prove that $n \cot A - m \cot B = (m+n) \cot \theta$.
6. Draw the straight lines represented by
   $$y - 3x + 3a = 0, \quad \text{and} \quad 2y - x + 2a = 0;$$
   find the co-ordinates of their point of intersection, and the angle between them.

7. Two forces, P and Q, act in opposite parallel directions, at two given points A and B. Find where the direction of their resultant cuts the line AB.

Explain your result, when $P=Q$.

8. A point which has an initial velocity v, moves with uniform acceleration in a straight line. Find the space passed over in t seconds.

A stone dropped from a balloon, which was rising at the rate of 5 miles an hour, reached the ground in 9 seconds. From what height was it dropped?

CCXLIX.

1. If $$x(b-c)+y(c-a)+z(a-b)=0,$$
show that

(i.) $$\frac{bz-cy}{b-c}=\frac{cx-az}{c-a}=\frac{ay-bx}{a-b},$$

(ii.) $$\frac{y-z}{b-c}=\frac{z-x}{c-a}=\frac{x-y}{a-b}.$$

2. If $$\log(1+x)^{\frac{1}{1-x}}=a_0+a_1x+a_2x^2+\ldots\ldots$$
prove that $$2n(a_{2n-1}-a_{2n})=1.$$

3. Sum the series :—

(i.) $$\left(\frac{6}{1+}\;\frac{6}{1+}\;\frac{6}{1+}\ldots\ldots\right)+\left(\frac{8}{2+}\;\frac{8}{2+}\;\frac{8}{2+}\ldots\ldots\right)$$
$$+\left(\frac{10}{3+}\;\frac{10}{3+}\;\frac{10}{3+}\ldots\ldots\right)+\ldots\ldots \text{ to } n \text{ terms.}$$

(ii.) $$\frac{1}{1\,.\,2}+\frac{3}{2\,.\,5}+\frac{5}{5\,.\,10}+\frac{7}{10\,.\,17}+\ldots\ldots \text{ to } n \text{ terms.}$$

4. Find the series for the expansion of $\cos\alpha$ in powers of α.

In what measure is the angle α expressed?

Prove that the series is convergent.

5. Find exponential values for $\cos\theta$ and $\sin\theta$.

Show that the coefficient of θ^{2n} in the expansion of $e^{\theta}\cos\theta$ is
$$\frac{2^n}{\lfloor 2n}\cos\frac{n\pi}{2}.$$

6. A hollow paper cone, whose vertical angle is 60°, is held vertex downwards, and in it placed a sphere of radius 2 inches. The upper portion is cut away along the line where the cone and sphere touch. Find the area of the remaining surface.

7. A uniform triangular lamina, of weight W, is just raised from a smooth horizontal table by a force applied at A. Find the force and the pressure on the table. Show that if the applied force be not vertical, the lamina will slide.

8. How is an impulsive force measured?

If a given impulse acting on a stone causes it to rise to a height h, how high will a stone of half the weight rise under the action of the same impulse?

CCL.

1. Sum the series :—

(i.) $\dfrac{1\,.\,r}{\lfloor 1+r} + \dfrac{2\,.\,r^2}{\lfloor 2+r} + \ldots\ldots$ to n terms,

(ii.) $\dfrac{3}{1\,.\,2\,.\,3} + \dfrac{5}{2\,.\,3\,.\,4} + \dfrac{7}{3\,.\,4\,.\,5} + \ldots\ldots$ to n terms.

(iii.) $4 + 5x + 7x^2 + 11x^3 + \ldots\ldots$ to infinity. $(x < 1.)$

2. If
$$\frac{x - \dfrac{yz}{x}}{1 - yz} = \frac{y - \dfrac{zx}{y}}{1 - zx},$$
then each of these is equal to
$$\frac{z - \dfrac{xy}{z}}{1 - xy} = x + y + z = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

3. If $$(1+x)^n = c_0 + c_1x + c_2x^2 + \ldots\ldots + c_nx^n$$

prove that

(i.) $$(c_0 - c_2 + c_4 - c_6 + \ldots)^2 + (c_1 - c_3 + c_5 - \ldots\ldots)^2$$
$$= c_0 + c_1 + c_2 + \ldots\ldots = 2^n,$$

(ii.) $$\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \ldots\ldots = \frac{n(n+1)}{2}.$$

4. Prove De Moivre's Theorem for a negative index.

Show that the expression

$$\sqrt[n]{a+b\sqrt{-1}} + \sqrt[n]{a-b\sqrt{-1}}$$

has n real values, and find those of

$$\sqrt[3]{1+\sqrt{-3}} + \sqrt[3]{1-\sqrt{-3}}.$$

5. Show that

$$\tan(\alpha+\beta+\gamma+\ldots) = \frac{s_1 - s_3 + s_5 - \ldots\ldots}{1 - s_2 + s_4 - \ldots\ldots},$$

where s_r denotes the sum of the products of $\tan\alpha$, $\tan\beta$, etc., taken r together.

6. Find the equation of a circle which touches the y-axis on the positive side of the x-axis, and cuts the x-axis at distances a, $4a$, from the origin.

Show that the equations of the tangents to the circle at the points of intersection with the axis are

$$3x + 4y = 3a \quad \text{and} \quad 3x - 4y = 12a.$$

7. Find the equation of the straight line through $(5, 6)$, which has its intercepts on the axis,

(i.) equal in magnitude and both positive,

(ii.) equal and of opposite signs.

8. Find the locus of the middle points of a system of parallel chords in an ellipse.

CCLI.

1. If $(1+x)^n = 1 + c_1x + c_2x^2 + c_3x^3 + \ldots\ldots + c_nx^n$, prove that

   (i.) $1 - c_1\dfrac{1+x}{1+nx} + c_2\dfrac{1+2x}{(1+nx)^2} - c_3\dfrac{1+3x}{(1+nx)^3} + \ldots\ldots = 0.$

   (ii.) $1 + 3c_1 + 5c_2 + \ldots\ldots + (2n+1)c_n = (n+1)2^n.$

2. If $$y = x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \ldots\ldots \textit{ ad inf.}$$ find the series for x in terms of y as far as 4 terms.

3. Find the value of
$$\frac{1}{b+}\,\frac{1}{a+}\,\frac{1}{b+}\,\frac{1}{a+}\ldots\ldots$$
Show that if
$$u = a + \frac{b}{a+}\,\frac{b}{a+}\ldots\ldots \quad \text{and} \quad v = a + \frac{1}{a+}\,\frac{1}{a+}\ldots\ldots,$$
then
$$v = \frac{u^2-b}{u} + \cfrac{1}{\cfrac{u^2-b}{u}+}\;\cfrac{1}{\cfrac{u^2-b}{u}+}\ldots\ldots.$$

4. Sum the series :—

   (i.) $\sin\alpha - \sin(\alpha+\beta) + \sin(\alpha+2\beta) - \ldots\ldots,$

   (ii.) $\sin\alpha + \sin\beta\sin 2\alpha + \sin^2\beta\sin 3\alpha + \ldots\ldots.$

5. From a ship the angle between 2 forts A and B is 30°; the ship sails 4 miles towards A, and then the angle is 48°; find the distance of the ship from B, at the second observation.

6. A uniform wire is bent into the shape of an equilateral triangle; how must it be suspended to hang with one side vertical?

7. Construct 2 systems of pulleys by which weights of 6 lbs. and 7 lbs. respectively can be balanced by a downward force of 1 lb.; friction being neglected and the weight of the pulleys taken into account.

8. A particle is projected from a point O, and two lines OA, OB represent in magnitude and direction the velocity of projection and that due to gravity at the end of a second. Construct geometrically the positions of the particles at the end of the 1st, 2nd, and 3rd seconds.

CCLII.

1. Sum the series :—

(i.) $$\frac{1}{a^2(a^2+4)}+\frac{3}{(a^2+4)(a^2+4.2^2)}+\frac{5}{(a^2+4.2^2)(a^2+4.3^2)}+\ldots\ldots,$$

(ii.) $1.3.5+3.5.7+5.7.9+\ldots\ldots,$

(iii.) $1+5+12+22+\ldots\ldots$, all to n terms.

2. Prove that

$$a(b-c)(1+ab)(1+ac)+2 \text{ similar expressions}$$
$$\equiv abc(b-a)(a-c)(c-b).$$

3. Eliminate x and y from

$$y^2-x^2=ay-\beta x, \quad 2xy=ax+\beta y, \quad x^2+y^2=1.$$

4. In the middle of a field in the shape of an equilateral triangle there is a tower 200 feet high. From the top of the tower each side subtends an angle $\cos^{-1}\frac{1}{10}$. Find a side of the field.

5. Assuming De Moivre's Theorem for integral indices, prove it for a fraction.

Hence find the fifth roots of unity.

6. A hollow right prism stands on a base which is an equilateral triangle. The vertical faces of the prism are squares, whose side is 1 foot. The prism is filled with water, and the largest possible sphere immersed in it. Find the amount of water left in the prism.

7. Write down the general equation for a circle in rectangular co-ordinates; and show from it that if a straight line OPQ be drawn from the origin to meet the circle in P and Q, the rectangle OP, OQ is constant for all directions of OPQ.

8. Show that $$y + mx - 2am - am^3 = 0$$ is the equation of a normal to the parabola $$y^2 = 4ax.$$

Find the equation of the locus of the fourth vertex of the rectangle, of which three vertices are the vertex of the parabola and the intersections of a normal with the axis and the tangent at the vertex.

CCLIII.

1. Prove that

$$\sqrt{p - \sqrt{p - \sqrt{p - \sqrt{p - \ldots\ldots}}}}\ \textit{ad inf.} = \frac{m^2 - mn + n^2}{2mn}$$

if $$p = \frac{m^4 + m^2n^2 + n^4}{4m^2n^2}.$$

2. Prove that

(i.) $\left(1 + \frac{x}{n}\right)^n > \left(1 + \frac{x}{n-1}\right)^{n-1}$,

(ii.) $n+1$ is not $> (r+2)(n-r)$, if $r < n-1$.

(iii.) $(n+1)^{n-1} < (\underline{|n})^2$.

3. The radix of the scale in which 49 is a square is of the form $$(r+1)(r+4).$$

4. Sum the series :—

(i.) $\sin\theta + \sin 2\theta + \sin 3\theta + \ldots\ldots$,

(ii.) $\sin 2\theta \sec\theta \sec 3\theta + \sin 4\theta \sec 3\theta \sec 5\theta$
$+ \sin 6\theta \sec 5\theta \sec 7\theta + \ldots\ldots$,

each to n terms.

5. Eliminate θ and ϕ from

$$\sin^2\theta \cos\gamma = \cos\alpha, \quad \sin^2\phi \cos\gamma = \cos\beta,$$
$$\tan\theta \tan\alpha = \tan\phi \tan\beta.$$

6. An iron cylinder, 12 feet long, 5 feet in diameter, and weighing 1000 cwt., is lying on the ground: find (i.) the least force directly applied, (ii.) the least work, required to raise it to the vertical.

7. Investigate the relation between the power and weight in the case of a screw, friction being neglected.

   Prove that for a given amount of work done, the power varies inversely as the distance through which its point of application moves.

8. A mass P hanging freely, draws another mass Q along a smooth horizontal table, by means of a string passing over a pulley at the edge. Find the acceleration (f) and the tension of the string. If the table be rough, and the coefficient of friction μ, the acceleration will be diminished by $\mu(g-f)$.

CCLIV.

1. Prove that $n^2 - n + 1$ cannot be a square, if n is an integer.

   If n be odd $n(n^2-1)$ is divisible by 24.

2. If $$(1+x)^n = c_0 + c_1 x + c_2 x^2 + \ldots\ldots + c_n x^n,$$
   prove that

   (i.) $(c_0 + c_1)(c_1 + c_2)\ldots\ldots(c_{n-1} + c_n) = \dfrac{(n+1)^n}{\lfloor n} c_0 c_1 \ldots\ldots c_n,$

   (ii.) The sum of the first $\overline{r+1}$ coefficients in the expansion of $(1-x)^{-2n}$ is

$$\frac{\lfloor 2n+r}{\lfloor 2n \ \lfloor r}.$$

3. Sum the series :—

(i.) $\frac{0 \cdot 2}{1 \cdot 2} + \frac{1 \cdot 2^2}{2 \cdot 3} + \frac{2 \cdot 2^3}{3 \cdot 4} + \ldots\ldots,$

(ii.) $\frac{1}{1 \cdot 3} + \frac{2}{1 \cdot 3 \cdot 5} + \frac{3}{1 \cdot 3 \cdot 5 \cdot 7} + \ldots\ldots.$

4. If θ be the circular measure of an angle in the first quadrant, prove that

$$\cos\theta < 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24}.$$

If $$\frac{\sin\theta}{\theta} = \frac{2165}{2166},$$

show that $\theta = 3°$ nearly.

5. Prove that the quantity e is incommensurable.

Show that

$$1 + \frac{2}{1} + \frac{3}{\lfloor 2} + \frac{4}{\lfloor 3} + \ldots\ldots = 2e.$$

6. Prove that $$\frac{l}{r} = 1 + e\cos\theta,$$

and $$\frac{l}{r} = \cos(\theta - a) + e\cos\theta$$

are the polar equations respectively of a conic, and of a straight line which only meets the conic in one point.

7. Find the equation of the normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at any point.

If CP, CD be conjugate semi-diameters, and $CP = \sqrt{ab}$, show that the normal at D touches the circle

$$x^2 + y^2 = (a - b)^2.$$

8. Trace the loci :—

(i.) $(2x + y)^2 - (x + a)(x - a) = 0,$

(ii.) $x^2 + y^2 + \frac{13}{6}y(x - 1) - 2x + 1 = 0.$

CCLV.

1. If
$$a+\frac{y^2-z^2}{b-c}=b+\frac{z^2-x^2}{c-a},$$
then each $=c+\dfrac{x^2-y^2}{a-b}$.

2. Show that if $\dfrac{e^x}{1-x}=1+a_1x+a_2x^2+\ldots\ldots$

then
$$a_n-a_{n-1}=\frac{1}{\lfloor n}.$$

3. Prove that

(i.) $$\left(\frac{1}{b+}\,\frac{1}{a+}\,\frac{1}{b+}\,\frac{1}{a+\ldots}\right)\left(a+\frac{1}{b+}\,\frac{1}{a+}\,\frac{1}{b+}\,\ldots\right)=\frac{a}{b},$$

(ii.) $$\frac{a_1}{a_1+}\,\frac{a_2}{a_2+}\,\frac{a_3}{a_3+\ldots}\;+\frac{a_n}{a_n}\equiv\frac{1}{1+}\,\frac{1}{a_1+}\,\frac{a_1}{a_2+}\ldots+\frac{a_{n-2}}{a_{n-1}}.$$

4. If n be a positive integer expand $(\cos\theta)^{2n+1}$ in a series of cosines of multiples of θ.

Express $\cos^5\theta\sin^2\theta$ in terms of cosines of multiples of θ.

5. Show that
$$\tan\theta+\tfrac{1}{3}\tan^3\theta+\tfrac{1}{5}\tan^5\theta+\ldots=\tfrac{1}{2}\log\frac{\cos\left(\theta-\dfrac{\pi}{4}\right)}{\cos\left(\theta+\dfrac{\pi}{4}\right)}.$$

6. A hollow shell, 12 inches in diameter, is placed in a conical vessel, whose vertical angle is 60°, and water poured into it, until it just covers the shell, and fills the cavity in it. When the shell, emptied of the water in it, is removed, and a solid ball of the same diameter substituted, the water stands $\frac{1}{2}$ inch above it. Find the thickness of the shell.

7. A uniform rod AB rests against a smooth vertical wall AC, and is supported by a string BC. Show by a diagram the lines of action of the forces which keep it in equilibrium, and find the forces.

8. A stone A is thrown vertically upwards with a velocity of 96 feet per second. How high will it rise? After 4 seconds another stone B is let fall from the same point. When will A overtake B?

CCLVI.

1. If
$$x = \frac{1}{a_1 +}\,\frac{1}{a_2 +}\,\frac{1}{a_1 +}\,\frac{1}{a_2 + \ldots\ldots} = f(a_1, a_2),$$
$$y = f(2a_1, 2a_2), \quad z = f(3a_1, 3a_2),$$
then will
$$x(y^2 - z^2) + 2y(z^2 - x^2) + 3z(x^2 - y^2) = 0.$$

2. Prove that

(i.) $\left(\dfrac{a_1 + a_2 + \ldots\ldots + a_n}{n}\right)^n > a_1 a_2 \ldots\ldots a_n,$

(ii.) $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}$ is not $> \dfrac{a^2 + b^2 + c^2}{abc},$

and not $< \dfrac{\sqrt{a} + \sqrt{b} + \sqrt{c}}{\sqrt{abc}}$, a, b, c, being positive quantities.

3. If
$$y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots,$$
find a series for x in terms of y.

4. Establish the exponential expressions for $\cos\theta$ and $\sin\theta$.

If
$$C + iS = e^{x \cdot e^{i\theta}}$$
where $i = \sqrt{-1}$, find the values of C and S.

5. Prove that in any triangle
$$c^2 = (a-b)^2 \cos^2\frac{C}{2} + (a+b)^2 \sin^2\frac{C}{2},$$
and show how to adapt it to logarithmic computation.

6. Prove that
$$y = mx + c$$
is the equation of a straight line, and interpret the constants.

Two straight lines cut the x-axis at distances a and $-a$, and the y-axis at distances b and b' respectively, from the origin. Find their point of intersection.

7. If in the last example a number of lines be drawn with a the same, and $bb'=a^2$ in each pair, find the locus of the points of intersection of the pairs.

8. Find the equation of the polar of (h, k) with regard to the ellipse $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

CCLVII.

1. Prove that

(i.) $x+y>2\sqrt{xy}$,

(ii.) $\dfrac{(ax)^{\frac{1}{4}}+(by)^{\frac{1}{4}}}{(a+b)^{\frac{1}{2}}+(x+y)^{\frac{1}{2}}}$ is a proper fraction.

2. Sum the series :—

(i.) $1^2+3^2+5^2+\ldots\ldots$,

(ii.) $\dfrac{1}{2\,.\,5\,.\,8}+\dfrac{1}{5\,.\,8\,.\,11}+\ldots\ldots$,

(iii.) $1+3x+5x^2+7x^3+\ldots\ldots$.

3. Prove that

$$p^n+q^n=(p+q)^n-n(p+q)^{n-2}pq+\frac{n(n-3)}{\lfloor 2}(p+q)^{n-4}p^2q^2 - \ldots\ldots$$

4. Show that

(i.) $\sin\theta=\theta\left(1-\dfrac{\theta^2}{\pi^2}\right)\left(1-\dfrac{\theta^2}{2^2\pi^2}\right)\left(1-\dfrac{\theta^2}{3^2\pi^2}\right)\ldots\ldots$,

(ii.) $i\log\dfrac{x-i}{x+i}+2\tan^{-1}x=\pi$. $\quad (i=\sqrt{-1}.)$

5. Show how to find by the measurements of straight lines only the position of an inaccessible object on a horizontal plane; and the distance between two inaccessible objects in the plane.

6. A heavy truck of mass 16 tons is standing at rest on a smooth line of rails. A horse pulls steadily with a force equal to the weight of 1 cwt. How far will he move it in one minute?

7. A uniform beam AB, weighing 100 lbs., is held in position by the tension of a string PH, P being a point in BA, $\frac{1}{7}$ of the length of AB from B, and the angle $HPB = 75°$. The end A rests on a rough horizontal plane AC, so that the angle $PAC = 45°$. Find the tension of the string, and the least possible value for the angle of limiting friction.

   Solve this also graphically.

8. In a vertical circle find that radius down which a particle will fall from the centre to the circumference in double the time that it takes to fall down the vertical diameter.

CCLVIII.

1. If a_n be the coefficient of x^n in the expansion of
$$\frac{a+bx}{1+x+x^2},$$
then $a_n = a_{n-3}$.

   Hence write down the expansion.

2. Prove that
$$\left(n + \frac{1}{2n+}\,\frac{1}{2n+}\,\frac{1}{2n+\ldots}\right)^2 - \left(n - \frac{1}{2n-}\,\frac{1}{2n-}\,\frac{1}{2n-\ldots}\right)^2 = 2.$$

   If
$$x = \frac{y}{y^2+}\,\frac{y}{y^2+}\cdots\cdots,$$
then
$$y = \frac{x}{\frac{1}{x}-}\,\frac{x}{\frac{1}{x}-}\,\frac{x}{\frac{1}{x}-}\cdots\cdots$$

3. Show that

$$(2x+a)\sqrt{a-x}$$

has its greatest value when $x=\frac{a}{2}$.

4. Prove that in any triangle

$$r_a+r_b+r_c=4R+r.$$

Find all the elements of the triangle, given the base, radius of inscribed circle, and radius of circle escribed on the base.

Construct it also geometrically.

5. Discuss the following data for a triangle :—

(i.) $A=43°\ 15'$, $AB=36{\cdot}5$, BC 20,

(ii.) $A=43°\ 15'$, $AB=36{\cdot}5$, BC 30,

(iii.) $A=43°\ 15'$, $AB=36{\cdot}5$, BC 45.

6. Find the cost of the canvas, 2 feet wide, at 3s. 6d. a yard, required to make a conical tent, 12 feet in diameter, and 8 feet high.

7. Find the equation to a chord of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

in terms of the eccentric angles of its ends.

If the difference between the eccentric angles be 60°, show that the intercepts α and β of the chord on the axes are connected by the equation

$$\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}=\frac{4}{3}.$$

8. Explain how to transform from polar to rectangular co-ordinates, and *vice versa*.

Find the equation of the straight line through the point (1, 2), which cuts off equal intercepts on the axes.

CCLIX.

1. Prove that

(i.) $(a+b+c)^3 > 27abc \quad < 9(a^3+b^3+c^3)$,

(ii.) if a, b, c are in H.P., then $a^n + c^n > 2b^n$.

2. Show that every square number is of the form $5m$, or $5m \pm 1$.

If $a^2 + b^2 = c^2$, one of the three is divisible by 5.

3. Show that every integer is the sum of a number of terms (not necessarily consecutive) taken from the series

$$1,\ 2,\ 2^2,\ 2^3, \ldots\ldots$$

4. A statue on the top of a pillar, on level ground, subtends the greatest angle (α) at the eye of an observer, when his distance from it is c feet; prove that the length of the statue is $2c \tan \alpha$ feet, and having given the height above the ground of the observer's eye, find the height of the pillar.

5. Prove that the limit of $n \tan \dfrac{\alpha}{n}$, when n is large, is α.

Having given the radius r of a circle inscribed in a regular polygon of n sides, find the area of the polygon, and hence deduce an expression for the area of a circle.

6. The beam of a balance is 6 feet long, and it appears correct when empty. A body placed in one scale weighs 120 lbs., and in the other 121 lbs. Find the position of the fulcrum.

7. Find the amount of foot-pounds of work expended in drawing a weight of 100 lbs. up a slope of 1 in 10, for a distance of 100 feet, the coefficient of friction being $\frac{1}{6}$, and the force parallel to the plane.

8. Four forces acting along, and represented by, the sides of a parallelogram form two couples of opposite senses. Prove that they are in equilibrium.

CCLX.

1. Given $$(1+r)^x = 1 + r + rx,$$ find an approximate value of r,
   (i.) when r is small, (ii.) when r is large.

2. Every cube is of the form $7m$ or $7m \pm 1$.
   Show that $$n^4 - 4n^3 + 5n^2 - 2n$$ is divisible by 12.

3. If $x < 1$, then
$$\frac{1}{x} + \frac{1}{\log(1-x)} < 1.$$

4. Prove that
$$\text{(i.)}\quad \theta = \tan\theta - \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} - \ldots\ldots$$
$$\text{(ii.)}\quad \left(3^{\frac{1}{2}} - 3^{-\frac{1}{2}}\right) - \frac{1}{3}\left(3^{\frac{3}{2}} - 3^{-\frac{3}{2}}\right) + \frac{1}{5}\left(3^{\frac{5}{2}} - 3^{-\frac{5}{2}}\right) - \ldots\ldots = \frac{\pi}{6}.$$

5. Show that
$$\cos\theta + i\sin\theta = e^{i\theta}, \quad (i = \sqrt{-1})$$
and hence prove that
$$\sin(\theta + i\phi) = \sin\theta \,.\, \frac{e^{\phi} + e^{-\phi}}{2} + i\cos\theta \,.\, \frac{e^{\phi} - e^{-\phi}}{2}.$$

6. Find the polar equation of a circle.
   Show that for any given value of θ there are two values of r and find the condition which connects them.
   The points $(1, 60°)$, $(2, 30°)$ are the ends of the diameter of a circle. Show that its equation is
$$r^2 - r\{\cos(\theta - 60°) + 2\cos(\theta - 30°)\} + \sqrt{3} = 0.$$

7. Prove that the length of the subnormal in the parabola is constant.

8. Find the equation of the tangent to a parabola, in Cartesian co-ordinates.

Show that the parabolæ

$$y^2 = 8x, \quad x^2 = 27y$$

cut one another at an angle

$$\tan^{-1}\frac{9}{13}.$$

CCLXI.

1. Prove that $n^5 - n$ is divisible by 30, and if n be odd by 120.

If $a^2 + b^2 = c^2$, then abc is divisible by 60.

2. Show that $4xy - 3(x^2 - y^2)^{\frac{2}{3}}$ is $>$ or <1 according as $(x+y)^{\frac{2}{3}} - (x-y)^{\frac{2}{3}}$ is $>$ or <1.

3. Find the value of

$$\sqrt{3 + \sqrt{\frac{2}{\sqrt{3 + \frac{2}{\sqrt{3 + \frac{2}{\sqrt{3 + \ldots\ldots}}}}}}}}$$

4. Show that $$\cos\frac{m}{n}\theta + \sqrt{-1}\sin\frac{m}{n}\theta$$

is one of the values of

$$(\cos\theta + \sqrt{-1}\sin\theta)^{\frac{m}{n}}.$$

What are the other values?

5. Prove that

$$\frac{\pi}{4} = 4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}.$$

Hence deduce the value of π to 5 places of decimals.

6. A well, 5 feet in diameter, and 30 feet deep, is to have a lining of bricks fitting close together, without mortar, 9 inches thick. A brick $9 \times 4\frac{1}{2} \times 3$ inches weighs 5 lb. Find the total weight approximately in tons.

7. If a body be let fall from a height of 64 feet at the same instant that another is sent vertically upwards from the foot of the height with a velocity of 64 feet, find when they will meet.

8. The sides BC, CA, AB, of a triangle are 3, 4, and 5 feet respectively; find the magnitude and direction of a force acting at C, whose moments about A and B are 7 and 5 foot-pounds respectively.

CCLXII.

1. Between a and b $(a<b)$ insert n means,

$$a_1, a_2 \ldots\ldots a_n,$$

such that $a_1 - a, a_2 - a_1, a_3 - a_2 \ldots\ldots b - a_n$, form an A.P. whose common difference is d; and find between what limits d must lie.

2. Prove that the numerator and denominator of the n^{th} convergent to $a + \dfrac{b}{2a+}\;\dfrac{b}{2a+}\ldots\ldots$ are the coefficients of x^n, in the expansion of

$$\frac{1-ax}{1-2ax-bx^2} \quad \text{and} \quad \frac{x}{1-2ax-bx^2}.$$

3. Show that

$$2^2 - \frac{3^2}{\underline{|2}} + \frac{4^2}{\underline{|3}} - \ldots\ldots = 1 + e^{-1}.$$

4. In any triangle show that

(i.) $\cos\dfrac{A-B}{2} = \dfrac{(a+b)\sin\theta}{2\sqrt{ab}}$ where $\cos\theta = \dfrac{a-b}{c}$,

(ii.) area $= Rr(\sin A + \sin B + \sin C)$.

5. Find the roots of the equation

$$x^{2n} - 2x^n \cos n\theta + 1 = 0.$$

6. Define the eccentric angle of any point on an ellipse. The sum of the eccentric angles of two points is 2β; show that the straight line joining them is parallel to the tangent at the point whose eccentric angle is β.

7. Find the condition that the circles

$$(x-a)^2 + (y-b)^2 = c^2$$

$$\text{and} \quad (x-b)^2 + (y-a)^2 = c^2$$

should intersect, and prove that if θ be the angle between the two tangents to one of the circles at the points of intersection, then

$$\sin\frac{\theta}{2} = \frac{a-b}{c\sqrt{2}}.$$

Explain the result if $a - b = c\sqrt{2}.$

8. Show that the equation

$$y = mx + a\left(m + \frac{1}{m}\right)$$

represents a straight line touching the parabola

$$y^2 = 4a(x+a).$$

CCLXIII.

1. Evaluate the determinant

$$\begin{vmatrix} 1, & 1, & 1, & 1, & 1 \\ 1, & 2, & 3, & 4, & 5 \\ 1, & 3, & 6, & 10, & 15 \\ 1, & 4, & 10, & 20, & 35 \\ 1, & 5, & 15, & 35, & 69 \end{vmatrix}.$$

2. Hendrik, Elias, and Cornelius, and their three wives, Katrine, Gertrude, and Anna, went to buy sheep. H. bought 23 more than K., and E. 11 more than G.

Each man spent £3 3s. more than his wife, and each man and woman paid as many shillings for each sheep as the number of sheep he or she bought. Who was each man's wife, and how many sheep did each buy?

3. Show that if $x < 1$, then

$$(1 + 9x^2 + 25x^4 + \ldots\ldots \textit{ ad inf.})^2$$
$$- (4x + 16x^3 + 36x^5 + \ldots\ldots \textit{ ad inf.})^2 = (1 - x^2)^{-2}.$$

4. Show that the distance between the centres of the inscribed and circumscribed circles of a triangle is

$$\sqrt{R^2 - 2Rr}.$$

If they coincide the triangle must be equilateral.

5. Sum the series :—

(i.) $\cos \alpha + \cos 2\alpha + \cos 3\alpha + \ldots\ldots,$

(ii.) $\tan \alpha + 2 \tan 2\alpha + 2^2 \tan 2^2\alpha + \ldots\ldots.$

6. A body of weight w lies on a smooth horizontal table. Find what horizontal force will cause it to move with unit acceleration.

7. A body, moving from rest with uniform (unknown) acceleration, acquires a velocity v in time t; find the space described.

8. Two weights, joined by a string passing over a pulley, are at rest on two smooth inclined planes of the same height, and whose highest points coincide at the pulley. Show by the principle of *virtual velocities* that the weights vary as the lengths of the planes, and that, however they are displaced, their centre of gravity moves horizontally

CCLXIV.

1. If $x + y + z = 0$, then

$$\left(\frac{y-z}{x} + \frac{z-x}{y} + \frac{x-y}{z}\right)\left(\frac{x}{y-z} + \frac{y}{z-x} + \frac{z}{x-y}\right) = 9.$$

2. If s_r be the sum to n terms of a G.P., whose first term is 1, and ratio r, prove that

$$s_1 + s_2 + 2s_3 + 3s_4 + \ldots\ldots + \overline{n-1}\,s_n = 1^n + 2^n + \ldots\ldots + n^n.$$

3. Sum to infinity

$$1 + \frac{1+x}{\underline{|2}} + \frac{(1+x)(1+2x)}{\underline{|3}} + \frac{(1+x)(1+2x)(1+3x)}{\underline{|4}} + \text{ etc.}$$

4. If D be the middle point of the side BC of a triangle, and S be the area of the triangle, then

$$\cot ADB = \frac{AC^2 - AB^2}{4S}.$$

5. If p_1, p_2, p_3 be the perpendiculars of a triangle, show that

$$\frac{1}{r} = \frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}.$$

6. A cathedral has two spires and a dome: each of the former consists, in the upper part, of a pyramid, 60 feet high, on a square base of 20 feet side. The dome is a hemisphere of 40 feet radius. Find the cost of covering the three with lead at $7\frac{1}{2}$d. per square foot.

7. Prove that $\qquad y^2 = 4a(x+a)$

is the equation of a parabola.

If two points be taken on the axis equidistant from the origin, the difference of the squares of the perpendiculars from them on any tangent is constant.

8. Find the equation of the straight line drawn through C the middle point of the line AB, and perpendicular to that line, the co-ordinates of A and B being (x_1, y_1) and (x_2, y_2), and the axes rectangular.

If A and B are on the axes of co-ordinates OX and OY, respectively, and the perpendicular through C meets the line bisecting the angle XOY in P, prove that $CP = \frac{1}{2}AB$.

CCLXV.

1. If
$$\frac{(2x-y-z)^3}{x}=\frac{(2y-z-x)^3}{y}$$
then each
$$=\frac{(2z-x-y)^3}{z}.$$

2. Find the coefficient of x^n in the expansion of
$$\frac{2x-3}{x^2-3x+2}.$$

3. Show that
$$ab(a+b)+ac(a+c)+bc(b+c)>6abc \text{ and } <2(a^3+b^3+c^3).$$

4. If $e^{i\theta}$ is a root of
$$x^3-px^2+qx-r=0$$
prove that
$$\sin 3\theta - p\sin 2\theta + q\sin\theta = 0. \quad (i=\sqrt{-1}.)$$

5. Sum the series :—

(i.) $\cos\alpha+\frac{1}{2}\cos 2\alpha+\frac{1}{3}\cos 3\alpha+\ldots\ldots,$

(ii.) $\sin\frac{\pi}{2n+1}-\sin\frac{3\pi}{2n+1}+\sin\frac{5\pi}{2n+1}-\ldots\ldots$

6. Find the equation of the normal to a parabola
$$y^2=4ax,$$
in the form $\quad y=mx-2am-am^3.$

If normals be drawn from (h, k) to the parabola, they meet it in points situated on the curve
$$y^2(2a-h)+4ax^2=2aky.$$

7. Show that the equation of a circle in terms of the co-ordinates of the ends of a diameter is
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0.$$

If ρ be the distance of (x, y) on the circle, from (x_1, y_1), show that
$$\rho^2=(x-x_1)(x_2-x_1)+(y-y_1)(y_2-y_1).$$

8. Investigate the loci represented by—

$$\text{(i.)}\ x^2+y^2+6x-4y=12,$$
$$\text{(ii.)}\ b^2x^2+a^2y^2-a^2b^2=0,$$

and (iii.) $r=a\operatorname{cosec}^2\frac{\theta}{2}$, by giving to θ the successive values 0°, 60°, 90°, etc.

CCLXVI.

1. If $x(y^2-z^2)=(b-c)yz$, and $y(z^2-x^2)=(c-a)zx$, then will $z(x^2-y^2)=(a-b)xy$.

2. Prove that

(i.) $(1+x)^n(1+x^n)>2^{n+1}x^n$,

(ii.) $8(a^3+b^3+c^3)>3(a+b)(b+c)(c+a)$.

3. Prove that $2^{2n}-3n-1$ is divisible by 9.

4. The circumference of a semi-circle of radius a is divided into n equal arcs. Show that the sum of the distances of the several points of section from the extremity of the diameter is $a\left(\cot\frac{\pi}{4n}-1\right)$.

5. If $\cos(\alpha+i\beta)=a-ib$, and $\sin(\alpha+i\beta)=c+id$,

show that $\alpha=\tan^{-1}\sqrt{\frac{bc}{ad}}$, and $\beta=\frac{1}{2}\log\dfrac{1-\sqrt{\frac{bd}{ac}}}{1+\sqrt{\frac{bd}{ac}}}$.

Prove that $\sin\alpha=\alpha-\frac{\alpha^3}{\lfloor 3}+\frac{\alpha^5}{\lfloor 5}-\text{etc.}$

6. Out of a uniform rectangular lamina $ABCD$ there is cut an isosceles triangle APD, with AD as base. The remainder balances about P in neutral equilibrium. Find the altitude of the triangle compared with AB.

7. If the unit of space be 2 feet, what must be the unit of time, that a weight of 1 lb. may be the unit force?

8. A body is dropped over the side of a vessel travelling at the rate of 10 miles an hour from a point 10 feet 1 inch above the sea. Find the velocity and direction of motion of the body when it strikes the water.

CCLXVII.

1. Prove that

   (i.) $2^{2n+1} - 9n^2 + 3n - 2$ is divisible by 54.

   (ii.) that $\dfrac{m^5}{120} - \dfrac{m^3}{24} + \dfrac{m}{30}$ is an integer.

2. If a, b, c are the sides of a triangle, show that
$$2(bc + ca + ab) > a^2 + b^2 + c^2.$$

3. If $a \propto b$, $b^2 \propto ac$ and $c \propto \sqrt[3]{a^2 d} + \sqrt[3]{ab^2}$ then
$$m\sqrt[4]{ad} - n\sqrt[4]{bc} \propto p\sqrt{a} + q\sqrt{d}.$$

4. If a, b, c be the sides of a triangle, and C the circular measure of the angle opposite to c, show that
$$\log c = \log a - \frac{b}{a}\cos C - \frac{b^2}{2a^2}\cos 2C - \frac{b^3}{3a^3}\cos 3C - \text{etc.}$$

5. Sum to n terms :—

   (i.) $\sin\theta\sin 3\theta + \sin 2\theta\sin 6\theta + \sin 4\theta\sin 12\theta + \text{etc.},$

   (ii.) $\cos^3\alpha + \cos^3(\alpha+\beta) + \cos^3(\alpha+2\beta) + \text{etc.}$

6. A parabola with focus S touches each of the axes (rectangular); if the equation of OS be
$$x\sin\theta - y\cos\theta = 0,$$
show that the equation of the directrix is
$$x\cos\theta + y\sin\theta = 0.$$

7. The sides of a right-angled triangle which contain the right angle are $3a$ and $4a$; find the equation of the circle circumscribing the triangle, referred to these sides as axes of X and Y respectively. Prove that the equation of the tangent at the origin is $3x + 4y = 0$.

8. Find the equation of an ellipse referred to a pair of conjugate diameters as axes. If through a fixed point on a given diameter of an ellipse any chord PQ be drawn, the tangents at P, Q, will always intersect on a straight line parallel to the conjugate diameter.

Prove this also geometrically by the properties of pole and polar.

ANSWERS.

I.

1. 192 groschen. **2.** ·833008. **3.** $3x^2+5x-8$.

4. $k+1+\frac{1}{k}$. **5.** (i.) $\frac{x^2-4xy-y^2}{(x^2-y^2)^2}$; (ii.) 1. **6.** 23 persons.

7. $(x+y+a-b)(x+y-a+b)(x-y+a+b)(-x+y+a+b)$.

8. (i.) Euc. I. 11, 31; (ii.) 4 points.

II.

1. 83. **2.** 1 acre 3 roods.

3. B wins by 50 yards. Time, 4 min. 45 secs.

4. (i.) $x=30$, $y=20$; (ii.) $x=-\frac{2}{3}$, 3, or $-\frac{1}{2}$.

5. $\frac{abc}{a+b}$ miles each way. **6.** $\frac{x+y+z}{2}$.

7. $4x^2-5x+8$. **8.** Euc. I. 8, 27.

III.

1. A, 12 days; B, 15 days; C, 18 days. **2.** $7\frac{71}{91}$.

3. $16x^4-24x^3+36x^2-54x+81$. **4.** $(a^2+b^2)(a^6-b^6)$

5. (i.) 2; (ii.) -3; (iii.) $\frac{3x-5}{(x-1)(x-2)}$.

6. (i.) $x=\frac{b}{a}$: (ii.) $x=\frac{-b\pm\sqrt{b^2-ac}}{a}$; (iii.) $x=\sqrt[3]{3^3}$, or $\sqrt[3]{\frac{1}{3^3}}$.

7. 26. **8.** Euc. I. 1, 31

IV.

1. £9000. **2.** £28 11s. 6d.

3. (i.) $2x^5 + 6x^3 - \frac{1}{4}x^2 - \frac{3}{4}$; (ii.) $0 \times 1\frac{1}{2} = 0$.

4. (i.) $\dfrac{1}{x-3}$; (ii.) $\left(\dfrac{a}{b}\right)^{\frac{1}{3}}$, or $\sqrt[3]{\dfrac{a}{b}}$.

5. G.C.M., $3x - 2y$;
L.C.M., $5(3x-2y)(4x+5y)(5x+4y)(x+y)$.

6. (i.) $x = 12, y = 15$; (ii.) $x = 3$, or $-\frac{26}{9}$; (iii.) $x = 2$, [or -7].*

7. 24 ft. long by 21 ft. wide. **8.** Euc. I. 4, 27.

V.

1. $3\frac{67}{504}$. **2.** £1 7s. $3\frac{3}{8}$d.

3. $x^2 + (a-2b)x + a^2 + 2ab + 4b^2$.

4. $x - \frac{2}{3} + \dfrac{3y}{2x}$. **5.** $5x - y$.

6. (i.) $x = 8$; (ii.) $x = 3, y = -4$; (iii.) $x = 64$, or 25, $y = 4$, or 1. **7.** 400 inches. **8.** Euc. I. 39.

VI.

1. Friday, 26th June, 1761. **2.**

3. £586 2s. $0\frac{24}{59}$d. **4.** $\dfrac{\sqrt{m(m+3n)}}{4}$.

5. $b^2x^2(m+1) + abx(m+1)(n+1) + a^2(n+1)$.

6. (i.) $\dfrac{3x^2 + 7x - 12}{(x^2-16)(x^2-9)}$; (ii.) $\dfrac{1}{(x-3)(x-4)}$.

7. (i.) $x = -\frac{575}{48} = -11\frac{47}{48}$; (ii.) $x = b$, or $\dfrac{b}{\sqrt[3]{5}}$; $y = a$, or $-\dfrac{a}{\sqrt[3]{5}}$.

8. Euc. I. 32; III. 20.

* An answer enclosed thus in square brackets is obtained by taking the negative value of a square root.

VII.

1. 10·189875. **2.** He would gain 5 per cent.
3. £17135. **4.** $18a^2b^2(a^6 - b^6)$. **5.** (i.) q; (ii.) p^2.
6. $x = 9$, or -3. **7.** $32\frac{8}{11}$ mins. past 12. **8.** 1.

VIII.

1. £1820. **2.** $56\frac{2}{3}$ per cent. **3.** £12. ($57\frac{3}{5}$ dollars.)
4. (i.) $x = 6$, or 4; (ii.) $x = 3$, or -1; $y = 2$, or -6.
5. 5, 10, 15. **6.**
7. 30 and 50 miles an hour. **8.** 2.

IX.

1. 139 yds. 5 ins. (5·046768 in., more exactly).
2. £586 2s. $0\frac{24}{59}$d. **3.** £3 0s. $1\frac{7}{11}$d. per cwt.
4. (i.) $\dfrac{x^2 + y^2}{(x + y)^2}$; (ii.) $\dfrac{1}{(x - 1)(x - 2)(x - 3)}$. **5.** $a^2 + b^2$. **6.**
7. (i.) $x = 4\frac{2}{3}$; (ii.) $x = \pm 4$, $y = \pm 2$. **8.** 30 and 40.

X.

1. 75 days. **2.** $\frac{3773}{4216}$. **3.** £46 6s. $10\frac{775}{896}$d.
4. (i.) $\dfrac{3}{x(x^3 - 1)}$; (ii.) $\dfrac{x^2 - 4xy + y^2}{x^2 + y^2}$.
5. (i.) $x = 3$; (ii.) $x = \dfrac{3b}{2}$, $y = -\dfrac{a}{2}$.
6. $4(x^2 - 9)(9x^2 - 4)$. **7.** 1, -1, -3, -5, -7.
8. Euc. I. 13, 32.

XI.

1. 11 more. **2.** $2\frac{3}{5}$. **3.** 25·856(5).
4. 25 mins. to 10. **5.** 59. **6.** $x^2 - x - 2 = 0$.
7. $\dfrac{2x + 10}{2} - x \equiv 5$. **8.** (i.) $x = -\frac{3}{2}$, $y = \frac{12}{7}$; (ii.) $x = 0$, $y = 1$.

XII.

1. £9584 6s. $6\frac{3}{8}$d. **2.** ·01875. **3.** $414\frac{78}{113}$ feet.

4. 5. **5.** (i.) 1; (ii.) $\dfrac{8x-2}{x(x-1)(x+2)}$; (iii.) $\dfrac{a^2-b^2}{x^2-y^2}$.

6. $144a^2b^2(a^2-x^2)$. **7.** $336\frac{2}{3}$. **8.** Euc. I. 1, 31.

XIII.

1. 1·58113883...... **2.** $\frac{2}{3}$. **3.** £200.

4. (i.) $x=15a$; (ii.) $x=\frac{3}{2}$, $y=\frac{2}{3}$. **5.** $2x^2-3$.

6. $\dfrac{100x+10y+z}{9}\equiv 11x+y+\dfrac{x+y+z}{9}$. **7.** (i.) $\dfrac{3}{y}$; (ii.) $5x^3$.

8. Euc. I. 8, 34.

XIV.

1. 8 ft. **2.** £29 10s. $1\frac{7}{17}$d. better. **3.** $1\frac{11}{19}$ hours.

4. $x^2+1+\dfrac{1}{x^2}$. **5.**

6. (i.) $x=\dfrac{-25\pm5\sqrt{41}}{8}$; (ii.) $x=5$, or -4; $y=4$, or -5.

7. $\left(\dfrac{y^2-x^2}{x}\right)^2$. **8.** Euc. I. 47.

XV.

1. (i.) 9s. $8\frac{1}{2}$d.; (ii.) £58 13s. 3d. **2.** $1\frac{1}{6}$.

3. L C.M. $(x+1)(x+3)(x-10)(x^2+5x+10)$; G.C.M. x^2-2.

4. $\frac{6}{7}$. (Divide numerator and denominator by the factor $x-2$.)

5. (i.) $x=2$, or -8, [or $-3(1\pm\sqrt{5})$]; (ii.) $x=2, 5$; $y=5, 2$.

6. Man, 3s.; Boy, 2s. **7.** $\dfrac{x+y}{xy}$.

8. Euc. I. 3, 31, 34.

XVI.

1. £60. **2.** $\frac{5}{9}$. **3.** (i.) 11·7578125; (ii.) £11757 16s. 3d.

4. (i.) $x=13$; (ii.) $x=\dfrac{2a+b}{2}$, $y=\dfrac{2a-b}{2}$.

5. (i.) 0; (ii.) $2\frac{1}{3}$.

6. (i.) $x^{2r}+x^r y^s+\dfrac{y^{2s}}{4}$; (ii.) $1+\dfrac{b}{a}+\dfrac{b^2}{a^2}+\dfrac{b^3}{a^3}$.

7. **8.** Euc. I. 29, 32.

XVII.

1. 729 yards. **2.** 192000.

3. 6336000 cub. ft.; $176785\frac{5}{7}$ tons.

4. (i.) $-\dfrac{1}{a}$; (ii.) $\dfrac{4x^2-4x+1}{4x^2-3x-1}$. **5.** a^6-4a^3+1.

6. (i.) $x=5$; (ii.) $x=7$; (iii.) $x=5$, $y=7$.

7. (i.) $(\frac{3}{5})^{17}$; (ii.) $\frac{125}{18}\{1-(\frac{3}{5})^{20}\}$. **8.** 15 and 24.

XVIII.

1. 3 hours. **2.** £557 16s. 3d. **3.** 42238274625.

4. **5.** $x=\frac{1}{3}\{a+b+c\pm\sqrt{a^2+b^2+c^2-bc-ca-ab}\}$.

6. 27. **7.** (i.) $x^{\frac{1}{6}}y^{\frac{7}{6}}$; (ii.) $\dfrac{6x^2+10x+2}{x(x+1)(x+2)}$.

8. Euc. I. 9, 4, 29, 6.

XIX.

1. £3 15s. **2.** $\cdot 4\dot{8}$. **3.** 16 weeks.

4. (i.) $x=\frac{1}{3}$; (ii.) $x=\dfrac{2}{a-b+c}$, $y=\dfrac{2}{a+b-c}$, $z=\dfrac{2}{-a+b+c}$;

(iii.) $x=15$. **5.** $3x^2-4x+1$.

6. $(b-a)(ab^2+a^2+a^2b+2ab+b^3)$.

7. $a^{\frac{4}{5}}-3a^{\frac{3}{5}}x^{\frac{1}{3}}+9a^{\frac{2}{5}}x^{\frac{2}{3}}-27a^{\frac{1}{5}}x+81x^{\frac{4}{3}}$. **8.** Euc. I. 8, 4.

XX.

1. (i.) $7\frac{1}{2}$ per cent.; (ii.) £50 less. **2.** 3907. **3.** $4\frac{1}{21}$.
4. (i.) $x=3$, or $-\frac{7}{3}$; (ii.) $x=\frac{7}{5}$, [or $-\frac{7}{5}$].
5. $\frac{1}{3}\left\{5^3-\frac{2^{10}}{5^7}\right\}$. **6.** $5\frac{5}{17}$ hours.
7. (i.) $\frac{a}{b(a-c)}$; (ii.) $\frac{x^2-y^2}{2(x^2+y^2)}$. **8.** Euc. I. 26, 38.

XXI.

1. £17135. **2.** $189\frac{1}{27}$, or $189\cdot\dot{0}3\dot{7}$.
3. £11·2784546875, or £11 5s. 6·829125d.
4. 9, 27, 81. **5.** 22. **6.** $(x+1)(x-1)(x-2)(x-3)$.
7. 3. **8.** Euc. I. 47; III. 31.

XXII.

1. £16 11s. $8\frac{11}{12}$d. **2.** ·0145. **3.** £3676 9s. $4\frac{16}{17}$d.
4. $2x-a$. **5.** (i.) $x=15$; (ii.) $x=3$; (iii.) $x=\frac{1}{2}$, $y=2$.
6. Cart, $\frac{2}{3}$ ton; waggon, $1\frac{1}{2}$ tons.
7. $10\frac{10}{11}$ mins. past 1. **8.** Euc. I. 47; II. 5.

XXIII.

1. 203. **2.** $\frac{2}{5}$. **3.** 64·77(6)
4. A, 28 yrs.; B, 14 yrs. **5.** The 102nd.
6. $\frac{2a}{b+a^2}$ and $\frac{2a}{b-a^2}$, where a is the difference of the reciprocals, b the difference of their squares. **7.** $2x-5$.
8. (i.) $x=6$; (ii.) $x=-2$; (iii.) 2, and 8.

XXIV.

1. (i.) ·30625; (ii.) ·00091875. **2.** £24 15s.
3. £3000. **4.** $\frac{3}{5}$. **5.** $b^2=4ac$.
6. (i.) $x=4$; (ii.) $x=2$, or 1. **7.** $-b$.
8. Euc. I. 23, 32.

XXV.

1. £1040. 2. £120.
3. (i.) $15\frac{25}{72}$ cub. ft., or 15 cub. ft. 600 cub. in.;
(ii.) $44\frac{55}{72}$ sq. ft., or 44 sq. ft. 110 sq. in.
4. £20 13s. 4d. 5. (i.) $\dfrac{10(x^2+1)}{(x^2-1)^2}$; (ii.) a.
6. (i.) $-52\frac{1}{2}$; (ii.) $\frac{6841}{686}(=9\frac{667}{686})$.
7. L.C.M. $(a^2-x^2)(a^2-4x^2)$; G.C.M. $x+3$.
8. Euc. I. 38, 39. Cf. 'Progressive' Euclid, App. vii.

XXVI.

1. ·0782(7)...... 2. 13 cwt. 0 qrs. 24 lbs. 8 oz.
3. 1·092857. 4. 8d. and 9d. per doz. 5. $x-2$.
6. (i.) $3x^2+2x+1$; (ii.) $321^2=103041$. 7. $\frac{2}{3}$.
8. (i.) Euc. II. 5; (ii.) If $ab=cd$, and $a+b=c+d$, then $a-b=c-d$, whence $a=c$, and $b=d$.

XXVII.

1. 998. 2. The $4\frac{3}{4}$ Per Cents. (As 26 : 25.)
3. £45. 4. $38\frac{2}{11}$ mins. past 7.
5. (i.) $\frac{1}{3}\{1-(-\frac{1}{2})^n\}$; (ii.) $\frac{1}{3}$. 6. $a^2-3a^{\frac{4}{3}}b^{-\frac{2}{3}}+3a^{\frac{2}{3}}b^{-\frac{4}{3}}-b^{-2}$.
7. (i.) $x=11$, or -3; (ii.) $x=35$, $y=12$.
8. Euc. I. 14, 32. Cf. Euc. I. 16.

XXVIII.

1. ·134. 2. £168 4s. $11\frac{1}{16}$d. 3. 9450 links.
4. (i.) $x=\frac{1}{7}$, or -2; (ii.) $x=\sqrt[3]{8}$, or $\sqrt[3]{-1}$; (iii.) $x=10$.
5. (i.) $ab\,.\sqrt[3]{a^2}$; (ii.) $6\sqrt{5}$; (iii.) $a^{\frac{2}{n^2-1}}$; (iv.) $a^{\frac{7}{2}}$.
6. $a+bx+cx^2$. 7. $\dfrac{x+4}{x^2+2x-2}$.
8. Euc. I. 29, 6. The perimeter will be equal to twice a side of the triangle.

XXIX.

1. 288120 sq. units.
2. 1057 revolutions.
3. £573 19s. 2d.
4.
5. Assume it to be the square of $x^2+\frac{p}{2}x+\sqrt{s}$, and equate coefficients of x^2 and x.
6. (i.) $x=0$, or $\frac{5}{3}$; (ii.) $x=7$, $y=2$.
7. $4e6547$.
8. Euc. I. 26.

XXX.

1. £3 11s. $11\frac{11}{173}$d.
2. 877.
3. £1345·9942125, or £1345 19s. 10·611d.
4. $(x+1)^2$.
5. (i.) $x=8$; (ii.) $x=\frac{1}{9}$, or 1.
6. $\dfrac{a^2+2a+3}{a^2+a+1}$.
7.
8. Euc. I. 32; or see "Progressive" Euclid, App. xiv.

XXXI.

1. At $10\frac{10}{11}$ mins. and $43\frac{7}{11}$ mins. past 5.
2. 18s. 9d. a gallon.
3. 30·72d. (Interest, £16 2s. 6·72d.; discount, £16).
4. (i.) $x=\frac{1}{2}$, $y=2$; (ii.) $x=9$, or 2; $y=2$, or 9.
5. $x^{3m}+x^{2m}y^m+x^my^{2m}+y^{3m}$.
6. 142857.
7. 5050.
8. Euc. I. 3, 4, 20.

XXXII.

1. £64.
2. $283\cdot\dot{5}$.
3. (i.) $x=\dfrac{ac}{b}$; (ii.) $x=4$, $y=9$, $z=16$, $u=25$.
4.
5. $a+1$.
6. 3.
7. $\dfrac{x^{3r}}{y^{3s}}-\dfrac{y^{3s}}{x^{3r}}$.
8. Euc. I. 23, 31, 34.

XXXIII.

1. 326·498...... yards. 2. $18\frac{9}{10}$. 3. $\frac{29}{36}$.

4. The condition gives $x - y + k - l + klxy\left(\frac{1}{y} - \frac{1}{x} + \frac{1}{l} - \frac{1}{k}\right) = 0$, hence y and l are interchangeable.

5. (i.) $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$;
(ii.) $(x^2+xy+y^2)(x^2-xy+y^2)$.

6. (i.) $x=106$; (ii.) $x=3$, $y=\frac{1}{3}$; (iii.) $x=0$, [or $-\frac{24}{25}$].

7. $\dfrac{x+2}{x+1}$. 8. Euc. I. 32, 28.

XXXIV.

1. $37\frac{43}{81}$ yds., = 37 yds. 1 ft. $7\frac{1}{9}$ in. 2. $3\frac{1}{2}$ per cent.
3. 3 days. 4. (i.) $x = -\frac{5}{2}$; (ii.) $x=\frac{3}{7}$, $y=\frac{7}{3}$.
5. $a-b$. 6. 12422311.
7. 60 farthings. 8. Euc. I. 10.

XXXV.

1. 3s. 11d. 2. £3 14s. 3. £11 18s. 6d.

4. (i.) $x=\dfrac{2l}{b+c}$, $y=\dfrac{2m}{a+c}$, $z=\dfrac{2n}{a+b}$; (ii.) $x=\dfrac{ab}{a+b}$;
(iii.) $x=4$, or ∞.

5. (i.) $\dfrac{a+x}{ax}$; (ii.) $\dfrac{x+1}{2x+1}$; (iii.) x^2.

6. $x^2-ax+2a^2$. 7. $a^2+1+\dfrac{1}{a^2}$.

8. Join the vertex to a point in the base. Euc. I. 16, 13.

XXXVI.

1. 8401·4 lbs. 2. 2122.
3. A £162, B £324, C £405, D £486, E £648.
4. $2x^4-x^2-2$. 5. $x^{\frac{1}{3}}+y^{\frac{1}{3}}$. 6. (i.) x^2; (ii.) 2.
7. (i.) $x=2$, or 1; (ii.) $x=2$, $y=-1$. 8. Euc. I. 11, 9, 32.

XXXVII.

1. 1s. 3d. per lb. **2.** Tuesday in 1960, Monday in 2060.
3. 4s. $4\frac{1}{2}$d. **4.** 3715. **5.** (i.) 212; (ii.) $127\frac{1}{2}$.
6. 5, 7, 9. **7.** (i.) $x=7$; (ii.) $x=y=6$; (iii.) $x=6$, or -3.
8. $2\left(\frac{x}{y}-\frac{y}{x}\right)$.

XXXVIII.

1. 531. **2.** (i.) $\frac{1}{(x+1)(x+3)(x+5)}$; (ii.) $\frac{1}{x}$; (iii.) $ab^{\frac{5}{6}}$.
3. 96. **4.** x^2+4x+4. **5.** (i.) $x=3$; (ii.) $x=8$, $y=12$.
6. $a^{\frac{2}{3}}-b^{\frac{2}{3}}$. **7.** $a^2+ab-ac-bc$. **8.** £9 13s.

XXXIX.

1. (i.) $3\frac{111}{374}$; (ii.) $\frac{37}{320}$. **2.** 2·288(1).
3. 1·706666($\dot{6}$). **4.** 2, $1\frac{5}{6}$, $1\frac{2}{3}$, etc.
5. 0. **6.** A in $44\frac{4}{19}$ days, B in $93\frac{1}{3}$ days.
7. (i.) $x=\frac{a}{m}$, or $\frac{b}{n}$; (ii.) $x=1$, or -4; (iii.) $a=\frac{8}{5}$ or 2.
8. Euc. I. 44, 41.

XL.

1. 2325. **2.** £49 5s. 6d. **3.** £9 9s.
4. **5.** 70 at £3 each.
6. (i.) $a-b$; (ii.) $x-2a$. **7.** (i.) $\frac{n(a+l)}{2}$; (ii.) $4\frac{1}{2}$.
8. Euc. II. 5. See 'Progressive' Euclid, App. xxviii.

XLI.

1. $109{\cdot}990917-109{\cdot}990907={\cdot}00001$.
2. (i.) 75 miles an hour; (ii.) $257\frac{1}{7}$ revolutions.
3. $-\frac{1}{2}$; 1; $-\frac{7}{4}$. **4.** x^2-4x+5.
5. (i.) $x=-\frac{45}{37}$; (ii.) $x=y=-3$. **6.** 12 large, 10 small.
7. $\frac{x^2-1}{x^2-4}$. **8.**

XLII.

1. 10 days. 2. $1\frac{153}{244}$. 3. ·0017(4).
4. (i.) $x=8$, or $-\frac{9}{2}$; (ii.) $x=3$, or $-\frac{5}{4}$, $y=\frac{3}{4}$, or 5; (iii.) $x=14$, [or 2]. 5. 6.
7. $\frac{n}{2}\{2a+(n-1)b\}$; (i.) 265; (ii.) -5; (iii.) $\dfrac{16\{1-(\frac{5}{8})^n\}}{3}$, $\dfrac{16}{3}$. 8. 1; $\dfrac{1}{a^m}$.

XLIII.

1. (i.) $687007\frac{7}{999}$; (ii.) $752999\frac{110}{863}$. 2. ·866(3).
3. ($£\frac{745}{80}-£\frac{745}{81}=$) 2s. $3\frac{16}{27}$d. 4.
5. (i). $\dfrac{a(r^n-1)}{r-1}$; (ii.) $p-1$. 6. $2x^2-x-3$.
7. (i.) $\dfrac{2}{a-b}$; (ii.) $\dfrac{2ax}{1+x^2}$.
8. Euc. II. 12, 13. See 'Progressive' Euclid, App. xxvi.

XLIV.

1. 1. 2. 138·36(2). 3. (i.) 6·075 bush.; (ii.) £2 0s. 6d.
4. $(x^3-y^3)\div(x-y)\equiv x^2+xy+y^2$.
5. (i.) $x=\dfrac{ab^2+bc^2+ca^2}{a^2b+b^2c+c^2a}$; (ii.) $x=4$, $y=5$.
6. £800, £3200, and £1000.
7. $33\frac{1}{3}$ miles per hour; $48\frac{1}{3}$ miles.
8. Euc. I. 31, 34; III. 12, 16.

XLV.

1. $2\frac{14}{29}$ days. 2. £2 9s. $3\frac{1}{9}$d. per cwt.
3. £125. 4. 7 and 8. 5. $-p$.
6. (i.) -28; (ii.) $1\frac{41}{81}$, $1\frac{1}{2}$.
7. (i.) $x=\frac{3}{4}$, or -3; (ii.) $x=5$, $y=3$.
8. $z(y-x)(x+y-z)$.

XLVI.

1. In 12 hours. **2.** 2 minutes. **3.** ·0004(0).
4. (i.) $x=5\frac{1}{2}$; (ii.) $x=4$, $y=12$. **5.** 20 miles.
6. (i.) $\frac{17}{3}$ or $\frac{4}{3}$; (ii.) 1 or -3. **7.** 4·85409.
8.

XLVII.

1. ·000734(3). **2.** £14 11s. 7d. **3.** ·463467(2).
4. (i.) $x=-4\frac{3}{4}$; (ii.) $x=3$, $y=6$; (iii.) $x=2$, $y=4$.
5. £1. **6.** (i.) $x^2-x-6=0$; ii. $2b^2=9ac$.
7. (i.) $\dfrac{a(r^n-1)}{r-1}$; (ii.) $\dfrac{r^n-1}{ar^{n-1}(r-1)}$. **8.** Euc. I. 47, 41.

XLVIII.

1. 2s. $2\frac{122}{273}$d. (£4 15s. – £4 12s. $9\frac{151}{273}$d.).
2. (i.) 11·7578125; (ii.) £11757 16s. 3d.
3. ·367879(1).
4. (i.) $x=\frac{5}{2}$, or $-\frac{4}{3}$; (ii.) $x=\dfrac{-1\pm\sqrt{5}}{2}$, or ∞.
5. 15 miles; 3, $2\frac{1}{2}$, and 4 miles per hour.
6. The second expression is symmetrical in a, b, c, d.
7. (i.) 0; (ii.) $\dfrac{x^2-x+1}{x-1}$; (iii.) 3. **8.** Euc. I. 4, 13, 32.

XLIX.

1. 288. **2.** (i.) $2\frac{71}{91}$; (ii.) 5. **3.** ·00000547(9).
4. (i.) $x=4$, or $-\frac{3}{2}$; (ii.) $x=3$, or $\frac{5}{2}$;
(iii.) $x=3$, $\frac{1}{3}$, or $\dfrac{-15\pm\sqrt{221}}{2}$.
5. Subtracting we get $x=a-5$; then substitute for x.
6. (i.) 3280; $\dfrac{3^n-1}{2}$; (ii.) $2(2^8-1)-8$, or 502; $2(2^n-1)-n$.
7. 1.

8. $x^2+y^2>2xy$, $\because x^2-2xy+y^2$ is positive. (ii.) If a straight line be divided into any two parts, the squares on the two parts are together greater than twice the rectangle contained by the parts.

L.

1. £4 1s. $6\frac{23}{27}$.
2. £1416.
3. 22 lb. nitre, $4\frac{2}{3}$ lb. charcoal, $3\frac{1}{3}$ lb. sulphur.
4. $x=(a+b)^2$, $y=(a-b)^2$.
5. $\frac{5}{3}\sqrt{3}-2$.
6. $77\frac{1}{2}$, $\dfrac{n(3n+1)}{4}$.
7. $\dfrac{x+4a}{x+2a}$.
8. Euc. I. 9.

LI.

1. 16 tons 15 cwt. 3 qr. 0 lb. $2\frac{2}{3}$ oz. or $16\frac{2117}{2688}$ tons.
2. $\frac{70}{467}$.
3. 90; £465.
4. $x=\pm 2$, $[\pm\sqrt{-1}.]$
5. $-\dfrac{1}{x^2}$.
6. $1+x-x^3-x^4$.
7. 2, $\frac{3}{2}$, $\frac{6}{5}$, 1, $\frac{6}{7}$, $\frac{3}{4}$.
8. Euc. I. 26, 39.

LII.

1. 8876.
2. 39 barges.
3. (i.) £1; (ii.) $\cdot\dot{7}4\dot{0}$.
4. $\dfrac{5x\sqrt{6}}{2}$.
5. $-7\frac{1}{2}$.
6. (i.) $x=\pm 5$, $y=\pm 7$; (ii.) $x=c$.
7. 6·4, or 4·6.
8. Euc. I. 1, 32.

LIII.

1. By 20.
2. $4\frac{23}{64}$.
3. 767232 gallons.
4. $\frac{2}{3}\{1-2^{10}\}$, $=-682$.
5. 152.
6. (i.) $x=0$, 1, or $\dfrac{1\pm\sqrt{-7}}{2}$; (ii.) $x=\dfrac{cd-ab}{a+b-c-d}$.
7. $x(4x+3)(3x-1)(2x+1)$.
8. Euc. I. 21.

LIV.

1.

8 doz. knives, - - -	£14	0	0
6 doz. do., - - -	7	10	0
12 prs. carvers, - -	7	4	0
5 prs. do., - -	3	7	6
10 prs. do., - -	6	0	0
5 steels, - - -	1	2	6
	£39	4	0

2. 1.

3. (i.) $(3x-2)(5x+1)$; (ii.) $(7x-a)(2x-5a)$;
(iii.) $(x-y)(x+y)(3x^2+2xy+y^2)$.

4. (i.) $\frac{a-x}{a+x}$; (ii.) $\frac{2xy}{x^2+y^2}$.

5. $4x+2y$.

6. (i.) $x=27\frac{2}{3}$; (ii.) $x=7$, or $-\frac{3}{2}$;
(iii.) $x=\frac{15}{2}$, or -4, $y=4$, or $-\frac{15}{2}$.

7. Man, 3s. 6d; boy, 2s.

8. Euc. I. 1, 9, 31.

LV.

1. $\cdot 321745(3)$.

2. $\cdot 875$.

3. 74.

4. (i.) $\frac{1}{x+a+b}$; (ii.) $\frac{x}{a}+3a+\frac{y}{2}$.

5. (i.) $x=4\frac{1}{3}$, or ∞; (ii.) $x=\frac{7}{2}$, or $\frac{5}{2}$; $y=\frac{5}{2}$, or $\frac{7}{2}$.

6. 10 mins. and 15 mins. respectively.

7. $\frac{49}{10}\left\{1-\frac{2^n}{7^n}\right\}$; $4\frac{9}{10}$.

8. Euc. III. 31.

LVI.

1. $\frac{633}{2000}$.

2. 2601.

3. (i.) $\left(\frac{1}{x}\right)^2-\left(\frac{1}{z}\right)^2$; (ii.) -3.

4. (i.) $(x-4a-4b)(x+a+d)$;
(ii.) $(x+y+2k-2l)^2(x+y-2k+2l)^2$.

5. $x(x^2+3x+11)$.

6. 643 and 1929.

7. $x^{\frac{3}{p}}-(y+2z)x^{\frac{2}{p}}+(2yz+z^2)x^{\frac{1}{p}}-yz^2$.

8. Euc. III. 31 ; I. 38.

LVII.

1. No difference. Both £144.

2. 4 days.

3. (i.) $\dfrac{xy}{x^2-y^2}$; (ii.) $\pm(x+2)$; (iii.) $\dfrac{b}{a^2}$.

4. $2x^2+2=(x+1)^2+(x-1)^2$.

5. 1.

6. (i.) $(k^4-k^2l^2+l^4)(k^2-kl+l^2)(k^2+kl+l^2)$;
(ii.) $(a+b)(b+c)(c+a)$.

7. 13 terms.

8. Euc. I. 23, 31. 4 solutions.

LVIII.

1. $33\frac{1}{3}$ days.

2. $\frac{142}{405}$.

3. $(x+1)^2-(x-1)^2=2(x+1+x-1)$.

4. x^2-y^2.

5. (i.) $-p$; (ii.) q.

6. (i.) $x=-a$, or $-b$; (ii.) $x=\pm 7$, $y=\pm 3$.

7. $r=2$; $s=47\frac{1}{4}$.

8. Euc. I. 32.

LIX.

1. $1\frac{1}{2}$.

2. $\frac{1}{36}$ in.

3. $\dfrac{k^2}{3}-kl+3l^2$.

4. 5 yds., and 9 yds.

5. 1.

6. (i.) $x=5$, or $-2\frac{3}{4}$; (ii.) $x=4$, or 2; $y=2$, or 4.

7. (i.) $x^3+4x^2+x-6=0$; (ii.) $x^3-(k-l)x^2-klx=0$.

8. Euc. I. 26, 47. Cf. Euc. IV. 4.

LX.

1. 5 yds. 4 in. 2. £534 6s. 6d.
3. (i.) The shares, in the ratio 1750 : 1563; (ii.) £136762 10s. in each.
4. (i.) $5-2\sqrt{2}$; (ii.) $1+\sqrt{3}$. 5. $\dfrac{1}{b^3\sqrt{a}}$, or $a^{-\frac{1}{2}}b^{-3}$.
6. (i.) $x=4$, or 2; (ii.) $x=10$, or -6; $y=\frac{12}{5}$, or -4.
7. $x=8$. 8. Euc. I. 37.

LXI.

1. $2\frac{1}{2}$ per cent. 2. $50\frac{2}{7}$ lbs. 3. 3·605(5).
4. $m(x^3+1)+n(x^2+x)$. 5. (i.) $\sqrt{\dfrac{b}{a}}$; (ii.) 3.
6. (i.) $x=7$; (ii.) $x=12$, $y=6$.
7. $2x-3y$. 8. Euc. I. 37.

LXII.

1. As 40 : 53. 2. 8004; $3\frac{1}{2}$.
3. In 120 days. One will be at 11 h. 46 secs. and the other at 12 h. 16 secs.
4. $5\frac{173}{180}$ grs. 5. $25\sqrt{3}$.
6. 45. 7. (i.) 300; (ii.) $x=7$.
8. (i.) $x=7$ or -4; (ii.) $x=\pm\sqrt{6}$, $y=\pm 2$.

LXIII.

1. ·001. 2. $11\frac{2}{3}$ days. 3. 65.
4. Dividing by $x-h$, the remainder is $ah^2+bh+c=0$.
5. $\dfrac{a^2(x-a)}{x+a}$. 6. (i.) $57\frac{1}{6}$; (ii.) $-10\frac{1}{2}$; (iii.) $12\left(1-\dfrac{1}{2^n}\right)$.
7. (i.) $x=5$, or 1; (ii.) $x=5$, $y=3$.
8. 13 and 2.

LXIV.

1. £$899\frac{251}{1446}$.
2. 1728000 times.
3. 148 feet.
4. $x^3 - 9x^2 + 20x = 0$.
5.
6. $\frac{2}{5}, \frac{2}{7}, \frac{2}{9}, \frac{2}{11}$.
7. 3.
8. (i.) $x = \pm\frac{a}{\sqrt{2}}$; (ii.) $x = -\frac{1}{2}$, or -2; $y = -2$, or $-\frac{1}{2}$.

LXV.

1. 36 yds. and 24 yds.
2. *t376e0e.*
3. 18,495,000.
4. (i.) $x = \frac{18}{7}$, $y = \frac{8}{3}$; (ii.) $x = -1$, or $\frac{5 \pm \sqrt{-15}}{4}$;

   (iii.) $x = 3$, or $-\frac{1}{2}$.
5.
6. $10x + y - (10y + x) = 9(x - y)$
7. $\frac{(n+100)(n-99)}{2}$
8.

LXVI.

1. £2 7s. 11d.
2. £$147\frac{1}{17}$, $=$ £147 1s. $2\frac{2}{17}$d.
3. $7908\frac{102}{121}$ miles.
4. 40 mins. later.
5. 3 and 12.
6. $x = 0$, *or* 2; $y = 0$, *or* 6.
7. 10, 20, 40.
8. Euc. III. 31.

LXVII.

1. By $x - y$, $x^4 - y^4$ and $x^5 - y^5$,
   by $x + y$, $x^3 + y^3$ and $x^4 - y^4$,
   by $x^2 - y^2$, $x^4 - y^4$.
2. (i.) $x = 3$, or $-\frac{5}{7}$; (ii.) $x = \pm 5$, $\left[\text{or} \pm \sqrt{\frac{59}{2}}\right]$;

   $y = \pm 3$, $\left[\text{or} \pm \frac{3}{\sqrt{2}}\right]$.
3. 460 feet. 24 posts.
4. 299 adults, 54 children, 13 infants.
5. $\frac{43}{49}, \frac{37}{49}, \frac{31}{49}, \frac{25}{49}, \frac{19}{49}, \frac{13}{49}$.
6. $\frac{125}{648}$; $\frac{12}{5}\{1 - (\frac{5}{6})^n\}$; $\frac{12}{5}$.
7.
8. Euc. I. 19, 37.

LXVIII.

1. 417 ; $11\frac{8}{13}$.
2. £23 9s. 4d.
3. B walks 1 mile in $13\frac{1}{4}$ mins., and loses by $11\frac{1}{4}$ mins., or $\frac{45}{53}$ mile.
4.
5. (i.) $x=17$, $y=19$; (ii.) $x=a\pm\frac{1}{a}$.
6. In the scale of 11.
7. $a^{-1}+a^{-\frac{1}{2}}b^{-\frac{1}{2}}+b^{-1}$.
8. Euc. I. 31, 34.

LXIX.

1. £37 0s. 8d.
2. $\frac{1}{9}$.
3. £491 8s.
4. $x=(b-a)^2$.
5. (i.) 1000; (ii.) $r^3+3r^2+3r+1=(r+1)^3$.
6. 1.
7. (i.) 2; (ii.) $\frac{31+6\sqrt{5}}{5\sqrt{5}}$.
8. Euc. II. 5, 6. See "Progressive" Euclid, page 121.

LXX.

1. $1\frac{11}{13}$ days.
2. £59 5s.
3. £7 6s. $6\frac{3}{4}$d.
4. (i.) -12; (ii.) $\frac{n(n+1)(2n+1)}{6}$.
5. (i.) 51117344; (ii.) 131.
6. (i.) $x=20$; (ii.) $x=5\frac{1}{2}$.
7. $x^4-3x^3+4x^2-3x+1=(x-1)^2\{(x-1)^2+x\}$ which is positive when $x>0$.
8. Euc. I. 3, 23, 6.

LXXI.

1. £7524 11s. 3d.
2. £993 4s. 6d.
3. £30232 7s. $9\frac{3}{4}$d.
4. 3s. each.
5. (i.) $x=6$; (ii.) $x=\frac{1}{3}$ or -10.
6. $x^2+12x-540=0$.
7. $\sqrt[3]{9}$.
8. Euc. I. 10, 11.

LXXII.

1. 1s. 3d. per yard.
2. £1040.
3. 0.
4. $xy+ab+z^2$.
5. $(x-1)n-1+x=(n+1)(x-1)$.
6. (i.) $x=17$, or -4; (ii.) $x=\pm\dfrac{a^2}{\sqrt{a^2+b^2}}$, $y=\pm\dfrac{b^2}{\sqrt{a^2+b^2}}$.
7. $x^2-3x\sqrt{2}+4=0$.
8. Euc. I. 31, 37. Cf. "Progressive" Euclid, App. xviii.

LXXIII.

1. $104\frac{5}{12}$.
2. $30\frac{5}{64}$.
3. (i.) $x=\dfrac{2\pm\sqrt{109}}{3}$; (ii.) $x=\pm\frac{3}{2}$, $\pm\frac{1}{2}$; $y=\pm\frac{1}{2}$, $\pm\frac{3}{2}$.
4. $2a$, where a is the given number.
5.
6. $1+\sqrt{3}$.
7. $(x^2+xy+y^2)(x^2-xy+y^2)$;
   $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.
8. $\dfrac{x}{y^2}+\dfrac{y}{x^2}$ is the greater.

LXXIV.

1. £18 5s. $7\frac{1}{2}$d.
2. 16s. $0{\cdot}37\dot{5}01\dot{3}$d.
3. 278 inches square, $69\frac{1}{2}$ inches deep.
4. $2a+pb-ap-b$.
5. (i.) 19; (ii.) $\dfrac{4}{3}\left\{1-\dfrac{1}{4^n}\right\}$; (iii.) sum to infinity $=\dfrac{4}{3}$.
6. $x=-1\pm\sqrt{2}$.
7. $x=\pm\sqrt{11}$.
8. See "Progressive" Euclid, I. 26, Second Proof.

LXXV.

1. $5\frac{5}{24}$.
2. The latter in the ratio 1452 : 1225.
3. £50350 12s. 6d.
4. (i.) $x=\frac{a}{a+b}$, $y=\frac{b}{a+b}$; (ii.) $x=3$, $y=\frac{1}{3}$, $z=-1$.
5. $\frac{3x}{a}-1+\frac{a}{3x}$.
6.
7. £3.
8. A in m days, B in mn days.

LXXVI.

1. £273 15s.
2. ·491824(7).
3. $\frac{x}{y}-\frac{y}{x}$.
4. (i.) $\frac{p^2}{q^2}$; (ii.) $\frac{2(x^4-2)}{x(x^4-4)}$.
5. (i.) $x=5$; (ii.) $x=2$, $y=-1$;
   (iii.) $x=a\left(\frac{a+b+c}{abc}\right)$, $y=b\left(\frac{a+b+c}{abc}\right)$, $z=c\left(\frac{a+b+c}{abc}\right)$.
6. 6 dozen oranges, 21 dozen apples.
7. 0.
8. Euc. I. 47; III. 31.

LXXVII.

1. 12 hours a day.
2. $2^2\times 13^2\times 37$.
3. $6x^2-37x+6=0$.
4. (i.) $\frac{x^2+y^2}{x}$; (ii.) abc.
5. (i.) $x=0$, or $-2\pm\sqrt{-1}$; (ii.) $x=\pm 2$, or ∞, $y=\pm 1$, or ∞;
   (iii.) $x=3$, 2, $-\frac{1}{3}$, or $-\frac{1}{2}$.
6. 8d. a dozen.
7. Between 8 and 9.
8. Euc. I. 26. See "Progressive" Euclid, App. xxv.

LXXVIII.

1. 41·4.
2. £132 0s. $7\frac{1}{2}$d.
3. a^2-a-1.
4. $3x^2+5x-2$.

5. $abc^{\frac{1}{3}}$.

6. (i.) $x=\dfrac{a+b}{3}$; (ii.) $x=a$, or $-\dfrac{1}{a}(a^2-b^2)$,

(iii). $\dfrac{1}{yz}=\dfrac{1}{2}\left(\dfrac{1}{b^2}+\dfrac{1}{c^2}-\dfrac{1}{a^2}\right)$ etc.

Hence $x=\dfrac{abc\sqrt{2\sqrt{a^2b^2+a^2c^2-b^2c^2}}}{\sqrt{(a^2b^2-a^2c^2+b^2c^2)(a^2c^2-a^2b^2+b^2c^2)}}$ etc.

7. $\dfrac{mn(a+b)}{mn-m-n}$ acres.

8. Euc. I. 34, 38. "Progressive" Euclid, App. viii.

LXXIX.

1. 1·707031(2). 2. $\frac{375}{869}$ in.

3. $a^3+a^2(x+\frac{1}{2}y-\frac{1}{2}z)+a\left(\dfrac{xy}{2}-\dfrac{xz}{2}-\dfrac{yz}{4}\right)-\frac{1}{4}xyz$;

$a^3+3a^2b+3ab^2+b^3$. 4. x^2-3x+1.

5. (i.) 1; (ii.) $(ac+bd)^2+(ad-bc)^2$.

6. (i.) $x=1$; (ii.) $x=1+\sqrt[3]{8}=3$, or two imaginary values.

7. A 6 days, B 3 days, C $3\frac{3}{5}$ days. 8.

LXXX.

1. 2596. 2. 3000 men.

3. G.C.M. $x+2y$; L.C.M. $(x+2y)(x^2-y^2)$.

4. $\dfrac{x}{3}-\dfrac{y}{4}+\dfrac{z}{5}$; $a^3-4ab+8c=0$.

5. (i.) $x=13$, 0, or -5; (ii.) $x=3$, $y=5$, $z=-7$

6. (i.) $x^2-x-12=0$; (ii.) $x^4-13x^2+36=0$;

(iii.) $x^2-2ax+a^2-b^2=0$.

7. $p=\pm 2$, ± 1, $q=\pm 1$, ∓ 2.

8. Euc. I. 31, 41. See "Progressive" Euclid, App. xxv.

LXXXI.

1. 15·300(5). **2.** £36 13s. 5·5725d. **3.** $a : d :: 7 : 16$.

4. $\frac{y^3}{x^3} - \frac{3y}{x} + \frac{3x}{y} - \frac{x^3}{y^3}$. **5.** (i.) 1; (ii.) $\frac{3a^4 + 10a^2b^2 + 3b^4}{a^2 - b^2}$.

6. **7.** A £48, B £40.

8. Euc. I. 10, 31. See "Progressive" Euclid, App. vii.

LXXXII.

1. 5 per cent. **2.** £511·5404125, or £511 10s. 9·699d.

3. $2a - 3x + 4y$.

4. (i.) $x = 0$, $-\frac{l}{k}$, or $\frac{l}{m}$; (ii.) $x = \frac{3}{4}$ or $\frac{1}{12}$;

(iii.) $x = 5$, or -2; $y = 2$, or -5.

5. $a_1x + b_1y + c_1z$. **6.** **7.**

8. Euc. I. 32, III. 31. Cf. Euc. VI. 8.

LXXXIII.

1. $10\frac{1}{2}$ yds. long, $3\frac{1}{2}$ yds. broad, 4 yds. high.

2. 406 ins. **3.** $\frac{69}{77}$ gal.

4. (i.) $x = 0$, [or $\frac{9}{16}$]; (ii.) $x = 5$. **5.** (i.) $\frac{3^{r+2}}{2^{2r+5}}$; (ii.) $2\frac{2}{3}$.

6. The equation gives $x = \frac{-7 \pm \sqrt{-11}}{6}$, which is imaginary.

7. $x = \frac{2a^2b}{a^2 + b^2}$. **8.** 5 and 7.

LXXXIV.

1. $27\sqrt{3}$ ins., $= 46·76537$ ins.

2. (i.) ·3891661; (ii.) 1·3502480.

3. $a^{\frac{4n}{5}} + a^{\frac{3n}{5}}b^{\frac{m}{5}} + a^{\frac{2n}{5}}b^{\frac{2m}{5}} + a^{\frac{n}{5}}b^{\frac{3m}{5}} + b^{\frac{4m}{5}}$. **4.** $x = 4c$.

5. $x = \frac{y^2 + 1}{2y}$. **6.** $-\frac{b(b^2 - 4ac)(b^2 - ac)}{a^4c}$.

7. 200,000. **8.** Euc. III. 21, 22.

LXXXV.

1. (i.) $x=1$, or $\dfrac{-7\pm\sqrt{-15}}{2}$; (ii.) $x=\frac{1}{2}$, or $\frac{1}{3}$; $y=\frac{1}{3}$, or $\frac{1}{2}$;
(iii.) $x=6$, or 1; $y=3$, or 7.
2. 3.
4. (i.) In the scale of 8; (ii.) *eee*.
5. n^{th} term $\dfrac{n(n+1)}{2}$; sum $=\dfrac{n(n+1)(n+2)}{6}$.
6. $(a-b)^2$ must be positive. 7. $n=4$.
8. (i.) $a\left\{\dfrac{n(n+1)}{2}+\dfrac{a^n-1}{a-1}\right\}$; (ii.) $\dfrac{1-a^n}{(1-a)^2}-\dfrac{na^n}{1-a}$.

LXXXVI.

1. ·7182818(2). 2. 34 ft. 6 in. 3. $\frac{117}{140}$.
4. 16 men. 5. $\frac{1}{3}\left\{\dfrac{5^6}{2^6}-1\right\}=81\frac{3}{64}$; $\frac{1}{5}$, $\frac{3}{5}$, 1, $1\frac{2}{5}$, etc.
6. (i.) $x=-\frac{1}{3}$, $y=-\frac{1}{2}$; (ii.) $x=\dfrac{3c}{a+b}$, or $-\dfrac{2c}{a+b}$;
(iii.) $x=-\frac{1}{2}$, -2, or $\dfrac{2\pm\sqrt{-21}}{5}$; $y=-2$, $-\frac{1}{2}$, or
$\dfrac{2\mp\sqrt{-21}}{5}$. 7. 8.

LXXXVII.

1. 8s. 5d. 2. £6 6s.
3. (i.) $x=4$, [or $\frac{1}{3}$]; (ii.) $x=a+b$, or $a+\dfrac{b}{4}$; $y=b$, or $-\dfrac{b}{2}$.
4. 18 and 81. 5.
6. (i.) $\pi=3·14159......$; (ii.) 180°.
7. 812 inches, or 22 yds. 2 ft. 8 in.
8. Euc. II. 11.

LXXXVIII.

1. A had 40s., B 42s.
2.
3. Take a as the greatest, then d is the least.
4. 20.
5. $\frac{10^{10}-1}{9}=1111111111$.
6. $\frac{49\pi a}{8000}$.
7. $BQ=PQ=1$ inch, $BP=\sqrt{3}$ inches.
8. Euc. I. 29, 13.

LXXXIX.

1. £9 18s. 9d.
2. $x=\frac{a^2 \pm a\sqrt{a^2-b^2}}{b}$.
3. $ax-a^4x^4$; -14.
4. £40.
5. 4d. a dozen.
6. (i.) $x=0$, or $\sqrt{\frac{mb^2-na^2}{m-n}}$; (ii.) $x=\sqrt[4]{1}$, or $\sqrt[4]{2}$;
(iii.) $x=\frac{-1\pm\sqrt{5}}{2}$, or $\frac{-1\pm\sqrt{-1}}{2}$.
7. $\frac{7363\pi}{36000}$.
8. $x=30^\circ$, or $n\pi+(-1)^n\frac{\pi}{6}$.

XC.

1. $2x^2-5x-3$.
2. One 20 poles by 8 poles, the other 16 poles by 10 poles.
3. $a^{\frac{3}{2}}+2ab^{\frac{1}{2}}+2a^{\frac{1}{2}}b+b^{\frac{3}{2}}$.
4. (i.) $7xy(3x+2y)(3x-2y)$; (ii.) $(a-b)^2(a+b)(a^2+b^2)$.
5. G.C.M. x^2-4x+4; L.C.M. $xy(x^6-y^6)$.
6. $-\frac{1}{\sqrt{2}}$, $-\frac{1}{\sqrt{2}}$, 1, $-\sqrt{2}$, $-\sqrt{2}$, 1.
7. Height, $160\sqrt{3}$ feet; distance, 320 feet.
8. "Progressive" Euclid, pp. 112-125.

XCI.

1. 2. x^2+1.

3. $(x^6-a^6)(x^2-a^2)$.

4. (i.) $x=1$; (ii.) $x=\pm 2$, $y=\pm 1$.

5. (i.) $\dfrac{1}{(x-1)(x-2)(x-3)}$; (ii.) $\dfrac{2}{a}$; (iii.) x^8+x^4+1.

6. Distance, 16 miles; A's rates, 4 and 3 miles per hour; B's rates, 4 and 5 miles per hour.

7. 8. The same as for 30°.

XCII.

1. £2160. 2. 30 sovereigns.

3. $x^2-3ax+a^2$.

4. (i.) $x=\frac{3}{2}$, or $\frac{4}{3}$; (ii.) $x=9$, [or $-\frac{18}{5}$]; (iii.) $x=3$, or -2; $y=2$, or -3.

5. (i.) Put $\dfrac{a}{b}=\dfrac{c}{d}=k$; (ii.) See Ex. lxxxviii. 3.

6. (i.) $\dfrac{n(n+1)}{2}$; (ii.) $7\sqrt{2}+14$. 7. $\dfrac{\sqrt{3}+2\sqrt{2}}{6}$.

8. (i.) $-\cos A$; (ii.) $\sin A$; (iii.) $\tan A$.

XCIII.

1. £610,260,560.

2. (i.) $x=4\frac{1}{2}$; (ii.) $x=\dfrac{2}{b+c-a}$, $y=\dfrac{2}{a-b+c}$, $z=\dfrac{2}{a+b-c}$; (iii.) $x=5$, -9, [or ± 7].

3. As 2 : 3.

4. 30 yds. long and 25 yds. wide.

5. $x^2+x\sqrt{2}+1$. 6. $\sqrt{3}-\sqrt{2}$, $=\cdot 3178\ldots$

7. $2(x^2+y^2)=(x+y)^2+(x-y)^2$. 8.

XCIV.

1. 10·17736.

2.

3. (i.) $\frac{x}{3}-\frac{y}{2}+\frac{z}{5}$; (ii.) $3\sqrt{3} \pm 2\sqrt{5}$.

4. (i.) $x=\frac{ak^2}{al+bm}$, $y=\frac{bk^2}{al+bm}$; (ii.) $x=0$, [or $-\frac{1}{9}$].

5. 5 men, 5 women, 90 children.

6. $\frac{A(r-n+1)+B(n-1)}{r}$.

7. $\frac{5\pi}{19}$.

8. $\theta=-15°$, *or* $\theta+\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{6}$.

XCV.

1. ·083333.

2. (i.) $x=7$, $y=8$, $z=9$; (ii.) $x=6$, or $-4\frac{1}{2}$; $y=12$, or -9.

3. $\frac{bk-al}{k-l}$.

4. (i.) $780a+760b$; (ii.) $4\frac{25}{27}$; $5\frac{2}{5}$; (iii.) $\frac{10}{57}$, $\frac{10}{59}$, $\frac{10}{61}$, $\frac{10}{63}$.

5. In 240 weeks.

6. $\frac{\pi}{64}$.

7. $\theta=0$, or $\frac{\pi}{2}$; or $\theta-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$.

8. $-\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$, -1, $-\sqrt{2}$, $\sqrt{2}$, -1.

XCVI.

1.

2. $x^2+2x^{\frac{3}{2}}y^{\frac{3}{5}}-2x^{\frac{1}{2}}y^{\frac{9}{5}}-y^{\frac{12}{5}}$; $k^{\frac{2}{3}}+k^{\frac{1}{3}}l^{\frac{1}{3}}+l^{\frac{2}{3}}$.

3. $\frac{3-\sqrt{7}}{2}$.

4. (i.) $x=1·57(6)$, or $-·95(1)$;
(ii.) $x=\pm 7$, or $\pm\sqrt{43}$, $y=\pm 1$, or 0; (iii.) $x=16$,

5. A has £500, and spends £560; B has £700, and spends £400.
6. $\cdot 0505\pi$.
7. $\theta = 30°$, or $90°$.
8.

XCVII.

1. 1351 rupees, 5 annas, 0·99 pice; £160 9s. 4·553203125d.
2. 66661.
3. $x = \pm\sqrt{3}$. ($x^2 - 3$ is the G.C.M.)
4. (i.) $\dfrac{yz + zx + xy - x^2 - y^2 - z^2}{(x-y)(y-z)(z-x)}$; (ii.) $10\frac{2}{3}$.
5. (i.) $x = \frac{23}{52}$; (ii.) $x = \frac{1}{4}$, $y = -\frac{35}{32}$.
6. $56\cdot25°$, $62\cdot5^g$, $\dfrac{5\pi}{16}$.
7. $x = 45°$, $y = 15°$.
8. $\theta = n\pi + \dfrac{\pi}{4}$.

XCVIII.

1. 1 inch.
2. $1\frac{8}{9}$.
3. (i.) $\cdot45685(0)$; (ii.) 0; (iii.) $\dfrac{(x+2y)^2}{x-2y}$.
4. $\dfrac{3pq - p^3}{q^3}$.
5. (i.) $x = \dfrac{2abc}{ab+ac-bc}$, $y = \dfrac{2abc}{ab-ac+bc}$, $z = \dfrac{2abc}{bc+ac-ab}$;
(ii.) $x = 4$, or 10; $y = 2$, or 4.
6. $8a^3x^3(a-x)^3(a+x)$.
7. (i.) $\dfrac{\sqrt{3}+2\sqrt{2}}{6}$; (ii.) $\dfrac{4\sqrt{2}+7\sqrt{3}}{18}$.
8. $9n°$, or $10n^g$.

XCIX.

1. 60 miles; 20 and 30 miles per hour.
2. $3c - 2b$ and $3a - 2c$.
3. 0. [The value given is a root of the equation
$$(mx+n)(nx-m) = c^2.]$$
4. L.C.M. $a^3b^2(a^2-b^2)$; G.C.M. $x - y$.
5. (i.) $x = 3$, or 5; (ii.) $x = a - b$, $y = a^2 - b^2$;
(iii.) $x = -1$, $y = 1$, $z = 0$.
6.
7. $A = \frac{600°}{73}$, $B = \frac{540°}{73}$, $C = \frac{12000°}{73}$.
8. Euc. IV. 4; III. 20, 32.

C.

1. $x^4 - 2a^2x^2 + a^4 + 2ax^2 - 2a^3 - x^2 + a^2.$

2. (i.) $(a^{\frac{1}{2}} - b^{\frac{1}{3}})(a^{\frac{1}{2}} + b^{\frac{1}{3}})^2$; (ii.) $(3x+2)(2x-3)$; G.C.M. $x-a$; L.C.M. $60(x-a)(2x^2+2ax-a^2)(3x^2+5ax+5a^2)$.

3. (i.) $\dfrac{2x}{x^2-9}$; (ii.) a.

4. (i.) $x=16$; (ii.) $x=\dfrac{a}{a+b}$, $y=\dfrac{b}{a-b}$.

5. 8 pieces. **6.** 9.

7. $\dfrac{600^\circ}{\pi}$, $=190\dfrac{10^\circ}{11}$. **8.**

CI.

1. (i.) $x^4 - x^2(a+b) + ab$; (ii.) $\dfrac{x^2}{9} - \dfrac{ax}{3\sqrt{2}} + \dfrac{a^2}{2}$.

2. G.C.M. $2x-3$; L.C.M. $(x+2)^2(x-2)^3$.

3. (i.) $x=\dfrac{67a}{10}$; (ii.) $x=25$, $y=1$; (iii.) $x=7$, or $-\frac{1}{7}$;

(iv.) $x=y=z=5$. **4.** **5.**

6. 7370 feet. **7.** 1. **8.** Cf. Euc. IV. 11.

CII.

1. $a^2 - 4a^{\frac{3}{2}}b^{\frac{1}{2}} + 6ab - 4a^{\frac{1}{2}}b^{\frac{3}{2}} - 2ac^{\frac{1}{2}} + 4a^{\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{1}{2}} + b^2 - 2bc^{\frac{1}{2}} + c.$

2. **3.** (i.) $\dfrac{1}{a-b}$; (ii.) $\dfrac{4y(2x^2+y)^2}{y^3-x^6}$.

4. (i.) $x=\pm 1$, [or ± 3]; (ii.) $x=1, 1, \dfrac{1\pm\sqrt{13}}{6}$;

(iii.) $x=1$, or 2; $y=2$, or 1.

5. *A* to *B*, 85 miles; *A* to *C*, 81 miles; 51 and 45 miles per hour. **6.** 3. **7.**

8. $\sin\theta = \dfrac{1}{\sqrt{1+\cot^2\theta}}$; $\cos\theta = \dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$; (ii.) $\tan\theta = \cot 2a$.

CIII.

1. $2n=8$; $d=1\frac{1}{2}$.
2. $q=4$; $x=1$.
3. 3 miles an hour.
4.
5.
6. $120°$; $133^g\ 33^{`}\ 33^{``}\cdot\dot{3}$; $\dfrac{2\pi}{3}$.
7.
8.

CIV.

1. (i.) 0; (ii.) 1.
2. $ax^{-1}+1+a^{-1}x$; $2\sqrt{2}-\sqrt{3}$.
3. G.C.M. x^2-x+1; L.C.M. $x(x-1)(x-2)(x+3)(x^2+1)$.
4. (i.) $x=\dfrac{2ab}{a^2-b^2}$; (ii.) $x=2$, $y=1$; (iii.) $x=3$, or $-\frac{19}{8}$; (iv.) $x=1$, or -4; $y=2$, or -13.
5. $n=3$.
6. $22°\ 30'\ 1''\cdot 296$; $15°$.
7. $\frac{1}{3}$ or $\frac{5}{3}$.
8.

CV.

1. (i.) $\overline{4}\cdot 0969100$; (ii.) 5.
2. (i.) $x=2$, or $-\frac{2}{3}$; (ii.) $x=-\frac{1}{24}$, $y=\frac{13}{24}$, $z=\frac{5}{24}$; (iii.) $x=y=5$.
3. (i.) $\dfrac{19(\sqrt{3}-\sqrt{2})}{18}$; (ii.) 0.
4. $\dfrac{x^2}{9}-\dfrac{ax}{3\sqrt{2}}+\dfrac{a^2}{2}$.
5. $\dfrac{(\sqrt{a}-\sqrt{x})\sqrt{a-x}}{a-x}$.
6. $67\frac{1}{2}°$, 75^g, $\dfrac{3\pi}{8}$.
7. $x=52\frac{1}{2}°$, $y=7\frac{1}{2}°$.
8.

CVI.

1. (i.) $\frac{3}{4}(x-y)$; (ii.) 0; (iii.) Difference $=3$.
2. (i.) $x=y=12$; (ii.) $x=b+a$, $y=b-a$.
3.
4. $10\frac{1}{2}$ miles.
5. $n\pi+(-1)^n a$.
6. $x-\dfrac{\pi}{4}=2n\pi\pm A$.
7. $\overline{1}\cdot 8920946$.
8. Euc. I. 4.

CVII.

1. (i.) 0; (ii.) 3.
2. $8\cdot 3$; $12x^2+4y^3$.
3. (i.) $x=7a$, or $-a$; (ii.) $x=0$, or $\pm\sqrt{2}$.

4. 1 sov. : 3 half-sovs. : 6 half-crowns : 5 florins.

5. **6.** **7.** **8.** Euc. III. 28; VI. 3.

CVIII.

1. (i.) $x=\frac{1}{4}$, or $-\frac{5}{8}$; (ii.) $x=\pm\sqrt{3}$; (iii.) $x=\frac{1}{9}$, [or $\frac{1}{49}$].

2. Man, £3 10s.; woman, £1 15s.; child, 11s. 8d.

3. $8(\sqrt{2}-1)$ inches, $=3{\cdot}3137\ldots\ldots$ inches.

4. (i.) $s=153$, $l=-31$; (ii.) 5. **5.**

6. $\sec^2\theta=1+\tan^2\theta$; $\operatorname{cosec}^2\theta=1+\cot^2\theta$.

7. **8.** Euc. I. 32.

CIX.

1. $y-2$. **2.** Half-an-hour.

3. (i.) $x=4$, or 1; (ii.) $x=6$, or $\frac{5}{2}$; (iii.) $x=1$, [or 6].

4. $\frac{n}{2}\left\{2a+\overline{n-1}b\right\}$; (i.) n^2; (ii.) $\dfrac{n(6-n)}{5}$;

(iii.) $\dfrac{49}{2}\left\{1-\left(\dfrac{5}{7}\right)^n\right\}$; (iv.) 6 terms. **5.**

6. $\sin=\dfrac{1}{c}$, $\cos=\dfrac{\sqrt{c^2-1}}{c}$, $\tan=\dfrac{1}{\sqrt{c^2-1}}$, $\sec=\dfrac{c}{\sqrt{c^2-1}}$,

$\cot=\sqrt{c^2-1}$; (ii.) $\frac{12}{37}$, $\frac{35}{37}$, $\frac{12}{35}$, $\frac{37}{12}$, $\frac{37}{35}$, $\frac{35}{12}$.

7. $-\dfrac{2}{\sqrt{3}}$; $-\infty$; $2-\sqrt{3}$.

8. Cf. Euc. III. 31, 33; III. 21, I. 16.

CX.

1. $709\frac{1}{2}$ lbs.

2. (i.) $\dfrac{31\sqrt{5}+30}{25}$; (ii.) n^{th} term $=(8n-7)(8n+1)$;

$$s=64\,\frac{n(n+1)(2n+1)}{6}-24n(n+1)-7n$$

$$=\frac{64n^3+24n^2-61n}{3}.$$

3. (i.) $x=1$, 1, or $\dfrac{-3\pm\sqrt{5}}{2}$; (ii.) $x=\pm 3$, or $\pm\dfrac{1}{\sqrt{2}}$;

$y=\pm 2$, or $\mp\dfrac{5}{\sqrt{2}}$.

4.

5. $\dfrac{\lfloor 8}{\lfloor 2 \lfloor 2}$, $=10080$.

6. -1, $\dfrac{1\pm\sqrt{-3}}{2}$.

7. $\sin x=\dfrac{-\cos\alpha\pm\sqrt{4-3\cos^2\alpha}}{2}$.

8. $\dfrac{1}{41\times 61}$.

CXI.

1. 10s. $11\frac{1}{4}$d.
2. $p=13$.
3. ± 2, or $\pm\dfrac{\sqrt{-1}}{2}$.
4. 36 companies.
5. $2:3$ is the less.
6.
7. 83·19735 feet.
8.

CXII.

1. (i.) $x=1$; (ii.) $x=7$.
2. $v_1t - v_2t \propto t$.
3. $\dfrac{(a^2+1)^2}{2a(a-1)^2}$.
4. $2y^2$.
5.
6. a.
7. $-\frac{1}{2}$; $\frac{1}{2}$; $\sqrt{3}$.
8. The tangents from an external point to a circle are equal.

CXIII.

1.
2. (i.) $x=\dfrac{a\{a+b-1\}}{a+b}$, $y=\dfrac{b\{a+b-1\}}{a+b}$;

(ii.) $x=0, 5, 0$; $y=0, 4, \frac{1}{4}$; $z=0, 3, \frac{1}{2}$.
3. $\left(\dfrac{a}{b}\right)^{\frac{2}{3}}+\left(\dfrac{b}{a}\right)^{\frac{2}{3}}=1$.
4. $\dfrac{30\sqrt{2}(\sqrt{3}-1)}{\pi}$.
5. 7 pike, 5 eels.
6. (i.) $\operatorname{cosec}\theta-\sec\theta$; (ii.) $\sec A$.
7.
8. Euc. III. 22.

CXIV.

1. 2. $x=2$; amount $=$ £5 5s. 3. $x=3$, 5, or 7.

4. 5. (i.) $A=60°$; (ii.) $\theta=n\pi\pm\frac{\pi}{6}$.

6. 2·0969100; $\bar{4}$·6989700; 9·6989700.

7. Radius, 1·3 inches; perimeter, 8·16816 inches.

8. Euc. II. 14.

CXV.

1. As 155 : 147.

2. $3^{\frac{5}{2}}+3^2 . 5^{\frac{2}{3}}+3^{\frac{3}{2}} . 5^{\frac{4}{3}}+3 . 5^2+3^{\frac{1}{2}} . 5^{\frac{8}{3}}+5^{\frac{10}{3}}$. 3.

4. (i.) $x=7$; (ii.) $(a+b-c)(a-b+c)(b+c-a)=1$.

5. 6. $\phi=n\pi\pm\frac{\pi}{4}$, or $n\pi\pm\frac{\pi}{6}$.

7. $\frac{11\pi}{675}$ ft., or ·614 inches nearly. 8. Euc. III. 36.

CXVI.

1. $4\frac{1}{2}$ minutes past noon. 2. $x=18$, $y=30$.

3. (i.) £46 5s. 11·111d.; (ii.) $\cdot\dot{1}$d.

4. $\frac{3}{2}$; $\pm\sqrt{2}$; $\frac{4}{3}$; $\pm\frac{8\sqrt{2}}{9}$; $\frac{32}{27}$.

5. $(a-2b)x^2+(a^2-b^2)x+a^3-b^3$. 6. (i.) 75°; (ii.) $\frac{6}{5}$.

7. $\tan\theta=\frac{1}{a}$, or $-\frac{1}{b}$. 8.

CXVII.

1. $82\frac{7}{24}$ per cent.

2. (i.) $4x^2-7x+3$; (ii.) $2+\sqrt{-1}$.

3. (i.) $\frac{2(a+1)}{a^2+a+1}$; (ii.) $\frac{a-c}{1+ac}$.

4. (i.) $x=144$; (ii.) $x=\pm1$, $\pm\frac{7}{2}$; $y=\pm3,\mp12$.

5.

6. $\frac{1}{400}(\pi \pm 200)$.

7. $\theta = n\pi \pm \frac{\pi}{4}$.

8. Square and add.

CXVIII.

1. £450 10s. $1\frac{1}{16}$d.

2. 4 or 14.

3. $9\frac{3}{5}$ days.

4.

5. (i.) $\sqrt{\frac{x}{y}} - 1 + \sqrt{\frac{y}{x}}$; (ii.) $\sqrt{\frac{x}{y}}$.

6. $22\frac{1}{2}°$ and $67\frac{1}{2}°$.

7. $\cos x = \frac{\sqrt{5}-1}{2}$; $x = 2n\pi \pm \frac{2\pi}{5}$.

8. Euc. I. 10, 12; II. 12, 13. See "Progressive" Euclid, App. xxiv.

CXIX.

1. $30\frac{5}{64}$ per cent.

2. (i.) $x = 0, \frac{3}{4}$, or -3; (ii.) $x = 1, 1, -1 \pm \sqrt{5}$;
(iii.) $x = \frac{b\sqrt{a}}{\sqrt{a} \pm \sqrt{b}}$, $y = \frac{a\sqrt{b}}{\pm\sqrt{a} + \sqrt{b}}$.

3. (i.) $3\frac{1}{8}$; (ii.) $\frac{x(x^n - 1)}{x - 1} + \frac{n(n+1)}{2}a$.

4. $\frac{2}{3}, -\frac{1}{2}, \frac{3}{8}, -$ etc.

5. As $2:1$.

6. $-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 1, -\sqrt{2}, -\sqrt{2}, 1$.

7. $a = n\pi + \frac{3\pi}{4}$.

8. Euc. III. 20, 21, I. 32.

CXX.

1. 173877; 444·44.

2. $8x^2z = (x+z)^3$.

3. $x = 2, 6$, or $2\frac{1}{4}$.

4.

5. 9°.

6. 4·4471580; $\bar{5}$·5563025; $\bar{2}$·4559320.

7. $\frac{4\pi r^3}{3}$; $4\pi r^2$; $\frac{2\pi r^3}{3}$.

8. Euc. II. 11. (See "Progressive" Euclid, p. 135.)

CXXI.

1. $18\frac{1}{3}°$ Cent.
2.
3. 4, 1, 6, -1, etc.; $5n$.
4. 7 terms.
5. 20°, and 50°.
6. $(2n+1)\pi$.
7. $0,\ 1-\frac{1}{\sqrt{2}},\ 1,\ 1+\frac{1}{\sqrt{2}},\ 2$.
8. "Progressive" Euclid, App. ix.

CXXII.

1. 66 lbs. $10\frac{181}{324}$ oz.
2. 2, 4, 6, 8......, and 1, 3, 9, 27......; $n^2+n+\frac{3^n-1}{2}$.
3.
4. $ab-xy-ck-cl$.
5. ·33537.
6. $\cos(2\theta+2\phi)$.
7. 45°, 135°, 60°, 120°.
8.

CXXIII.

1. 4·54(3) litres.
2. (i.) $x=10$ or $-\frac{5}{7}$; (ii.) $x=1$, 2, or $\frac{3\pm\sqrt{17}}{2}$;

   (iii.) $x=\pm 6$, or $\pm\frac{14\sqrt{-1}}{\sqrt{3}}$; $y=\mp 10$, or $\pm\frac{2\sqrt{-1}}{\sqrt{3}}$;

   $z=\pm 4$, or $\mp\frac{16\sqrt{-1}}{\sqrt{3}}$.
3. $x^3=4(y+1)^2$.
4.
5.
6. $\frac{\sqrt{3}-1}{2\sqrt{2}},\ -\frac{\sqrt{3}+1}{2\sqrt{2}},\ \sqrt{3}-2,\ \frac{2\sqrt{2}}{\sqrt{3}-1},\ -\frac{2\sqrt{2}}{\sqrt{3}+1},\ -2-\sqrt{3}$.
7. $10°,\ 11\frac{1}{9}^g,\ \frac{\pi}{18}$.
8. Euc. I. 23, III. 11, 12.

CXXIV.

1. As 27951 : 12500. 2. $x=3,\ -\frac{7}{3},\ \left[\text{or } \frac{1\pm\sqrt{181}}{3}\right]$.

3. (i.) $5\sqrt{3}-2\sqrt{7}$; (ii.) 10.

4. $\dfrac{3abc}{2(ab+bc+ca)}$. 5.

6. $\dfrac{50d^\circ}{77}$, and $\dfrac{27d^\circ}{77}$. 7. $\dfrac{200}{\sqrt{3}}$ yds., $= 115\cdot470\ldots$ yds.

8.

CXXV.

1. (i.) $\dfrac{7\sqrt{3}}{2}$; (ii.) $\dfrac{3n-m}{2}$. 2. 75, 60, 48.

3. $x=3,\ -6,\ 12$, or -24.

4. $\tan\alpha=\dfrac{-1\pm\sqrt{1+\tan^2 2\alpha}}{\tan 2\alpha}$. 5. $\theta=\dfrac{n\pi}{8}$, or $\dfrac{2n\pi}{3}\pm\dfrac{\pi}{9}$.

6. 4 inches. 7. $1165\cdot699$.

8. Euc. II. 5; III. 3, 35. (Join C to the centre. See "Progressive" Euclid, App. xxiv.).

CXXVI.

1. 2. 17.

3. (i.) $x=1$, or 9; $y=9$, or 1; (ii.) $x=-3$, or 5; $y=0$, or 4; (iii.) $x=9,\ 14,\ 19,\ldots\ldots$; $y=7,\ 15,\ 23,\ldots\ldots$

4. (i.) $\dfrac{12\,.\,11\ldots\ldots7}{\lfloor 6}(20)^6\,.\,(3b)^6$;

(ii.) $a\left\{1+\dfrac{1}{3}\,\dfrac{b^3}{a^3}-\dfrac{1\,.\,2}{3^2\lfloor 2}\,\dfrac{b^6}{a^6}+\dfrac{1\,.\,2\,.\,5}{3^3\lfloor 3}\,\dfrac{b^9}{a^9}-\dfrac{1\,.\,2\,.\,5\,.\,8}{3^4\lfloor 4}\,\dfrac{b^{12}}{a^{12}}+\ldots\right\}$.

5. 6. (i.) $\theta=2n\pi-\dfrac{\pi}{6}\pm\dfrac{\pi}{3}$; (ii.) $A=n\pi+\dfrac{\pi}{6}$.

7. $40\sqrt{3}$ feet. 8. Euc. I. 37, III. 21.

CXXVII.

1. (i.) $x+1+\frac{1}{x}$; (ii.) $5+3\sqrt{6}$.

2. (i.) $x=3\frac{1}{2}$, or $-\frac{2}{3}$; (ii.) $x=5$, or -4; $y=2$, or $-\frac{5}{2}$; (iii.) $x=a$.

3. (i.) 180; (ii.) $1\frac{1}{8}$.

4. $\left(\frac{p}{q}\right)^{p+q}$

5. $x^{\frac{2}{3}}+x^{\frac{1}{3}}y\sqrt{2}+y^2$.

6.

7. The expression $=\cos 2\beta$.

8. $\frac{180(3-\sqrt{2})^\circ}{7}$, $\frac{180(3\sqrt{2}-2)^\circ}{7}$, $\frac{360(3-\sqrt{2})^\circ}{7}$, etc.

CXXVIII.

1. 1.

2. (i.) $x=2a$; (ii.) $x=\frac{3}{2}$, or $\frac{1}{2}$; $y=\frac{1}{2}$, or $\frac{3}{2}$.

3. Put each $=k$.

4. x^6-1.

5. $\frac{ap-\beta q}{p-q}$; $\frac{ap+\beta q-2aq}{p-q}$.

6. $\frac{3\pi}{4}$.

7.

8. ·4771213.

CXXIX.

1. (i.) $\frac{a^3(a^2+b^2)}{a^6-b^6}$; (ii.) $\frac{ab(a^2-1)}{a^2b+a-b}$.

2. (i.) $x=\frac{93}{53}$, $y=-\frac{219}{53}$; (ii.) $x=\frac{5}{2}$, or -1.

3. (i.) $-\frac{171}{2}$; (ii.) 24.

4. 24 ft. long, 18 ft. wide.

5.

6. $44^g\ 37'\ 3''\cdot 7\dot{0}\dot{3}$.

7.

8. $\theta=n\pi+\frac{\pi}{2}$.

CXXX.

1. Between 8 and -2; $p=0$.

2. A, 18 miles; B, 12 miles.

3. $2p^3-9pq+27r=0$.

4.

5. $x=-8$, or $\frac{1}{4}$.

6. 133·785 feet.

7. Between 37 and 38 years.

8. Euc. I. 37, 38, 39.

CXXXI.

1.

2. (i.) $x = \pm 1$, or $(\pm 1 \pm \sqrt{2})\sqrt{-1}$;

(ii.) $x = 5$, [or 11]; $y = \pm 4$, [or $\pm\sqrt{112}$];

(iii.) $x = 4$, $y = 5$, $z = 6$.

3. 99, $52\frac{4}{5}$, 36.

4. The squares.

5. 240 and 360.

6. $\frac{4\pi}{3}$.

7.

8. See Ex. CXX. 4 (ii.).

CXXXII.

1. (i.) $-\dfrac{(a^2+b^2)(a^2-ab+b^2)}{(a+b)(a^3-b^3)}$;

(ii.) $\dfrac{aceg + aceh + adg + adh + beg + beh + acf + bf}{ceg + ceh + dg + dh + cf}$.

2. $\dfrac{9P+Q}{90}$.

3. $(a+b+c)^2$; $x^{\frac{1}{3}} + a^{\frac{1}{6}}x^{\frac{1}{6}} + a^{\frac{1}{3}}$.

4. (i.) $\dfrac{1}{\sqrt{6}}(\sqrt{2}-1)$; (ii.) $2x^2 - 2xy - y^2$.

5. G.C.M. x^2+x+1; L.C.M. $(x-1)(x-2)(x-3)$.

6. 120 acres, 35s. an acre.

7. $x = \dfrac{2n\pi}{3}$, or $\dfrac{n\pi}{4}$.

8. $\dfrac{81^\circ}{50\pi}, = 30'\ 55\dfrac{7''}{11}$.

CXXXIII.

1. 30,000 sq. mètres.

2. $x^2 - 9x + 20 = 0$; $b^2 = 4ac$.

3. 1, 2, 4, 8, etc.

4.

5. (i.) $x = \dfrac{-1 \pm \sqrt{17}}{2}$, or $\dfrac{-1 \pm \sqrt{2}}{2}$;

(ii.) $x = 2, -3, 1$, or 1; $y = 1, 1$, or $\dfrac{1 \pm \sqrt{17}}{2}$.

6. $a^2 - b^2$.

7. $\frac{1}{4}(\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}})$.

8. $\theta = \dfrac{n\pi}{3}, \dfrac{n\pi}{2}$, or $n\pi \pm \cos^{-1}\sqrt{\dfrac{2}{3}}$.

CXXXIV.

1. $\frac{26}{111}$.
2. 2s. 10d., and 1s. 4d.
3. (i.) $\dfrac{b^2-2ac}{a^2}$, and p^2-2q; (ii.) $\dfrac{b^3-3abc}{a^3}$, and $p^3-3pq+3r$.
4. $x=\dfrac{n\pi\pm\sqrt{n^2\pi^2+16}}{4}$.
5. (i.) $\dfrac{18}{\pi}$ feet; (ii.) 5729·6 yards.
6. 11·00525 cwt.
7. $x=1{\cdot}556$.
8. Euc. I. 41.

CXXXV.

1. £3475.
2. (i.) $x=7$; (ii.) $x=19$, $y=31$.
3. (i.) -48; (ii.) 3^n-1; (iii.) 50.
4. (i.) $\dfrac{6a^2+ab+b^2}{3a-b}$; (ii.) $\dfrac{\sqrt{a^2-x^2}+x-a}{x}$.
5.
6.
7. ·6532125.
8. Euc. III. 28.

CXXXVI.

1. If $m=n$ the inequality becomes an equality.
2. $a^n r^{\frac{n(n-1)}{2}}$; ar^m.
3. (i.) $\dfrac{1}{x+y}$; (ii.) x.
4. 20 men, 30 women.
5. 1s. 4d.
6.
7. 1·4807254.
8. Euc. III. 32, 21, 22.

CXXXVII.

1. £1700.
2. Halves.
3. 20, 30, 45.
4. (i.) $\dfrac{-3\pm\sqrt{-7}}{8}$; (ii.) $x=\pm\sqrt{3}$, or $\pm\sqrt{\frac{3}{19}}$;

   $y=0$, or $\pm 6\sqrt{\frac{3}{19}}$.
5.
6. $B=\dfrac{n\pi}{5}+(-1)^n\dfrac{\pi}{20}$, $=9°$, $27°$, $81°$, $99°$, $153°$, etc.
7. $\bar{4}{\cdot}3180633$.
8. Euc. I. 37.

CXXXVIII.

1. (i.) $x=\frac{16}{13}$, [or 0];

(ii.) $x=6$, or $\frac{9}{4}$; $y=\pm\frac{1}{\sqrt{3}}$, or $\pm 2\sqrt{-\frac{1}{3}}$.

2. 12. 3. $x^2-(1-x)^2+1=2x$.

4. 160 acres arable, 60 acres pasture. 5.

6. 8·496253. 7. 8. Euc. I. 24, 47.

CXXXIX.

1. (i.) $\frac{a-2b}{a^2+ab+b^2}$; (ii.) $\frac{2}{x-3y}$.

2. (i.) $x=-\frac{71}{120}$; (ii.) $x=\left\{\frac{3a^2\pm\sqrt{12ab^3-3a^4}}{6a}\right\}^3$,

$y=\left\{\frac{-3a^2\pm\sqrt{12ab^3-3a^4}}{6a}\right\}^3$.

3. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. 4.

5. 12 feet. 6.

7. 1·34484. 8. Euc. III. 21, 22.

CXL.

1. (i.) $x=3$, [or $-\frac{4}{5}$]; (ii.) $x=0$, $y=\infty$, $(xy=1)$; or $x=-\frac{1}{2}$, $y=-1$.

2. $1\frac{5}{6}$ hours. (Original pace, 3 miles an hour).

3. 4. $s=\frac{n(a+l)}{2}$.

5. 6. 2·174773. √

7. (i.) $\sin\theta=\frac{-1\pm\sqrt{5}}{4}$; (ii.) $\theta=\frac{n\pi}{2}$, or $\frac{n\pi}{3}$.

8. Cf. 'Progressive' Euclid, App. xxiv.

CXLI.

1. (i.) $x = \pm\sqrt{\frac{n}{n-2}}$, or $\pm\sqrt{\frac{n-1}{n+1}}$;
(ii.) x and $y = a\frac{-b^2 \pm b\sqrt{2a^2 - b^2}}{a^2 - b^2}$. 2.
3 £528. 4. $3(\sqrt{2}+1)$. 5.
6. 7. $\sin\theta = \pm\frac{\sqrt{3}}{2}$, or $\pm\frac{1}{\sqrt{2}}$.
8. Euc. II. 11.

CXLII.

1. 2. $-\frac{15}{17}$; $(2a+b-c$ is a common factor).
(ii.) $(ac \mp bd)^2 + (bc \pm ad)^2$.
3. $n = \frac{2a-b-c}{a-b}$; $a+c=0$. 4. Result $= 2xy(x^2 - y^2)$.
5. 6. 1·26765. 7. $\frac{1-\tan^8 A}{\tan^4 A}$.
8. Euc. III. 26.

CXLIII.

1. 600. The votes would have been 300, 200, 100.
2. $3\frac{1433}{1442}$ mins. to 12. 3.
4. $x = 2$, or $\frac{1}{108}$; $y = 3$, or 18. 5. $\frac{n(n+1)(2n+7)}{6}$.
6. $3\frac{x^2}{y} - 2x + 3y$. 7. $\theta = (2n+1)\frac{\pi}{4}$.
8. $\theta = \tan^{-1}(1+a)$; $a = -\frac{2}{3}$, 2, and ∞ respectively.

CXLIV.

1. B, 83 yards; C, 132 yards. 2.
3. 4.
5. When $A = B = C$. 6. $\theta = \sin^{-1}\frac{1 \pm \sqrt{7}}{4}$.

7. $\sin A = \dfrac{2ab}{a^2+b^2}$, and $\sin 2A = \dfrac{4ab(a^2-b^2)}{(a^2+b^2)^2}$.

8. Cf. Euc. IV. 4.

CXLV.

1. $a:b:c :: 2:3:4$.

2. $(x^4+1)(x+1)^2(x-1)$; $x^{\frac{1}{2}}y^{\frac{1}{4}} + 2y^{\frac{1}{3}} + x^{-\frac{1}{2}}$.

3. (i.) $x=\frac{1}{2}$; (ii.) $x=y=z=0$, or $\dfrac{1}{yz} = \frac{1}{2}\left(\dfrac{1}{c^2}+\dfrac{1}{b^2}-\dfrac{1}{a^2}\right)$, etc.,

whence $x = \dfrac{abc\sqrt{2}\sqrt{a^2b^2+a^2c^2-b^2c^2}}{\sqrt{(a^2b^2-a^2c^2+b^2c^2)(a^2c^2-a^2b^2+b^2c^2)}}$, etc.

4. $x^3-px^2+qx-r=0$. 5. 167.

6. (i.) $\theta = n\pi + (-1)^n\dfrac{\pi}{3}$; (ii.) $\alpha = (2n+1)\dfrac{\pi}{4}$, $\beta = n\pi \pm \dfrac{\pi}{6}$.

7. $8(\sin^4 A - \sin^2 A)+1$. 8. Euc. I. 37.

CXLVI.

1.

2. (i.) $x=4$, or $-\frac{1}{5}$; $y=5$, or $-\frac{1}{4}$; (ii.) $x = \dfrac{-b}{a+(ab-a^3)^{\frac{1}{3}}}$.

3. $\frac{1}{6}$. 4. 864.

5. n^{th} term $= \dfrac{(A-B)n+Ba-Ab}{a-b}$;

$s = \dfrac{2n(Ba-Ab)+(A-B)(n^2+n)}{2(a-b)}$. 6.

7. $AD = 20\sqrt{6}$, area $= 100(6+\sqrt{6})$. 8. 4 miles.

CXLVII.

1. (i.) $x=36$; (ii.) $x = \left(\dfrac{a^n}{b^q}\right)^{\frac{mp}{np-mq}}$, $y = \left(\dfrac{a^m}{b^p}\right)^{\frac{nq}{mq-np}}$.

2. 3. 13 and 7, *or* $10\sqrt{2}$ and $3\sqrt{2}$.

4. 6 ac. 3 ro. 37 po. $10\frac{3}{4}$ yds. **5.** $\dfrac{1-a^2}{1+a^2}$, and $\dfrac{2a}{1+a^2}$.

6. $x = n\pi \pm \dfrac{\pi}{3}$. **7.** $\dfrac{l}{2\sqrt{2}}$. **8.** 16.

CXLVIII.

1. (i.) $\dfrac{c^2-b^2}{2(a-c)}$; (ii.) Indeterminate, the equation becomes an identity. **2.** 1.

3. (i.) $x=1$; (ii.) $x = \pm 5, \pm 3$; $y = \pm 3, \pm 5$.

4. $2292\frac{3}{8}$ cub. ft.; 173 tons 19 cwt. 1 qr. $27\frac{3}{4}$ lbs.

5. $4 \sin 2A \sin \dfrac{60-A}{2} \sin \dfrac{60+A}{2}$. **6.** $16\frac{4}{5}$ feet.

7. Height $\dfrac{a(3+\sqrt{3})}{2}$, Breadth $\dfrac{a(1+\sqrt{3})}{2}$.

8. Euc. II. 4.

CXLIX.

1. (i.) $x = 2(\sqrt{3}-2)$, or $2-\sqrt{3}$;

(ii.) $\dfrac{x-a}{a-b} = \dfrac{y-b}{b+c} = \dfrac{z-c}{-c-a} = \dfrac{2c(a-b)}{a^2+b^2+c^2+bc+ac-ab}$.

2. 140 miles; 20 miles an hour. **3.**

4. $p(b-c)+q(c-a)+r(a-b)=0$; 1, 2, 3.

5. $6\sqrt{3}$ miles. **6.** $4\sqrt{3}$ miles.

7. 1·3979400; 2·9030900. **8.**

CL.

1. **2.** $x = -1, 1, 3$, or 5. **3.**

4. (i.) $x+\dfrac{1}{x}$, $x+\dfrac{2}{x}$, $x+\dfrac{3}{x}$, etc.; (ii.) If $a=b=c$.

5. 39° 53′ 46″. (ABO is a right angle.)

6. $10(10-3\sqrt{11})$ ft., $=$ ·50125 ft. **7.**

8.

CLI.

1. (i.) $y=\frac{a+b}{a-b}$, $z=\frac{a-b}{a+b}$; (ii.) $x=y=1$, or 0,

   or, $x=\frac{3\pm\sqrt{-3}}{2}$, $y=\frac{3\mp\sqrt{-3}}{2}$; (iii.) $x=\sqrt{a}$.

2. $a:b:c::2:3:4$.
3. $4\frac{2}{5}$ miles; 22 mins.
4. $\frac{x}{2}$.
5. $4\sqrt{2}$ miles.
6. (i.) $x=\frac{n\pi}{3}$, or $\frac{n\pi}{2}\pm\frac{\pi}{12}$; (ii.) $2A=n\pi+(-1)^n\frac{\pi}{3}$.
7.
8. Sine, 1 and -1; tangent, none; secant cannot lie between 1 and -1.

CLII.

1. $\cdot356207$.
2. (i.) $x=0, 0, 0, \frac{2}{3}$; $y=0, 0, 0, -2$;

   (ii.) $x=3, -\frac{7}{2}$, $\left[\text{or } \frac{-3\pm\sqrt{1357}}{12}\right]$.
3.
4. $3x^2-2a^2x-a^4=0$.
5. (i.) 7·44(4) in.; (ii.) 3·38(5) in.
6.
7. $1, \frac{1}{2}, \frac{\sqrt{3}}{2}$.
8. $\frac{1}{2}(\sqrt{3}+1)$ miles.

CLIII.

1. (i.) $\frac{x^4-x^3-x^2-2x-1}{(x+1)(x-1)^2}$; (ii.) $4\sqrt{x(x-1)}$.
2. 0, 3, 6, 9,...... 30 fourpenny pieces; 42, 38, 34,...... 2 threepenny pieces.
3. $3+\frac{1}{7+}\,\frac{1}{15+}\,\frac{1}{1+}\,\frac{1}{25+}\,\frac{1}{1+}\,\frac{1}{7+}\,\frac{1}{4}$;

   $\frac{3}{1}, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{9208}{2931}, \frac{9563}{3044}, \frac{76149}{24239}, \frac{314159}{100000}$.

4. $a^{\frac{1}{2}} - \frac{1}{2} - \frac{1}{8}a^{-\frac{1}{2}} - \frac{3}{8}a^{-1}$.

5. £900, and £800.

6. $\dfrac{3000a + 50b - 2700p - 27q}{270000}$.

7. $a(2c^2 - d^2) = bcd$.

8. $\frac{1}{2}$; $2 - \sqrt{3}$.

CLIV.

1. $t4592$; 215102.

2. 4, 8, 16.

3. (i.) $x = 4(a+b)$, [or 0]; (ii.) $x = 4$, or 3; $y = 3$, or 4; (iii.) $x = 101, 88, 75, \ldots\ldots 10$; $y = 7, 16, 25, \ldots\ldots 70$.

4. $\dfrac{x}{y} + 1 - \dfrac{y}{x}$.

5. When $b^2 < 4ac$.

6. $\overline{1}\cdot4660179$.

7. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

8. Euc. I. 39.

CLV.

1.

2. (i.) $x = 1$, 1, or $-3 \pm 2\sqrt{2}$; (ii.) $x = 5\frac{1}{2}$.

3. $\lambda = -(\sqrt{A} \pm \sqrt{B})^2$.

4. The condition gives $(b+c)(c+a)(a+b) = 0$. Hence $b = -c$, or $c = -a$, or $a = -b$.

5. $\pm\dfrac{2\sqrt{15}}{13}$, $\left[\text{or } \pm\dfrac{2\sqrt{2}}{13}\right]$.

6. $\tan^2\dfrac{x}{2} = \dfrac{a-1}{a+1}$; hence a must be > 1, or < -1.

7.

8. Euc. I. 37, 38, 39. See "Progressive" Euclid, App. vii., viii.

CLVI.

1.

2. $y = 7, 3, 1, 0, -1, -2, -3, -5, -9$.

3. (i.) $x = 4$, $y = 1$; (ii.) $x = a$, or $a\dfrac{-1 \pm \sqrt{5}}{2}$; (iii.) $x = \pm 36$, or ± 3; $y = \mp\frac{23}{2}$, or ± 5.

4.

5. £165·532. (Arithmetically, £165·548145). 6.

7. $x=\frac{n\pi}{16}$. 8. $\cos\theta=\frac{4}{5}$, $\tan\theta=\frac{3}{4}$.

CLVII.

1. $ab+bc+ca+2\left(\frac{a+b+c}{3}\right)^2$. 2. $a^2-\sqrt{2-m}\,.\,ab+b^2$.

3. (i.) $y^2+2cy=2a$, or $\frac{2a(a-c)}{a+c}$; hence

$$y=-c\pm\sqrt{2a+c^2}, \text{ or } -c\pm\sqrt{\frac{2a^2-2ac+ac^2+c^3}{a+c}};$$

(ii.) $x=\pm 2, \pm 1, \pm 2i, \pm i$; $y=\pm 1, \pm 2, \pm i, \pm 2i$. $(i=\sqrt{-1})$. 4. 5.

6. $6+2\sqrt{3}$ inches. 7. 81° 45′ 1·3″.

8. (i.) $x=\frac{\log(1\pm\sqrt{5})-\log 2}{\log a}$; (ii.) -1.

CLVIII.

1. 8, 12, 16. 2. $x=\cdot 63093$.
3. A, 3 galls.; B, 8 galls. 4. x^2y-z^3.
5. 6.
7. 8. Euc. III. 20, I. 47.

CLIX.

1. (i.) $\frac{(a+b)^3(b-a)}{2a^2b^4}$; (ii.) 0. 2.

3. 10 miles an hour.

4. (i.) $\sqrt{7}+\sqrt{5}$; (ii.) $\sqrt{3}+\sqrt{2}$; (iii.) $\frac{5}{3}(8+\sqrt{15})$.

5. 6. $x=3\cdot 97$. 7. 17·1099 miles.

8. N.B. $mn=1$. Substitute $\frac{1}{m}$ for n.

CLX.

1. The Six Per Cents., in the ratio 1176 : 1135.
2. £2 14s. $4\frac{1}{2}$d.
3. (i.) $x = \dfrac{b\sqrt{a}}{\sqrt{a} \pm \sqrt{b}}$; (ii.) $x = \frac{3}{2}$, or $-\frac{1}{2}$; $y = \frac{1}{2}$, or $-\frac{3}{2}$.
4. -7; 3; 10.
5. $\dfrac{n}{2}\left\{(n+1)\sqrt{2} - n + 3\right\}$.
6. (i.) 0; (ii.) $\frac{26}{9}$.
7. $\dfrac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{\lfloor r}$.
8. I. 8, 4. The line joining the centres passes through the point of contact.

CLXI.

1. 2·5563025; 1·8061800; ·6989700.
2. (i.) $x = 4$, or -6; $y = 2$, or 12; (ii.) $x = \frac{4}{9}$, or $-\frac{2}{3}$; (iii.) $x = 3$, $y = 9$, $z = 8$.
3.
4.
5. (i.) 220; (ii.) 55.
6. $3y^{\frac{4}{3}} + x^{-\frac{1}{8}} + 2x^{-\frac{9}{8}}y$.
7. $\dfrac{162^\circ}{\pi}$, $= 51\frac{6}{11}^\circ$.
8.

CLXII.

1.
2. (i.) $x = 1$, $y = \sqrt{2}$, $z = \sqrt{3}$;
   (ii.) $x = \pm 2$, $\pm\frac{1}{2}$, or $\pm\sqrt{\frac{17}{8}}$; $y = \mp\frac{1}{2}$, ∓ 2, or $\pm\sqrt{\frac{17}{8}}$.
3. $\dfrac{100(bmp + anq)}{ab(p+q)}$.
4. (i.) $\left\{\dfrac{n(n+1)}{2}\right\}^2$; (ii.) $\dfrac{a(1-a^n)}{(1-a)^2} - \dfrac{na^{n+1}}{1-a}$.
5.
6.
7. 4 and 3; or, 8 and 4; or, 20 and 5.
8.

CLXIII.

1. (i.) $x = \pm\dfrac{\sqrt{9 \pm \sqrt{33}}}{2\sqrt{3}}$, $y = \pm\dfrac{\sqrt{15 \pm \sqrt{33}}}{2\sqrt{3}}$;
(ii.) $x = \sqrt[2n]{8}$, or $\sqrt[2n]{-(\frac{8}{3})^3}$. 2.

3. $\dfrac{x}{y} - \dfrac{y}{x} + \dfrac{\sqrt{-1}}{2}$. 4. 5.

6. 28. 7. (i.) $\theta = \tan^{-1}\dfrac{a \pm \sqrt{a^2 + 4b^2}}{2}$; (ii.) $x = 5$.

8. Euc. I. 31, 34.

CLXIV.

1. (i.) $2 - \sqrt{3}$; (ii.) $x^3 + x + 1$.
2. (i.) $3\frac{1}{2}$; (ii.) $3\frac{29}{32}$; (iii.) 5. 3.
4. a by a, and $2a$ by $\frac{1}{2}a$. 5. $n\pi + \alpha$; $\theta = n\pi$.
6. $2 \sin A \cos A$; $\dfrac{2 \tan A}{1 - \tan^2 A}$; $\sqrt{\dfrac{1 - \cos A}{2}}$.
7. 8. $\dfrac{a(p_1 + p_2 + p_3)}{2} =$ area; where a is a side.

CLXV.

1. $x = 7$. 2. (i.) -1; (ii.) 1. 3.
4. $p^2 - 2q$; $\dfrac{p\sqrt{p^2 - 4q}}{q}$. 5. $2^n - 2$.
6. (i.) $x = b$, b, $\dfrac{-1 \mp \sqrt{3}}{2}b$; $y = a$, a, $(1 \pm \sqrt{3})a$.
(ii.) $x = \infty$, 2, or $\dfrac{-2 \pm 2\sqrt{7}}{3}$. 7.
8. $x = \dfrac{2n\pi}{3}$, or $2n\pi + \dfrac{\pi}{2}$.

CLXVI.

1. $x = 0, 2, 4, 6, 8, 10$; $y = 29, 24, 19, 14, 9, 4$.
2. $9e21$; te.
3. (i.) $\dfrac{60 \,.\, 59 \,.\, 58 \,.\, 57}{\lfloor 4}$; (ii.) $\dfrac{59 \,.\, 58 \,.\, 57 \,.\, 56}{\lfloor 4}$.

4. (i.) $x=a$, or $-\dfrac{c(a+b)}{2(a+c)}$;

(ii.) $x=\sqrt[3]{-1}$, or $\sqrt[3]{\frac{169}{38}}$; $y=\sqrt[3]{1}$, or $\dfrac{10}{\sqrt[3]{494}}$;

(iii.) $yz=2$, or $\frac{1}{2}$, $zx=3$, or $\frac{1}{3}$, $xy=6$, or $\frac{1}{6}$; whence
$x=6,\ 3,\ 2,\ 1,\ 1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{6}$;
$y=1,\ 2,\ 3,\ 6,\ \frac{1}{6},\ \frac{1}{3},\ \frac{1}{2},\ 1$;
$z=\frac{1}{2},\ 1,\ \frac{1}{6},\ \frac{1}{3},\ 3,\ 6,\ 1,\ 2.$

5. $n\pi+(-1)^n\dfrac{\pi}{6}$. 6.

7. $(a\sin\alpha+b\cos\alpha)^2=2b(a+b)$.

8. Euc. I. 23, 9, 31. See "Progressive" Euclid, p. 155.

CLXVII.

1. 2. $\frac{7}{13}$.

3. 149 sq. ft., 128 sq. in.; or 105t8 sq. in.

4. (i.) $x=\pm 2$, or ± 5; (ii.) $x=7$, $y=13$;
(iii.) $x=\pm 1$, $\pm\sqrt{-1}$.

5. (i.) $3^{10}.x^{10}+20.3^9x^9a+180.3^8.x^8a^2+960.3^7.x^7.a^3$
$+3360.3^6.x^6.a^4$;

(ii.) $\dfrac{1}{2^5x^5}+\dfrac{25a}{2^6.x^6}+\dfrac{375a^2}{2^7x^7}+\dfrac{35.5^3a^3}{2^8x^8}+\dfrac{70.5^4.a^4}{2^9x^9}$. 6.

7. 2600^g; $173^g\ 33^{\grave{}}\ 33^{\grave{}\grave{}}\cdot\dot{3}$. 8. Euc. III. 21, I. 11.

CLXVIII.

1. $ax+by+cz$.

2. (i.) $x=3$, $y=5$; (ii.) $x=2$, $\frac{1}{2}$, or ∞.

3. (i.) $a^2x^2-(b^2-2ac)x+c^2=0$;
(ii.) $a^2x^2+a(b-2c)x+c(a-b+c)=0$.

4. 9509900499; 3^8. 5. $1\frac{11}{24}$ mins. past 12.

6. sec and cosec never <1, tan and cot, any values.

7. 120 feet. 8. Euc. II. 14, I. 11, III. 37.

CLXIX.

1. $x=2$. 2. 2^n.

3. (i.) $x=4$, or $-\frac{37}{4}$; $y=5$, or $-\frac{61}{9}$;
(ii.) $x=0, \frac{1}{3}, \frac{1}{4}$; $y=0, \frac{1}{4}, \frac{1}{3}$. 4.

5. $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}-y^{\frac{1}{3}}z^{\frac{1}{3}}-z^{\frac{1}{3}}x^{\frac{1}{3}}$.

6. (i.) $\dfrac{4abc}{(b+c-a)(c+a-b)(a+b-c)}$; (ii.) 1.

7. $n\pi+(-1)^n\dfrac{\pi}{6}$. 8. $\dfrac{\sqrt{2-\sqrt{2}}}{2}$, $\dfrac{\sqrt{2+\sqrt{2}}}{2}$, $\sqrt{2}-1$.

CLXX.

1. (i.) $\dfrac{x^2+y^2}{x^2+xy+y^2}$; (ii.) $(x+1)(x+2)(x-2)(x+3)$.

2. $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; (i.) ∞, and $-\dfrac{c}{b}$; (ii.) ∞, ∞;
(α) $x=2$, -8, [or $-3\pm3\sqrt{5}$];
(β) $x=\frac{1}{2}\left\{b\pm\sqrt{\dfrac{4a^3-b^3}{3b}}\right\}$, $y=\frac{1}{2}\left\{b\mp\sqrt{\dfrac{4a^3-b^3}{3b}}\right\}$.

3. (i.) 27 ways; (ii.) 99 ways. 4. $\frac{6}{7}$; $1-x^{\frac{3}{4}}$.

5. Put $a=k\sin A=k\sin\overline{B+C}$, etc.

6. $\theta=n\pi\pm\dfrac{\pi}{4}$. 7. $\cdot02086675$.

8. Euc. VI. 3. Cf. Ex. CXLII. 8.

CLXXI.

1. $x+4$; $ab^{-1}+ba^{-1}+1$.

2. (i.) $x=\pm\sqrt{ab}$; (ii.) $x=5$, $y=12$.

3. Equate coefficients of x^{n+1} in the identity
$(1+x)(1+x)^{2n-1}\equiv(1+x)^{2n}$.

4. £375, £125. 5. 6. (i.) -1; $2\cdot199541$.

7. (i.) $\theta=(2n-1)\dfrac{\pi}{2}$; (ii.) $\sin 2x=\dfrac{4}{(2n+1)\pi}$. 8. 0.

CLXXII.

1.

2. Any number in the scale of r diminished by the sum of its digits, is divisible by $r-1$.

3. $\dfrac{3.7.11\ldots\ldots31}{4^8\,\lfloor\underline{8}}(3x)^{-\frac{3}{4}}\left(\dfrac{y}{6x}\right)^8.$

4. $3x-7$; $(x^3-8)(x+2)(x-3)$.

5.

6. $n\pi \pm \dfrac{\pi}{6}$.

7.

8. Euc. II. 12, 13. See "Progressive" Euclid, App. xxiv.

CLXXIII.

1. (i.) $n^2+2^{n+1}-2$; (ii.) $x^2\left\{\dfrac{1-x^{n-1}}{(1-x)^2}-\dfrac{(n-1)x^{n-1}}{1-x}\right\}$;

(iii.) $\dfrac{n(n+1)(2n+7)}{6}$.

2. The $\overline{n+1}^{\text{th}}$; $\frac{1}{2}\left\{2^{2n}-\dfrac{2n(2n-1)\ldots\ldots(n+1)}{\lfloor\underline{n}}\right\}$.

3. $\dfrac{\lfloor\underline{n}}{\lfloor\underline{p}\,\lfloor\underline{q}\,\lfloor\underline{r}}$; $n=7$.

4.

5. (i.) $(n-1)\theta=m\pi$, or $(n+1)\theta=m\pi+(-1)^m\dfrac{\pi}{2}$;

(ii.) $2\theta=n\pi+(-1)^n\dfrac{\pi}{3}$.

6. $\dfrac{\pi(60x+y-54z)}{10800}$.

7.

8.

CLXXIV.

1. $2\frac{1427}{3168}$ in. per minute.

2. $1{\cdot}8573325$; $2{\cdot}1303338$; $\bar{1}{\cdot}7406162$.

3. $\dfrac{3.5.7.9.11}{2^5\,\lfloor\underline{5}}$

4. 74, or 47.

5. $\dfrac{12-2\sqrt{6}}{7}$.

6. (i.) $\frac{5n(n+1)}{8}$; (ii.) $17\frac{727}{729}$; 18.

7. (i.) $\theta=(2n+1)\frac{\pi}{2}$, or $n\pi+(-1)^n\frac{\pi}{6}$;

(ii.) $\frac{\phi}{2}=n\pi$, or $2n\pi\pm\frac{\pi}{3}$. 8. Euc. III. 33, 21; I. 31.

CLXXV.

1. $1+\frac{p}{q}x+\frac{p(p+q)}{q^2\,\underline{|2}}x^2+\frac{p(p+q)(p+2q)}{q^3\,\underline{|3}}x^3$;

$\frac{p(p+q)\ldots\ldots(p+\overline{r-2}\,.\,q)}{q^{r-1}\,\underline{|r-1}}x^{r-1}$.

2. $\frac{n(n-1)\ldots\ldots(n-r+1)}{\underline{|r}}$; $\frac{m}{2}$, or $\frac{m}{2}+1$;

$$\frac{m(m-1)\ldots\ldots\left(\frac{m}{2}-1\right)}{\underline{\left|\frac{m}{2}\right.}}=2\frac{(m-1)\ldots\ldots\left(\frac{m}{2}-1\right)}{\underline{\left|\frac{m}{2}-1\right.}}.$$

3. (i.) $x=5$, or 3;

(ii.) $x=2$, or $\frac{8}{5}$; $y=3$, or $\frac{5}{2}$;

(iii.) Put $x=\sin\alpha$, $y=\sin\beta$, $z=\sin\gamma$.

$x=y=z=\pm 1$, or $x=\frac{a+bc}{\sqrt{(1-b^2)(1-c^2)}}$, etc.;

(iv.) Put $x=\tan\theta$, $y=\tan\phi$, and we get

$\tan(\theta+60^\circ)=-\tan 2\phi$, $\cot(\phi-15^\circ)=\tan 2\theta$,

whence $x=\frac{1}{\sqrt{3}}$, $-\frac{1}{\sqrt{3}}$, ∞,

$y=1$, $-2+\sqrt{3}$, $2-\sqrt{3}$.

4.

5. $R=\frac{abc}{4S}$.

6. $B=70^\circ\ 53'\ 36''\cdot 2$, $C=49^\circ\ 6'\ 23''\cdot 8$.

7.

8. $x=\pm 3$.

CLXXVI.

1. (i.) n^2; (ii.) $\frac{4}{15}\{1-(-\frac{3}{2})^n\}$; (iii.) 3 and 6.
2. (i.) $x=51$, $y=76$, $z=1$;

(ii.) $x=\dfrac{(a+b)(4ab-a^2-b^2)}{2ab}$, $y=\dfrac{(a-b)(a^2+b^2)}{2ab}$;

(iii.) $x=3$, 5, [or $4\pm\sqrt{10}$].

3. $\dfrac{3\,.\,1\,.\,1\,.\,3\,.\,5}{\underline{|5}}x^5$, $=\frac{3}{8}x^5$. 4. $\dfrac{\underline{|12}}{\underline{|3}\,\underline{|4}\,\underline{|5}}$.

5. $\dfrac{1}{\sqrt{2}}$, or $\dfrac{-1\pm\sqrt{3}}{2\sqrt{2}}$. 6. $\dfrac{100}{\pi}$.

7. $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$, $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$.

8. Euc. I. 31. Cf. "Progressive" Euclid, App. vii., xii.

CLXXVII.

1. $a^6-6a^5x+15a^4x^2-20a^3x^3+15a^2x^4-6ax^5+x^6$;

$$1-\tfrac{5}{4}x-\frac{5\,.\,3}{4^2\,.\,\underline{|2}}x^2-\frac{5\,.\,3\,.\,11}{4^3\,.\,\underline{|3}}x^3-\frac{5\,.\,3\,.\,11\,.\,17}{4^4\,.\,\underline{|4}}x^4-\ldots$$

2. (i.) $\dfrac{3\sqrt{3}+\sqrt{5}}{11}$; (ii.) $2a^{\frac{1}{3}}b^{\frac{1}{3}}$.

3. (i.) $x=0$, 1, or 3;

(ii.) $x=\sqrt[3]{\dfrac{b^2c^2}{a}}$, $y=\sqrt[3]{\dfrac{c^2a^2}{b}}$, $z=\sqrt[3]{\dfrac{a^2b^2}{c}}$. 4.

5. Fraction $=\dfrac{1}{1-x}+\dfrac{1}{1-\frac{x}{2}}-\dfrac{1}{1-\frac{x}{3}}$. 6. $-\dfrac{\sqrt{3}}{2}$, $\frac{1}{2}$, $-\sqrt{3}$. 7.

8. Let O and Q be circum-centre and orthocentre; OR, QS, perpendiculars on BC; and ON, QM, perpendiculars on AC. Then $CN=CR$, and $CM=CS$, etc.

CLXXVIII.

1. (i.) $3\frac{1}{2}$; (ii.) $\dfrac{1-r^n}{(1-r)^2}-\dfrac{nr^n}{1-r}$.

2. $8y^2 - 20a^3y - a^6 = 0$.

3. $a^5 - 10a^3x + 5ax^2 + (5a^4 - 10a^2x + x^2)\sqrt{-x}$;
(ii.) value $= a^5(176 \pm 80\sqrt{5})$. 4.

5. $\dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$. 6. 960·07 square feet.

7. $\bar{3}{\cdot}6020600$; 3·321928; 20 terms.

8. Euc. III. 20, 22.

CLXXIX.

1. 3^3. 2. (i.) $3 + 2\sqrt{2}$; (ii.) $\sqrt{b-c} + \sqrt{a-b}$.

3. (i.) $x = 3$, or -1; (ii.) $x = \dfrac{ac(a \pm \sqrt{a^2 - b^2})}{2b^2}$,
$y = \dfrac{c(a \pm \sqrt{a^2 - b^2})}{2b}$; (iii.) $x = 3$, or $-4\frac{1}{2}$.

4. $\dfrac{n(n-1)\ldots\ldots(n-r+1)}{\underline{|r}}x^{n-r}a^r$; 5922.

5. 5 and 6 miles per hour respectively.

6. $\dfrac{(n-2)\pi}{n}$.

7. (i.) $\theta = n\pi + \dfrac{\pi}{4}$; (ii.) $\theta = n\pi \pm \dfrac{\pi}{4}$, or $2n\pi \pm \dfrac{\pi}{2}$. 8.

CLXXX.

1. 3·314665; 3·548283.

2. $11111(\sqrt{10} + 1)$; $\dfrac{a - (2n-1)a^{n+1}}{1-a} - \dfrac{2a^2(a^{n-1} - 1)}{(a-1)^2}$.

3. 2, 6, 12, 20, 30……; $\dfrac{n(n+1)(n+2)}{3}$. 4.

5. 6. $a^0 = 1$. 7. $\dfrac{5773\pi}{21600}$; 20^g. 8.

CLXXXI.

1. £97 4s. 2. (i.) $262\frac{5}{6}$; (ii.) $46\frac{8}{9}$. 3.

4. (i.) 3, [or -3, or $\pm\sqrt{5}$]; (ii.) $a^{\frac{1}{3}}c^{-2}$.

5. (i.) $x=5$, $y=3$; (ii.) $x=57$, [or -23], $y=16$, [or -16].

6. $2\sin\alpha\sin^2\dfrac{\alpha+\beta}{2}$. 7.

8. If a be the distance between the points A and B, the locus is a circle, with centre distant $\dfrac{a}{3}$ from A, and radius $\dfrac{2a}{3}$.

Analytically, the equation is $3(x^2+y^2)+2ay-a^2=0$.

Geometrically. Find points satisfying the condition on AB, and AB produced. Then apply Euc. VI. 3, A., III. 31; *or*, it may be done by Book I. only, using Euc. I. 31, and Prog. Euc., App. vii., viii.

CLXXXII.

1. $7-\sqrt{6}$.

2. (i.) $x=2$, [or $5\frac{1}{5}$]; (ii.) $x=1$, or $\frac{5}{3}$; $y=2$, or $\frac{2}{3}$; (iii.) $x=0$, or -3. 3. 4. $a=3$.

5. $$a^{-\frac{10}{3}}+10a^{-\frac{13}{3}}b+\frac{10.13}{\lfloor 2}a^{-\frac{16}{3}}b^2+\frac{10.13.16}{\lfloor 3}a^{-\frac{19}{3}}b^3+\frac{10.13.16.19}{\lfloor 4}a^{-\frac{22}{3}}b^4.$$

6. No. 7. $-\cos A$; $-\cos A$; $\cot A$. 8.

CLXXXIII.

1. $\dfrac{\sqrt{2}}{2}\left\{(\sqrt{2}+1)^{\frac{3}{2}}+\sqrt{2}+1\right\}\left\{1-(\sqrt{2}-1)^{\frac{n}{2}}\right\}$.

2. $2^{-\frac{2}{3}}\left\{1+x+\frac{5}{4}x^2+\frac{5}{3}x^3+\ldots\ldots\right\}$.

3. 123456. 4. 3·279943.

5. (i.) $x=0,\ 3,\ \left[\text{or } \dfrac{3\pm\sqrt{-3}}{2}\right]$; $\theta = n\pi + \dfrac{3\pi}{4}$,

or, $\tan\theta = \dfrac{1\pm\sqrt{-3}}{2}$.

6. (i.) $x+3y=0$; (ii.) $x+y=1$; (iii.) $(\frac{3}{2},\ -\frac{1}{2})$.

7. 1; $-\frac{1}{2}$; 1.

8. 45 lbs. $\sin^{-1}\frac{45}{51}$, and $\cos^{-1}\frac{45}{51}$.

CLXXXIV.

1. 1·845818.

2. $x^{\frac{3}{2}}y^{-\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}} + 2y - x^{-\frac{1}{2}}y^{\frac{3}{2}}$;

$$\frac{a}{c} + x\left(\frac{ad}{c^2} - \frac{b}{c}\right) + x^2\left(\frac{ad^2}{c^3} - \frac{bd}{c^2}\right) + x^3\left(\frac{ad^3}{c^4} - \frac{bd^2}{c^3}\right) + \ldots\ldots$$
$$+ x^n\left(\frac{ad^n - bcd^{n-1}}{c^{n+1}}\right) + \ldots\ldots.$$

3. $\left(\dfrac{x+y}{x-y}\right)^{m-n}(-1)^m$. 4. $\dfrac{x+\sqrt{x^2-4}}{2}$.

5. 30 lbs. 6. $8\sqrt{3}$ inches, and $48\sqrt{3}$ sq. inches.

7. (i.) $x=\frac{5}{2}$, or -1; (ii.) $x=2,\ 3$; $y=-3,\ -2$;

(iii.) $\theta = n\pi \pm \dfrac{\pi}{4}$; (iv.) $\theta = 2n\pi + \dfrac{\pi}{4}$. 8. 144 feet.

CLXXXV.

1 Equate coefficients of x^{r+1} in the expansion of

$(1+x)^{n+2} \equiv (1+x)^2(1+x)^n$.

2. $1 + 2x + 5x^2 + \frac{40}{3}x^3 + \frac{110}{3}x^4$. 3. $x=2,\ 2$, or -3.

4. 200 ft.

5. (i.) $\theta = m\pi$, or $(n-1)\theta = m\pi - (-1)^m\dfrac{\pi}{6}$; (ii.) $x=\sqrt{3}$.

6. $\dfrac{\pi hr^2}{3}$; $10\frac{1}{2}$ and $31\frac{1}{2}$ ins. 7. 643685; 65 integers.

8. Euc. VI. 4. Drop perpendiculars on XYZ, from A, B, C.

CLXXXVI.

1. (i.) $x=2$, or $\frac{1}{2}$; (ii.) $x=4, 3$; $y=3, 4$;
(iii.) $x=5$, $-\frac{7}{3}$, [or $\frac{1}{6}(8\pm\sqrt{415})$].

2. 600 miles.
3. 11101001010, $10e6$.
4. $p^2(p^2+4)=q^2$.
5. $\dfrac{2\cos 2^n A+1}{2\cos A+1}=(2\cos 2^{n-1}A-1)(2\cos 2^{n-2}A-1)\ldots\ldots$
$\ldots\ldots(2\cos A-1)$.
6.
7. $90°$; 210 sq. ins.
8.

CLXXXVII.

1. $\cdot 0012644501$.
2. $\sqrt{3}$.
3. (i.) $x=\dfrac{-1\pm\sqrt{3}}{2}$, ∞, ∞; (ii.) $x=\dfrac{-3\pm\sqrt{17}}{2}$,
$\left[\text{or } \dfrac{-3\pm\sqrt{22}}{2}\right]$; (iii.) $x=4, 2$, $y=2, 4$.
4. 12π sq. ins.
5. Euc. II. 1.
6. $x^{\frac{5}{6}}-2x^{\frac{1}{2}}+x^{\frac{1}{3}}$.
7. $\dfrac{15°}{4\pi}$.
8. $90°$, $150°$, $120°$.

CLXXXVIII.

1.
2. $1+x+\dfrac{2x^2}{3}+\dfrac{10x^3}{27}+\ldots\ldots+\dfrac{120x^{14}}{3^{14}}+\ldots\ldots$
3.
4. (i.) $x=16$, or $\frac{1}{25}$; $y=25$, or $\frac{1}{16}$;
(ii.) $x=-a\pm a\sqrt{1+a}$; (iii.) $x=1$, or $\dfrac{-1\pm\sqrt{-3}}{2}$.
5 $78^g.15^`$
6. $2a=n\pi+(-1)^n\dfrac{\pi}{6}$.
7
8. $11x-5y=0$.

CLXXXIX.

1. $\dfrac{3\sqrt{3}-2\sqrt{2}}{6}$, or $\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{3}$.
2.

3. 4·8989795.

4. (i.) $x=1,\ 3,\ 27,\ 29$; (ii.) $x=9$, or 4; $y=4$, or 9; (iii.) $x=\frac{1}{5}$, $y=-\frac{1}{2}$, $z=-\frac{1}{3}$.

5. $\frac{a+b}{2}$; $\sqrt{ab}$. 6. 9·2 ft. per second.

7. 11. 8. $\frac{12}{\pi}$ ins.

CXC.

1. 81.

2. (i.) $\because (1-r^2)(1-r^4)>0$; (ii.) $\because (x-9)(\overline{x+4}^2+20)>0$.

3. (i.) $x=3$, or 2; $y=-2$, or -3; (ii.) $x=2$, [or $\frac{26}{5}$.]

4. 100 bushels. 5. 6. 7.

8. Find the 4th point of the Harmonic Range $ABCD$. Then the circle on BD as diameter is the required locus.

CXCI.

1. $n=4$. 2.

3. (i.) $x=a$, $y=b$; (ii.) $x=1$, $y=2$, $z=3$;

(iii.) $$\frac{x}{\begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} c_1 & d_1 & a_1 \\ c_2 & d_2 & a_2 \\ c_3 & d_3 & a_3 \end{vmatrix}} = \frac{z}{\begin{vmatrix} d_1 & a_1 & b_1 \\ d_2 & a_2 & b_2 \\ d_3 & a_3 & b_3 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}.$$

4. $br=a(n-r+1)$. 5.

6. $\frac{\theta}{2}=n\pi+\frac{\pi}{6}$. 7. $\theta=\frac{a}{2}$.

8. III. 37; II. 14; I. 17.

CXCII.

1. $\frac{1}{2}$.

2. $6+\frac{1}{1+}\ \frac{1}{5+}\ \frac{1}{1+}\ \frac{1}{12+}\ \frac{1}{1+}\ \frac{1}{5+\ldots}$; $\frac{6}{1}, \frac{7}{1}, \frac{41}{6}, \frac{48}{7}$.

3. (i.) $x=0, 2$, [or $1 \pm \sqrt{-7}$]; (ii.) $x= \pm \frac{\sqrt{a(a-1)}}{2}$.

4. $12x^2+4y^3$.

5. $a^{-1}-1$.

6. $\theta=(2n-1)\frac{\pi}{4}$, or $\frac{1}{7}\left(n\pi+(-1)^n\frac{\pi}{6}\right)$.

7. 5 lbs., and $5\sqrt{3}$ lbs.

8. Progressive Euclid, App. xiii.

CXCIII.

1. $n=\frac{5}{3}$.

2. $\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\ldots$

3. Expression $=\frac{\lfloor 2n}{\lfloor n \lfloor n}$.

4.

5. $\sin^{-1}\frac{\sqrt{6}+\sqrt{2}}{4}$, and $\sin^{-1}\frac{\sqrt{6}-\sqrt{2}}{4}$.

6. $A=40^\circ\ 53'\ 36''{\cdot}2$, $B\ 19^\circ\ 6'\ 23''{\cdot}8$.

7. 933·575 cub. in.; 1381·675 cub. in.

8. Euc. VI. 4.

CXCIV.

1. (i.) $x=\frac{(a-b)^2}{2a-b}$; (ii.) a, b, c are the roots of the equation $k^3-k^2z+ky-x=0$; hence $x=abc$, $y=ab+bc+ca$, $z=a+b+c$.

2. 252; $\frac{126}{252}$, $=\frac{1}{2}$.

3. $\frac{35 \cdot 3^4x^4y^4}{8}$.

4. 58 or 26 half-crowns, 5 or 10 Napoleons.

5. (i.) $x=(2n-1)\frac{\pi}{8}$, or $\frac{1}{3}\left(2n\pi\pm\frac{\pi}{3}\right)$; (ii.) $2\theta=n\pi+\frac{\pi}{4}$.

6. $\sin^2\frac{c}{2}$, and $-\cos^2\frac{c}{2}$, where $c=\alpha+\beta$.

7.

8.

CXCV.

1. (i.) $x = \pm\sqrt{\frac{1 \pm \sqrt{5}}{2}}$; (ii.) $x = 14$, or 10; $y = -10$, or -14;
(iii.) $x = \pm 2\sqrt{3}$, $\pm 3\sqrt{-1}$, ± 3, $\pm 2\sqrt{-3}$;
$y = \mp\frac{2}{\sqrt{3}}$, $\mp\sqrt{-1}$, ± 1, $\mp\frac{2}{\sqrt{-3}}$. $(x^4 = 3^4 \, . \, y^4.)$

2. 5; 8.　3. $\frac{3 + 3m + 12mn}{3 + 3m + 2n}$.　4. $\frac{2}{1}$, $\frac{3}{1}$, $\frac{5}{2}$, $\frac{8}{3}$.

5. (i.) $x = \frac{4}{3}$, or $-\frac{3}{8}$; (ii.) $\theta = n\pi + \frac{\pi}{4}$, or $n\pi$.　6.

7.　8. $x - y + a = 0$; It touches it at $(0, a)$.

CXCVI.

1. £114341·05.　2. 50 shillings, 20 half-crowns.
3.　4. $r = 4$.　5.　6.
7. $\theta = n\pi$, or $n\pi \pm \frac{\pi}{4}$.　8.

CXCVII.

1. 1260.　2.
3. $r \, . \, 5^{r-1} + (-1)^r 7^r$.
$\left(\text{Fractions are } \frac{1}{5(1-5x)^2} - \frac{1}{5(1-5x)} + \frac{1}{1+7x}\right)$.
4. (i.) $\cos\theta = \cos\alpha$, or $-2\cos\alpha$; (ii.) $x = \frac{5}{9}$.
5. As 39 : 33 : 25.
6. $A = 105° \, . \, 27' \, . \, 35''$, $B = 15° \, . \, 5' \, . \, 20''$, $c = 2·4484$.
7. 45°.　8. 1612·5 sq. ft.

CXCVIII.

1. $x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 1$; $ab - ac - bd + cd$.
2. (i.) $x = \pm 7$; (ii.) $x = 5$, or $-5\sqrt[3]{6}$; $y = 7$, or $7\sqrt[3]{6}$;
(iii.) $x = 9$, $y = 3$.

3. $\dfrac{1 \,.\, 4 \,.\, 7 \ldots\ldots(3r-2)}{\lfloor r}x^r$; The fourth, $\frac{323}{72}$.

4. £279620 5s. 4d.

5. 10·92064.

6. (i.) $\theta = \dfrac{(2n-1)\pi}{5}$, or $\dfrac{2n\pi}{3}$; (ii.) $x = \frac{1}{47}$.

7. 106 ft.

8. (i.) $r = a$; (ii.) $r\cos(\theta - \alpha) = a$.
(i.) A circle; (ii.) A straight line.

CXCIX.

1. (i.) $x = \dfrac{ac+bd}{a^2+b^2}$, $y = \dfrac{bc-ad}{a^2+b^2}$; (ii.) $x = 7$, or -2;
(iii.) $x = 3$, or -13; $y = 5$, or -19.

2. 20 per cent.

3. 49 and 121.

4. $4 + \dfrac{1}{8+}\,\dfrac{1}{8+}\ldots\ldots$; $\dfrac{4}{1}, \dfrac{33}{8}, \dfrac{268}{65}, \dfrac{2177}{528}$.

5. $\theta = n\pi + \dfrac{\pi}{4}$.

6. $x^2 + y^2 = a^2$.

7. $440\sqrt{6}$ yards.

8.

CC.

1. $y = a\left(1 - \dfrac{1}{2}\,\dfrac{x^2}{a^2} - \dfrac{1\,.\,1}{2^2\,\lfloor 2}\,\dfrac{x^4}{a^4} - \dfrac{1\,.\,1\,.\,3}{2^3\,\lfloor 3}\,\dfrac{x^6}{a^6} - \dfrac{1\,.\,1\,.\,3\,.\,5}{2^4\,\lfloor 4}\,\dfrac{x^8}{a^8} - \ldots\right)$.

2. (i.) $x = 5$, -3, or $1 \pm \sqrt{-10}$;
(ii.) $x = 5$, -3, or $1 \pm \sqrt{-22}$; $y = 3$, -5, or $-1 \pm \sqrt{-22}$;
(iii.) $x = 0, 15, 30$; $y = 15, 8, 1$.

3. £16 13s. 4d.

4. $x^2 + \dfrac{yz}{2} - z^2$.

5. (i.) $\theta = 2n\pi$; (ii.) $x = 2n\pi$, $y = 2m\pi$.

6.

7. 54° 10′ 55″·8, or 125° 49′ 4″·2.

8.

CCI.

1. 50 lbs. at 2s., 30 lbs. at 3s.

2. $3^{10}x^{10}+10\,.\,3^9\,.\,2\,.\,x^9a+\frac{10\,.\,9}{1\,.\,2}\,.\,3^8\,.\,2^2\,.\,x^8a^2$

$+\frac{10\,.\,9\,.\,8}{1\,.\,2\,.\,3}\,.\,3^7\,.\,2^3\,.\,x^7a^3$; $2^{-5}x^{-5}+5\,.\,2^{-6}\,.\,5x^{-6}a$

$+\frac{5\,.\,6}{1\,.\,2}\,.\,2^{-7}\,.\,5^2\,.\,x^{-7}a^2+\frac{5\,.\,6\,.\,7}{1\,.\,2\,.\,3}2^{-8}\,.\,5^3\,.\,x^{-8}a^3$.

3. (i.) $2+\sqrt{6}$; (ii.) $2+\sqrt{6}$.

4.

5. The scale of 11.

6. (i.) $x=\frac{1}{a}$, or $\frac{4a\pm(a^2-1)\sqrt{-3}}{a^2+3}$;

(ii.) $x, y, z=\pm 1$, or 6

(x, y, z are the roots of $v^3-6v^2-v+6=0$);

(iii.) 3 cows, 6 sheep.

7. 164·0467.

8.

CCII.

1. (i.) $\frac{3\,.\,8\,.\,13\ldots\ldots(5r-7)}{6^{r-1}\lfloor r-1}$; (ii.) The first.

2. 48.

3. $\lfloor 4\times 15\times 1555=559800$.

4. $3^{m+1}+3\,.\,2^{m+1}-9$.

5. (i.) $x=\frac{n\pi}{3}$, or $\frac{n\pi}{5}$; (ii.) $x=\frac{1}{3}$, or -2; $y=\frac{1}{2}$, or -3.

6.

7.

8.

CCIII.

1. (i.) 4th term = 5th term $=\frac{4^4\,.\,7\,.\,5}{3\,.\,13^3}$;

(ii.) Series $=(1-\frac{1}{3})^{-\frac{1}{2}}$.

2. $x=7$, 9, or -3.

3. $-1+x+5x^2+\ldots\ldots+(2^{n+1}-3)x^n\ldots\ldots$

$\left(\text{Fractions are } \frac{2}{1-2x}-\frac{3}{1-x}\right)$.

4.

5.

6. £12151·87.

7. (i.) π sq. ins.; (ii.) $(3\sqrt{3}-2\pi)$ sq. ins.

8. Euc. I. 10, 11, 9.

CCIV.

1. $x^{\frac{1}{3}} - \dfrac{x^{\frac{1}{6}} y^{\frac{1}{6}}}{\sqrt{2}} + y^{\frac{1}{3}}$.

2. 3·337728 acres; 3·343262 acres; 3·340495 acres.

3. (i.) $x=4$, [or $-3\frac{1}{5}$]; (ii.) $x=\pm\frac{1}{3}$, $y=1$, or 3;
(iii.) $x=\dfrac{\pm\sqrt{2}\pm\sqrt{-2}}{2}$.

4. (i.) $\left(\dfrac{2}{3}\right)^{r+1} - \dfrac{1}{2^{r+1}}$; (ii.) 5·0396842. 5. 1·486344.

6. $B=71^\circ\ 44'\ 29''\cdot 5$; $C=48^\circ\ 15'\ 30''\cdot 5$.

7. -2. 8.

CCV.

1. £1392 19s. $7\frac{1}{2}$d. nearly.

2. $\left\{\left(\dfrac{100+r}{100}\right)^n - 1\right\}\dfrac{100p}{r} \div \left(\dfrac{100+r}{100}\right)^n$. 3. 4.

5. (i.) $\dfrac{3\,.\,5\,.\,7\ldots\ldots(2r+1)}{2^r\,\underline{|r}}x^r$; (ii.) Series $=(1-\frac{1}{2})^{-\frac{3}{2}}$.

6. 576. 7.

8. Area $=r^2\left(\sqrt{3}-\dfrac{\pi}{2}\right)=r^2\times\cdot 16125\ldots\ldots$

CCVI.

1. 3·30277. 2. $\dfrac{7y}{6(y-x)}$ men.

3. (i.) $2\sqrt{-1}$, or $2\omega\sqrt{-1}$, or $2\omega^2\sqrt{-1}$, where ω is one of the imaginary cube roots of unity; (ii.) $\sqrt[4]{n}+\sqrt[4]{-n}$.

4. (i.) $x=4$, $y=5$, $z=6$; (ii.) $x=1$, [or $\frac{1}{81}$];
(iii.) $x=\pm 1$, $y=\pm 27$.

5. $-\frac{57}{1625}$. 6.

7. 8. 4 lbs.

CCVII.

1. $x=4$, or 6; $y=15$, or 10; (ii.) $x=2, \frac{1}{2}, \left[\text{or } \frac{5 \pm \sqrt{201}}{4}\right]$.

2. 18, 23, and 28 years.

3. 105. $(n=15.)$

4. (i.) 49; (ii.) $\frac{1}{18}(3^n - 1)$; (iii.) $\frac{n(2n^2+3n+13)}{6}$.

(iv.) $\frac{4a-3a^2}{(1-a)^2}$.

5.

6.

7. $p^{\frac{2}{3}}q^{\frac{2}{3}}\left(p^{\frac{2}{3}}+q^{\frac{2}{3}}\right)=1$.

8. (i.) $\frac{27}{2}$; (ii.) $x^2+y^2-17x-19y+50=0$.

CCVIII.

1. ·3107278.

2. In the scale of 6; 7775 and 1296.

3. $7+\frac{1}{14+}\;\frac{1}{14+}\ldots\ldots$

4.

5. (i.) $x=\frac{a^2-b^2}{ap-bq}$, $y=\frac{a^2-b^2}{aq-bp}$; (ii.) $\theta+\frac{\pi}{3}=2n\pi\pm\frac{3\pi}{4}$;

(iii.) $\theta=18°$ or $54°$.

6. $16\cos^5\theta-20\cos^3\theta+5\cos\theta$.

7. (i.) A parabola; (ii.) an ellipse.

8. 16 inches from the heavier weight.

CCIX.

1. n^3.

2. $a(1+b^3)$.

3. $2a^2+2ab(\cos\alpha-\sin\alpha)+b^2=c^2$.

4. (i.) $x=\pm a, \pm\frac{1}{a}, \pm ai, \pm\frac{1}{ai}$, where $i=\sqrt{-1}$;

(ii.) $x=a$, $y=b$, $z=c$.

5. $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3+\frac{35}{128}x^4$.

6. 3008; ·734375.

7.

8. A straight line parallel to the given lines.

CCX.

1.

2. $\left\{\frac{n(n+1)}{2}\right\}^2$; $\overline{n+1}^3 - 1$.

3.

4. (i.) 1; (ii.) $\frac{1.3.5\ldots\ldots\overline{2n-1}}{2^n \lfloor n}$;

(iii.) $\frac{1.1.3\ldots\ldots\overline{2n-3}}{(-2)^n \lfloor n} a^{\frac{1-2n}{2}}$; (iv.) $(3-2)^{\frac{3}{2}}$.

5.

6.

7.

8. A circle of radius $\frac{a}{2}$.

CCXI.

1. $\frac{n(n+1)}{2}$.

2.

3. $\frac{100\,A}{r}\left\{\left(1+\frac{r}{100}\right)^n - 1\right\} \div \left(1+\frac{r}{100}\right)^{2n}$.

4. $\frac{1.(n+1)(2n+1)\ldots\ldots\{(r-2)n+1\}}{n^{r-1}\lfloor r-1} a^{1-r-\frac{1}{n}} . x^{r-1}$;

(ii.) Equate coefficients of x^r, in the expansions of $(1+x)^{-1}(x+1)^n$ and $(1+x)^{n-1}$.

5. $\frac{m-m_1}{1+mm_1}$.

6.

7. 17·1005087 or 3·6841009.

8.

CCXII.

1. (i.) $\frac{\lfloor 11}{8}$; (ii.) $\frac{\lfloor 8}{\lfloor 3 \lfloor 5} + 21 = 77$.

2. (i.) $x = 5$, or -21; $y = \pm 4$, $\pm 20\sqrt{-1}$;

(ii.) $x = \pm 3$; $y = \pm 4$, ± 5; $z = \mp 5$, ∓ 4;

or $x = \pm 9$, $y = \frac{1}{2}(\pm 3 \pm \sqrt{-71})$, $z = \frac{1}{2}(\mp 3 \pm \sqrt{-71})$.

3. $(2+3)^5$.

4. At the end of the 8th and 15th days after A starts. After 12 days A begins to gain on B.

5. 6. $61° 17' 22''$. 7.

8. $1, \cos\frac{2\pi}{5} \pm i\sin\frac{2\pi}{5},\ \cos\frac{4\pi}{5} \pm i\sin\frac{4\pi}{5}$.

CCXIII.

1. $n^2 - 2n + 2,\ n^2 - 2n + 3, \ldots\ldots n^2;\quad s = n^3 + (n-1)^3$.

2. $\frac{a-b}{ab+1}$. 3.

4. $x = 1{\cdot}537243;\ 5\frac{1}{3}$. 5.

6. (i.) $x = \frac{n\pi}{3}$, or $\frac{n\pi}{2}$; (ii.) $x = \tan^{-1}\frac{b}{a} + \cos^{-1}\frac{1}{2}\sqrt{a^2+b^2}$,

$y = \tan^{-1}\frac{b}{a} - \cos^{-1}\frac{1}{2}\sqrt{a^2+b^2}$. 7. 8.

CCXIV.

1. $\sqrt{2}$, [or $\pm\sqrt{\frac{2}{3}}$, or $-\sqrt{2}$].

2. $x^2 - (p^2 - 2q)x + q^2 = 0$. 3

4. $\frac{a(r^n - 1)}{r-1};\ \frac{a}{r-1}\left\{r\frac{r^{2n}-1}{r^2-1} - n\right\}$.

5. (i.) $\theta = n\pi$, or $2n\pi \pm \frac{\pi}{3}$; (ii.) $\sin\theta = \sqrt{\frac{3 \pm \sqrt{5}}{8}}$.

6. 7. 8. $\frac{a^2}{6}$.

CCXV.

1. $\frac{ab(n+1)}{a+bn},\ \frac{ab(n+1)}{2a+\overline{n-1}b} \cdots\cdots\cdots\cdots \frac{ab(n+1)}{2b+(n-1)a},\ \frac{ab(n+1)}{b+an}$.

2. 3. $n = 5$.

4. Assume $x =$ one of the imaginary cube roots of unity. We then get $k + l\omega + m\omega^2 = 0$, whence equating coefficients of real and imaginary parts to 0, we get $k = l = m$. Then assume $x = 1$.

5. $z^2(a+b-y-e)=(x-e)(a-y)(y-b)$. 6.

7. $4\cos^3\alpha-3\cos\alpha$; $x=-\sqrt{2}\cos\alpha\pm\sqrt{6}\sin\alpha$

$$=2\sqrt{2}\cos\left(\frac{2\pi}{3}\pm\alpha\right).$$

8. $x^2+y^2=4(x+a)$. A circle.

CCXVI.

1. 2. $n(n-1)\ldots\ldots(n-r+1)$.

3. (i.) All numbers are of one of the forms $3m$, $3m\pm1$;
(ii.) All square numbers are of one of the forms $4n$, $4n+1$.

4. If first and third are $\frac{a}{b}$, $\frac{c}{d}$, second is $\frac{c-a}{d-b}$. 5.

6. $A=78°\ 17'\ 40''$; $B=49°\ 36'\ 20''$.

7. $x=3$, $y=4$. 8. Euc. III. 32, 22 (converse).

CCXVII.

1. $\frac{11}{1}$, $\frac{56}{5}$, $\frac{67}{6}$, $\frac{123}{11}$. 2. $c^2x^2-(b^2-2ac)x+a^2=0$.

3. a, and 0; or $a(2\pm\sqrt{2})$. 4. 1, $\frac{-1\pm\sqrt{-3}}{2}$.

5. 6. 1·9912261; $\bar{1}$·7279660.

7. 4·97971 or 3·00664 miles per hour. [$\angle B=71°\ 50'\ 55''\cdot3$, or $108°\ 9'\ 6''\cdot7$.]

8. $31x-27y=0$.

CCXVIII.

1. 64. 2. $x=5$, or $-\frac{5}{4}$.

3. $\frac{a(1-r^n)}{1-r}$; $\frac{1-x^n}{(1-x)^2}+\frac{nx^n}{1-x}$; $(1-x)^{-2}$; x must be a proper fraction.

4. $P(1+r)^n$; $PR^n-K\frac{R^n-1}{R-1}$, where $R=1+r$;

$$n=\frac{\log K-\log(P+K-PR)}{\log R}.$$

5. 27 feet from rest, 48 feet, 72 feet. 6. 7.

8. $x^2+y^2=k^2(\overline{x-a}^2+y^2)$. A circle, which cuts orthogonally the circle described on the line joining the given points, as diameter.

CCXIX.

1. "Casting out the nines."

2. $1\frac{2}{3}$ and $3\frac{1}{3}$ miles per hour. 3. (i.) $10\frac{2}{3}$; (ii.) 1.

4. $y=e^x-1=x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\ldots\ldots$ 5.

6. 7.

8. $Bx-Ay=0$; $r(A\cos\theta+B\sin\theta)+C=0$; $A\tan\theta=B$.

CCXX.

1. 2. 2, 4, 6, 8.

3. $x=\frac{e^{2y}-1}{e^{2y}+1}$. 4. 2925.

5. £8315 10s. nearly.

6. It touches the axis of x at the origin; $r=2a\sin\theta$.

7. 8. Euc. III. 21; VI. 3.

CCXXI.

1. (i.) $b^2=4ac$; (ii.) $b=0$. 2. $\lfloor 3=\frac{1}{2}\cdot\frac{\lfloor 4}{\lfloor 2}$.

3. 4. $\frac{3}{1}, \frac{22}{7}, \frac{355}{113}, \frac{3927}{1250}$. 5.

6. (i.) $\theta=\frac{n\pi}{4}\pm\frac{\pi}{16}$; (ii.) $\theta=\frac{2n\pi}{5}$, or $\frac{4n\pi}{7}\pm\frac{2\pi}{21}$.

7. 8. Euc. I. 37.

CCXXII.

1. 131·243.

2. (i.) $x=-4$, $y=-2$, $z=11$; (ii.) $x=\pm 8$, $y=\pm 6$; (iii.) $x=61, 138, \ldots\ldots$, $y=324, 733, \ldots\ldots$

3. $\dfrac{1}{(1-x)^2}+\dfrac{1}{2(1-x)}-\dfrac{1-x}{2(1+x^2)}$.

4. 6007.

5.

6. $\theta=(2m-1)\pi$, $\dfrac{4m\pi}{n-1}$, or $\dfrac{2(2m-1)\pi}{n+1}$.

7. $\frac{3}{5}\sqrt{5}$.

8. $x^2+y^2=\frac{1}{2}m^2-c^2$.

CCXXIII.

1. (i). $-\sqrt{-1}$; (ii.) $\pm(2+\sqrt{3})$; (iii.) $\pm(7-2\sqrt{3})$.

2. £11123·4.

3. $1+\dfrac{1}{2}x+\dfrac{3}{8}x^2+\dfrac{5}{16}x^3+\dfrac{35}{128}x^4+\ldots\ldots+\dfrac{1.3.5\ldots(2r-3)}{2^{r-1}\lfloor r-1}x^{r-1}$.

4. 2 ways; 8 half-guineas, and 3 half-crowns.

5. $a^2b^2+1=b^2+2ab\cos\theta$.

6.

7. Take the fixed lines as axes, and (h, k) the point. Then the locus is $2xy=hy+kx$.

8. $x=\dfrac{a+\sqrt{h^2+k^2}-\sqrt{h^2+k^2-2ah+a^2}}{2}$

$$y=\frac{\{a+\sqrt{h^2+k^2}-\sqrt{h^2+k^2-2ah+a^2}\}\{\sqrt{h^2+k^2}-h\}}{2k}$$

where $(a, 0)$, (h, k) are the points C and A.

(ii.) $y^2=cx(a-x)$, where $c=\tan\dfrac{B}{2}\tan\dfrac{C}{2}$.

CCXXIV.

1. $cx^2+x(cn+b-a)-an=0$.

2. (i.) $x=\pm1$; (ii.) $x=\pm1$, or $\dfrac{3\pm\sqrt{13}}{2}$; (iii.) $x=5$, or 1, $y=2$, or 5.

3. (i.) 225; (ii.) $\dfrac{512}{45}\left(1-\dfrac{5^n}{8^n}\right)$; $11\frac{17}{45}$.

4. $a^{11} - 11a^{10}x + 55a^9x^2 + 165a^8x^3 + \ldots\ldots;$

$$a^{\frac{3}{2}}\left\{1 + \frac{3}{2}\cdot\frac{x}{a} + \frac{3\,.\,1}{2^2\lfloor\underline{2}}\,\frac{x^2}{a^2} - \frac{3\,.\,1\,.\,1}{2^3\lfloor\underline{3}}\,\frac{x^3}{a^3} + \frac{3\,.\,1\,.\,1\,.\,3}{2^4\lfloor\underline{4}}\,\frac{x^4}{a^4} + \ldots\right\}.$$

5. 6. 7. 8.

CCXXV.

1. $\dfrac{n(n+1)}{\lfloor\underline{2}}x^{n-1}$; $\dfrac{m(m-1)\ldots\ldots(m-n+2)}{\lfloor\underline{n-1}}(-x)^{n-1}$;

Multiply by $(1-x)^{-1} = 1 + x + x^2 + \ldots\ldots$, and equate coefficients of x^r.

2. (i.) $n(n-1)\ldots\ldots(n-r+1)$; (ii.) $\lfloor\underline{5} \times \lfloor\underline{6} \div 2$, or $\lfloor\underline{5} \times \lfloor\underline{6}$, according as the direction of rotation is considered or not.

3. $112\cdot010\dot{1}$.

4. $p \sim q = pq$.

5. Expression $= \sin^2\beta$.

6. $\cdot 00002424067$.

7.

8.

CCXXVI.

1. (i.) $x = \pm a,\ \pm\dfrac{1}{a},\ \pm ai,\ \pm\dfrac{1}{ai}$, where $i = \sqrt{-1}$;

(ii.) $x = 4,\ y = 5,\ z = 2$, or $x = y = z = 0$;

(iii.) $x = a,\ y = b,\ z = c$.

2. $\dfrac{\lfloor\underline{n}}{\lfloor\underline{3}}$; $\dfrac{6\,.\,5}{\lfloor\underline{2}} \times \dfrac{10\,.\,9}{\lfloor\underline{2}}, = 675.$

3. $\dfrac{\lfloor\underline{2r}}{\{\lfloor\underline{r}\}^2}$; $1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{7}{16}x^3 - \frac{37}{128}x^4 - \ldots\ldots$

4. $x = 4,\ y = \frac{1}{2},\ n = 8$.

5. 22600; $\cdot 007te55022\ldots\ldots$; $21 \times \lfloor\underline{5} \times \dfrac{7^6 - 1}{6}$.

6. $x = \pm 1$.

7. 90°.

8. Euc. III. 22 (converse), 28, 27.

CCXXVII.

1. $n(n-1)\ldots\ldots(n-r+1)$; $n=15$.

2. 10007002·10035.
3. $\dfrac{b^4-4ab^2c+2a^2c^2}{a^4}$.

4. (i.) $x=\dfrac{1}{8}, -\dfrac{1}{72}, \left[\text{or } \dfrac{1\pm\sqrt{10}}{18}\right]$; (ii.) $x^2=\pm\dfrac{25}{4}$, $y^2=\pm\dfrac{9}{4}$; or, $x^2=\pm\sqrt{8}$, $y^2=\mp\dfrac{17}{\sqrt{8}}$; (iii.) $x=\pm 13$.

5. $\dfrac{\sec A \sec B}{1-\sqrt{(\sec^2 A-1)(\sec^2 B-1)}}$.
6.

7.
8.

CCXXVIII.

1. Every half-month; 2400 times.
2.

3. $\dfrac{1}{a^p}$; 1; x^n+1+x^{-n}.

4. (i.) $x=\dfrac{4}{3}$; (ii.) $x=\pm\left\{\dfrac{m+2a^2\pm 2\sqrt{a^4+a^2m-m^2}}{5}\right\}^{\frac{1}{2}}$, $y=\pm 2\left\{\dfrac{3a^2-m\mp 2\sqrt{a^4+a^2m-m^2}}{5}\right\}^{\frac{1}{2}}$; m^2 must lie between 0 and $\dfrac{3+\sqrt{5}}{2}a^4$.
5.

6.
7. $\dfrac{l}{\sqrt{2+2\sqrt{5}}}$.

8. (i.) The axes; (ii.) The bisectors of the angles between the axes.

CCXXIX.

1. $6:9::10:15$.

2. $-\dfrac{13\,.\,10\,.\,7\,.\,4\,.\,1\,.\,2}{\lfloor 6}x^6, = -\dfrac{91}{9}x^6$.

3. (i.) $x=5$, or 21; (ii.) $x, y, z, =1, 2, 3$, in any order.

4. $\dfrac{\lfloor n}{\lfloor p \lfloor q \lfloor r}$; $3+8+19+38+60+60=188$. **5.**

6. $24{\cdot}668801$; $x=5$. **7.** The point $(8, 9)$.

8. $\{(x-\alpha)^2+(y-\beta)^2-r^2\}\{(x'-\alpha)^2+(y'-\beta)^2-r^2\}$
$= \{(x-\alpha)(x'-\alpha)+(y-\beta)(y'-\beta)-r^2\}^2$.

CCXXX.

1. $x^2 \mp \sqrt{a^2-4b}\,.\,x-b=0$. The biquadratic is
$x^4-(a^2-2b)x^2+b^2=0$.

2. $1+x+\frac{3}{2}x^2-\frac{3}{2}x^3+\frac{3}{8}x^4\ldots\ldots$.

3. (i.) a; (ii.) b; (iii.) c.

4. $3\frac{9}{17}$ minutes. **5.** **6.**

7. $\sqrt{s(s-a)(s-b)(s-c)}$.

8. $x-y+1=0$; (ii.) $x^2+y^2-18y+2\lambda(x-y)=0$.

CCXXXI.

1. (i.) $1+x+\dfrac{2}{3}x^2+\dfrac{10}{27}x^3+\ldots\ldots+\dfrac{15\,.\,16}{3^{14}\,.\,2}x^{14}+\ldots\ldots$;

(ii.) Series $=(1-1)^n-n(1-1)^{n-1}\equiv 0$.

2. If the solution be $x=\alpha$, $y=\beta$, the other solutions are given by $x=\alpha+bt$, $y=\beta-at$; (ii.) In 9 ways.

3. If $A=30°$, there will be two possible triangles, provided c be $> a$ and $< 2a$.

4. (i.) $x=\dfrac{4ab\pm(a+b)\sqrt{6ab-a^2-b^2}}{(a-b)^2}$;

(ii.) $x=0$, 1, or $\frac{15}{22}$; $y=0$, 2, or $\frac{9}{22}$;

(iii.) $$\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\} = \left\{\begin{matrix} \dfrac{-1\pm\sqrt{6}}{2} \\ \dfrac{-1\mp\sqrt{6}}{2} \\ 2 \end{matrix}\right\} \text{ or } \left\{\begin{matrix} 0 \\ \dfrac{1\pm i}{2} \\ \dfrac{1\mp i}{2} \end{matrix}\right\} \text{ or } \left\{\begin{matrix} \dfrac{1\pm i}{2} \\ 0 \\ \dfrac{1\mp i}{2} \end{matrix}\right\}.$$

5. $\pi = 3{\cdot}1416(6)$. 6.

7. $A = 60^\circ,\ c = \dfrac{200}{\sqrt{3}},\ b = \dfrac{100}{\sqrt{3}}$.

8. $\{x - (y - b)\cot\alpha\}^2\tan^2\beta = (y - b + x\cot\alpha)^2$.

CCXXXII.

1. $\cdot 00170706$.

2. $(ax)^{-\frac{1}{5}}\left\{1 + \dfrac{1}{5}\dfrac{x}{a} + \dfrac{1.6}{5^2\,\underline{|2}}\dfrac{x^2}{a^2} + \dfrac{1.6.11}{5^3\,\underline{|3}}\cdot\dfrac{x^3}{a^3} + \dfrac{1.6.11.16}{5^4\,\underline{|4}}\dfrac{x^4}{a^4} + \dots\right\}$

3. (i.) $\dfrac{60.59\ldots\ldots55}{\underline{|6}}$; (ii.) $\dfrac{59\ldots\ldots54}{\underline{|6}}$.

4. $\dfrac{1}{de - cf}\left\{\dfrac{ad - bc}{c - dx} + \dfrac{be - af}{e - fx}\right\}$. 5.

6. $\cot^{-1}\left(\dfrac{b\cot\alpha - a\cot\beta}{a + b}\right)$.

7. $x(1 + m) = ma$, where $a = BC$.

8. $x - y - 1 = 0$, and $x - 4y + 2 = 0$.

CCXXXIII.

1. $164{\cdot}0467$.

2. (i.) $\dfrac{n(n-1)\ldots\ldots(n-r+1)}{\underline{|r}}$; (ii.) 2^n;

(iii.) $2^{n-1}(n+2) - n - 1$.

3. (i.) $\sqrt{10}$, [or $\sqrt{2}$]; (ii.) $\dfrac{2\sqrt{2} + \sqrt{6} - 3 - \sqrt{3}}{2}$.

4. (i.) $\dfrac{n\{2 + (n-3)\sqrt{x}\}}{2(1-x)}$; (ii.) $\dfrac{a(x^4 - a^8)}{x^4(ax^{\frac{1}{2}} + x)}$; (iii.) when $a^2 < x$.

5. (i.) $\theta = (2n-1)\dfrac{\pi}{2},\ (2n-1)\pi$, or $\dfrac{2n\pi}{5}$; (ii.) $\theta = n\pi \pm \dfrac{\pi}{4}$.

6. $(a^2 + b^2 - 1)\sin^2\theta + c^2 = 0$.

7. $x\sqrt{3} - y = 4a$.

8. $2r = c$, where c is the fixed length.

CCXXXIV.

1. $\dfrac{2}{x+1} - \dfrac{8}{(x+2)^2}$.

2. (i.) $\dfrac{n(n+1)(n+2)}{3}$; (ii.) $\dfrac{1-x^n}{(1-x)^2} - \dfrac{nx^n}{1-x}$; (iii.) 24.

3. $1 + \dfrac{1}{1+}\ \dfrac{1}{1+}\ \dfrac{1}{2+}\ \dfrac{1}{2}$; $1,\ 2,\ \dfrac{3}{2},\ \dfrac{8}{5},\ \dfrac{19}{12}$.

4. (i.) $x = 6,\ -3,\ \left[\text{or } \dfrac{3 \pm 3\sqrt{5}}{2}\right]$; (ii.) $x^2 - x + 1 = \pm\sqrt{2}$,

   whence $x = \dfrac{1 \pm \sqrt{-3 \pm 4\sqrt{2}}}{2}$; (iii.) $x = 5$, or $\dfrac{2}{3}$.

5. 6.

7. Take the 2 fixed lines as $y = 0$, and $y = mx$, and third fixed line as $y = nx$; then the equation is
$$x(1 + mn) + y(m - n) = 0;$$
or, taking the 2 fixed lines as axes,
$$x(n + \cos\omega) + y(1 + n\cos\omega) = 0.$$

8. $(k^2 - l^2)(x^2 + y^2) + l^2(2ay - a^2) = 0$; where a is the base, and $k : l$ the ratio of the sides.

CCXXXV.

1. (i.) General term is $\left(-\dfrac{1}{2} + \dfrac{1}{2^{n-1}} - \dfrac{1}{2 \cdot 3^{n-1}}\right)x^n$.

   (ii.) $1 - x + x^3 - x^4 + x^6 - x^7 + \ldots\ldots$

2. $x = 18,\ 35\ldots\ldots$, $y = 28,\ 57\ldots\ldots$ 3.

4. (i.) $x = 2n\pi \pm \alpha$, or $\cos x = -2\cos\alpha$; (ii.) $x = \dfrac{(2r+1)\pi}{2p}$,

   or, $\dfrac{(2r+1)\pi}{2(m+n+p)}$; (iii.) $x = n\pi \pm \dfrac{\pi}{4}$, or $n\pi \pm \dfrac{\pi}{6}$.

5. 6.

7. $hy + kx = 2xy$. 8. $\left(\dfrac{a}{m^2},\ \dfrac{2a}{m}\right)$.

CCXXXVI.

1. $1 + \log_e a \,.\, x + \dfrac{(\log_e a)^2}{\lfloor 2}x^2 + \ldots\ldots;\ \ 2e - 1.$

2. $x = 10$, or 2. **3.**

4. **5.**

6. $a \sin B = b \sin A$. **7.** **8.**

CCXXXVII.

1. 2421.

2. $13 + \dfrac{1}{2+}\,\dfrac{1}{2+}\,\dfrac{1}{2+}\,\dfrac{1}{26+}\,\dfrac{1}{2+\ldots\ldots};\ \ \dfrac{13}{1},\ \dfrac{27}{2},\ \dfrac{67}{5},\ \dfrac{161}{12}.$

3. (i.) $\dfrac{n(n+1)}{2} + \dfrac{a^n - 1}{a - 1}$; (ii.) $\dfrac{1}{4} - \dfrac{1}{2(n+1)(n+2)}$;

(iii.) $\left\{\dfrac{n(n+1)}{2}\right\}^2.$

4. 18 numbers; sum $= 33201$. **5.**

6. Angles $36°\ 52'\ 11''$, and $90°$, Side 95. **7.** $\dfrac{3}{\pi}$.

8. $y^2 = (y - mx - c)(y - m'x - c')$.

CCXXXVIII.

1. Both $= \dfrac{\lfloor m+n}{\lfloor m \lfloor n}$.

2. $\dfrac{A(R^n - 1)}{R^n(R-1)}$; $10R^{13} - 11R^{12} + 1 = 0$;

$\because R^{12}\{11 - 10R\} = +1$, R is $< \frac{11}{10}$.

3. (i.) $x = \frac{7}{2}, -\frac{3}{2}$, [or $1 \pm \sqrt{5}$]; (ii.) $x = 9, 3, 0$; $y = 3, 9, 0$.

4. $\dfrac{\beta}{a}$, and $\dfrac{a}{\beta}$.

5. If r_p is the outside radius of the p^{th} ring, we can shew that $r_p^2 = pr^2$. **6.**

7. The condition shows that the in-centre and circum-centre coincide. **8.**

CCXXXIX.

1. $p^{-1}=\frac{1}{x}\left(\frac{1}{1+x^{-2}}\right)$.

2. (i.) $x=\frac{1}{2}$, $y=1$, $z=\frac{1}{2}$; (ii.) $x=1, 1, 1, -7$;
(iii.) $x=-2, -2, 4$.

3. $v=5026{\cdot}544$ cub. ft.; $s=2513{\cdot}272$ sq. ft.; remainder $=4398{\cdot}22$ cub. ft.

4. Series $=\left(1-\frac{3}{2^2}\right)^{-\frac{1}{2}}=2$.

5.

6. $a=700$ ft., $b \doteq 800{\cdot}874$ ft., $c=95°\ 40'$.

7. The resultant is represented by $\overline{n+1}\,.\,OC$, where C is a point in AB such that $AC=n\,.\,BC$.

8. $\frac{169}{128}$ lbs. weight, or $42\frac{1}{4}$ poundals.

CCXL.

1.

2. (i.) $\frac{4}{3}$; (ii.) $\frac{18}{5}$; (iii.) $\frac{1}{\log_e a}$.

3. 78 ways.

4. $1+4x+\frac{4\,.\,5}{\lfloor 2}x^2+\frac{4\,.\,5\,.\,6}{\lfloor 3}x^3+\frac{5\,.\,6\,.\,7}{\lfloor 3}x^4+\frac{6\,.\,7\,.\,8}{\lfloor 3}x^5+\ldots\ldots$;
$\frac{n(n+1)\ldots\ldots(n+m-1)}{\lfloor m}$.

5. $A=69°\ 10'\ 10''{\cdot}37$, $B=46°\ 37'\ 49''{\cdot}63$; $c=173{\cdot}391$.

6. (i.) $\theta=\frac{1}{2}\sin^{-1}\frac{4}{5}$; (ii.) $\theta=n\pi\pm\frac{\pi}{3}$.

7. 50 feet.

8. $2xy=kx+hy$.

CCXLI.

1. (i.) $x=1, 0, \frac{3}{4}, \infty$; $y=0, 1, \frac{3}{4}, \infty$;
(ii.) $x=\infty$, $\frac{1\pm\sqrt{5}}{2}$, $\frac{1\pm\sqrt{-7}}{2}$;
(iii.) The hyperbola has one asymptote parallel to one of the straight lines, and they meet in three finite points.

2. 3. 3.

4. $m^2n^2(m^2+n^2+3)=1$.

5. $B=108° \; 12' \; 26''$, $C=49° \; 27' \; 34''$, $a=1{\cdot}00005$.

6. 7. Each $=\frac{1}{2} W \cot \frac{A}{2}$.

8. (i.) A circle, centre $(\frac{3}{2}, 2)$, radius $\frac{5}{2}$; (ii.) A straight line.

CCXLII.

1. (i.) $\frac{1}{12}\{80+(3n-1)(3n+2)(3n+5)(3n+8)\}$;
(ii.) $n^2(2n^2-1)$; A^c, $1-2x+x^2$; G^c, $1-x$.

2. 3.

4. $A=105° \; 38' \; 57''$; $B=15° \; 38' \; 57''$; $c=1{\cdot}38637$.

5. $x=\pm 1 \pm \sqrt{3}$.

6. The string is horizontal. $T=\frac{W}{\sqrt{3}}$, $R=\frac{2W}{\sqrt{3}}$.

7. $T=\frac{W}{2}\tan\frac{\theta}{2}$. 8. $\frac{264\sqrt{7}}{7}$ f.-s.

CCXLIII.

1. 2130·62 sq. ft.; 12016·59 cub. ft.

2. 2^n-1. 3.

4. 5.

6. $\sqrt{(s-a)(s-b)(s-c)(s-d)}$; $\cos B=\frac{a^2+b^2-c^2-d^2}{2(ab+cd)}$, etc.

7. 8.

CCXLIV.

1. Series = coefficient of x^{n-1} in expansion of $(1+x)^n(x+1)^n$,
$$=\frac{2n\,.\,(2n-1)\ldots\ldots n}{\underline{|n+1}}.$$

2. Because $(5\sqrt{2}+7)^{2m+1}(5\sqrt{2}-7)^{2m+1}=1$, and
$(5\sqrt{2}+7)^{2m+1}-(5\sqrt{2}-7)^{2m+1}=$ an integer.

3. Expression $=\dfrac{1-x^\infty}{1-x}=1+x+x^2+\ldots\ldots$ *ad. inf.*, if $x<1$.

4.

5. $a\sin B=70{\cdot}1583$, which is $>b$; hence the solution is impossible.

6. (i.) $x=\dfrac{2(\pi\pm\alpha)}{5}$, $y=-\dfrac{\pi\pm 6\alpha}{10}$; (ii.) $\theta=18°$, or $54°$.

7. (i.) A perpendicular to the initial line; (ii.) a circle; (iii.) a parabola; (iv.) a parabola; (v.) an ellipse.

8.

CCXLV.

1. $\dfrac{n(n-1)\ldots\ldots\left(\frac{n}{2}+1\right)}{\lfloor\frac{n}{2}}(-1)^{\frac{n}{2}}$. [Coefficient of x^n in the expansion of $(x+1)^n(1-x)^n$.]

If n be odd, the value $=0$.

2. $$\frac{x}{(1-x)^2-cx}\equiv\frac{x}{(1-x)^2}\left\{1-\frac{cx}{(1-x)^2}\right\}^{-1}$$
$$\equiv\frac{x}{(1-x)^2}\left\{1+\frac{cx}{(1-x)^2}+\frac{c^2x^2}{(1-x)^4}+\ldots\ldots\right\}.$$

3. $\sin n\theta=n\cos^{n-1}\theta\sin\theta-\dfrac{n(n-1)(n-2)}{\lfloor 3}\cos^{n-3}\theta\sin^3\theta+\ldots\ldots$

$\cos n\theta=\cos^n\theta-\dfrac{n(n-1)}{\lfloor 2}\cos^{n-2}\theta\sin^2\theta+\ldots\ldots$

If n be even, the last terms are $(-1)^{\frac{n}{2}}\sin^n\theta$, and $n(-1)^{\frac{n-2}{2}}\cos\theta\sin^{n-1}\theta$.

If n be odd, the last terms are $n(-1)^{\frac{n-1}{2}}\cos\theta\sin^{n-1}\theta$, and $(-1)^{\frac{n-1}{2}}\sin^n\theta$.

4. $B=38°\ 12'\ 47''{\cdot}6$, $C=21°\ 47'\ 12''{\cdot}4$.

5. $102902{\cdot}68$ cubic inches. 6.

7. $y=mx+\frac{a}{m}$. Putting $x=h$, $y=k$, we get a quadratic in m, which will have imaginary roots if k^2-4ah is negative.

8. Tension $=m\sqrt{2g}$; Horizontal velocity of $m'=\frac{m\sqrt{2g}}{m+m'}\sin\alpha$.

CCXLVI.

1. Equate coefficients of x^p in the identity

$$\{(1-x)^{-1}-1\}^n\equiv x^n(1-x)^{-n}.$$

2. $x^2+xy+y^2=(x-\omega y)(x-\omega^2 y)$, where ω is one of the imaginary cube roots of unity; and $(x-\omega y)^n$ may be put in the form $X-\omega Y$.

3. $\frac{n(n-2)(3n-1)(a+l)^2+4n(n+1)al}{24(n-1)}$.

4.

5. $\frac{2^{\frac{n}{2}}}{\lfloor n}\cos\frac{n\pi}{4}$.

6. $m^3y+(x-2a)m^2-a=0$.

7. Along FC.

8.

CCXLVII.

1. (i.) $\frac{3}{4}-\frac{2n+3}{2(n+1)(n+2)}$; (ii.) $1-\frac{1}{\lfloor n}$;

(iii.) Scale of relation $=1-2x-3x^2$,

$$\text{Sum}=\frac{7}{2}\cdot\frac{(3x)^n-1}{3x-1}+\frac{1}{2}\frac{(-x)^n-1}{x+1}.$$

2. (i.) Eliminate by determinant.

(ii.) $1-bc=\frac{x(x+y+z)}{(x+y)(x+z)}=\frac{x^2(x+y+z)}{a(x+y)(y+z)(z+x)}$, etc.

3.

4.

5. $x=\left\{\frac{a+\sqrt{a^2+b^2}}{2}\right\}^{\frac{1}{2}}$, $y=\left\{\frac{-a+\sqrt{a^2+b^2}}{2}\right\}^{\frac{1}{2}}$.

6. Distance $=2h$; 45°.

7. $R(3\mu+7)=14$, $348\mu R=7(h+5)$; 24 feet.

8. 3π cubic feet.

CCXLVIII.

1. (i.) Coefficient of x^n in $(1+x)^{2n}$; (ii.) $\dfrac{2^{n+1}-1}{n+1}$.

2. Integral part can be shown $=(3+\sqrt{5})^n+(3-\sqrt{5})^n$. Then proceed by induction.

3. The condition gives $(b+c)(c+a)(a+b)=0$, hence $a=-b$, or, etc.

4. $p=\dfrac{ab\sin C}{\sqrt{a^2+b^2-2ab\cos C}}$.

5.

6. $\left(\dfrac{4a}{5}, -\dfrac{3a}{5}\right)$; $45°$.

7. A couple.

8. $s=vt+\frac{1}{2}at^2$; 1230 feet.

CCXLIX.

1. $a(b-c)+b(c-a)+c(a-b)\equiv 0$
and $(b-c)+(c-a)+(a-b)\equiv 0$,
Proceed as if solving for $b-c$, $c-a$, $a-b$, by determinants.

2.

3. (i.) $2n$; (ii.) $1-\dfrac{1}{n^2+1}$. $\left[n^{\text{th}} \text{ term}=\dfrac{2n-1}{(\overline{n-1}^2+1)(n^2+1)}\right]$.

4. $\cos\alpha=1-\dfrac{\alpha^2}{\underline{|2}}+\dfrac{\alpha^4}{\underline{|4}}-\ldots\ldots$; (ii.) Circular measure.

5. $\cos\theta=\dfrac{e^{i\theta}+e^{-i\theta}}{2}$, $\sin\theta=\dfrac{e^{i\theta}-e^{-i\theta}}{2i}$.

6. 6π sq. in.

7. Force $=\dfrac{W}{3}$, pressure $=\dfrac{2W}{3}$.

8. $I=mv$; $4h$.

CCL.

1. (i.) $\dfrac{1}{\underline{|r-1}}-\dfrac{r^{n+1}}{\underline{|n+r}}$; (ii.) $\dfrac{5}{4}-\dfrac{4n+5}{2(n+1)(n+2)}$;
(iii.) $\dfrac{4-7x}{1-3x+2x^2}$.

2.

3. (i.) Expression $=(1+i)^n(1-i)^n=2^n$.

4. $2^{\frac{4}{3}}\cos\frac{\pi}{9},\ 2^{\frac{4}{3}}\cos\frac{7\pi}{9},\ 2^{\frac{4}{3}}\cos\frac{13\pi}{9}$. 5.

6. $x^2+y^2-5ax-4ay+4a^2=0$.

7. (i.) $x+y=11$; (ii.) $x-y+1=0$.

8. $\frac{x\cos\theta}{a^2}+\frac{y\sin\theta}{b^2}=0$.

CCLI.

1. (i.) Expression $=\left(1-\frac{1}{1+nx}\right)^n-\frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1}$;

(ii.) $2n(1+1)^{n-1}+(1+1)^n=2^n(n+1)$.

2. $x=y+\frac{y^2}{2}+\frac{y^3}{4}+\frac{y^4}{8}+\ldots\ldots$ 3. $\frac{-ab\pm\sqrt{a^2b^2+4ab}}{2b}$.

4. (i.) $\dfrac{\sin\left\{\alpha+\frac{(n-1)(\pi+\beta)}{2}\right\}\sin\frac{n(\pi+\beta)}{2}}{\sin\frac{\pi+\beta}{2}}$;

(ii.) $\dfrac{\sin\alpha-\sin^n\beta\sin(n+1)\alpha+\sin^{n+1}\beta\sin n\alpha}{1-2\sin\beta\cos\alpha+\sin^2\beta}$.

5. $2(\sqrt{5}+1)$ miles, $=6{\cdot}472$ miles nearly.

6. At a point in the side one-third way up from the base.

7. 8.

CCLII.

1. (i.) $\frac{1}{4}\left\{\frac{1}{a^2}-\frac{1}{a^2+4n^2}\right\}$;

(ii.) $\frac{1}{8}\{15+(2n-1)(2n+1)(2n+3)(2n+5)\}$;

(iii.) $\frac{n^2(n+1)}{2}$. $\left\{n^{\text{th}}\text{ term}=\frac{n(3n-1)}{2}\right\}$.

2. Expression vanishes when a is put equal to zero, or when we put $a=b$, etc.

3. $\alpha^2+\beta^2=1$. [Assume $x=\sin\theta$, $y=\cos\theta$.]

4. $300\sqrt{2}$ yards.

5. 1, $\cos\frac{2\pi}{5} \pm i\sin\frac{2\pi}{5}$, $\cos\frac{4\pi}{5} \pm i\sin\frac{4\pi}{5}$.

6. $\frac{\sqrt{3}}{4}\left(1 - \frac{2\pi}{27}\right)$ cub. ft., $= \cdot 3322$ cub. ft. nearly.

7. $x^2 + y^2 + 2gx + 2fy + c = 0$.

8. $ay^2 = x^2(x - 2a)$.

CCLIII.

1.

2. (i.) Expand by Binomial Theorem; (iii.) Follows from (ii.).

3. Assume the radix to be of the form $r^2 + ar + b$.

4. (i.) $\dfrac{\sin\frac{n+1}{2}\theta \sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$; (ii.) $\dfrac{\sec(2n+1)\theta - \sec\theta}{2\sin\theta}$.

5. $\cos\alpha + \cos\beta - \cos\gamma = \cos\alpha\cos\beta\cos\gamma$.

6. (i.) 500 cwt.; (ii.) 392000 foot pounds.

7.

8. (i.) $f = \frac{Pg}{P+Q}$, $T = P(g - f)$.

CCLIV.

1. It lies between n^2 and $(n-1)^2$.

2. (ii.) Equate coefficients of x^r in the expansions of $(1-x)^{-2n}(1-x)^{-1}$ and $(1-x)^{-(2n+1)}$.

3. (i.) $\frac{2^{n+1}}{n+1} - 2$; (ii.) $\frac{1}{2}\left\{1 - \frac{1}{1 \cdot 3 \cdot 5 \ldots\ldots (2n+1)}\right\}$.

4. (ii.) $\sin\theta = \theta - \frac{\theta^3}{6}$ nearly.

5.

6.

7. $ax\sec\phi - by\operatorname{cosec}\phi = a^2 - b^2$.

8. (i.) A hyperbola, touching $x \pm a = 0$, where $2x + y = 0$ meets them; (ii.) a hyperbola.

CCLV.

1. 2. Equate coefficients of x^n. 3.

4. (i.) $\cos^{2n+1}\theta = \frac{1}{2^{2n}}\Big\{\cos(2n+1)\theta + (2n+1)\cos(2n-1)\theta$

$$+\frac{(2n+1)2n}{\lfloor 2}\cos(2n-3)\theta + \dots + \frac{(2n+1)\dots(n+2)}{\lfloor n}\cos\theta\Big\}.$$

(ii.) $\frac{1}{2^6}\{5\cos\theta - \cos 3\theta - 3\cos 5\theta - \cos 7\theta\}$.

5. Series $= \frac{1}{2}\{\log(1+\tan\theta) - \log(1-\tan\theta)\}$.
6. 2·53406 inches.
7. If BD be perpendicular to AC, then $CA = AD$.
$R^2 : T^2 : W^2 :: 4r^2 - s^2 : 3s^2 : 4(s^2 - r^2)$; where $r =$ length of rod, $s =$ length of string.
8. (i.) 144 feet; (ii.) after 4 seconds more.

CCLVI.

1. 2.

3. $x = y + \frac{y^2}{\lfloor 2} + \frac{y^3}{\lfloor 3} + \dots\dots$

4. $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$, $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$;

$$C = e^{x\cos\theta}\cos(x\sin\theta), \quad S = e^{x\cos\theta}\sin(x\sin\theta).$$

5. $c = (a+b)\cos\phi$, where $(a+b)^2\sin^2\phi = 4ab\cos^2\frac{C}{2}$.

6. $m =$ tangent of the angle with the x-axis, $c =$ intercept on y-axis;

(ii.) $\left(\frac{(b-b')a}{(b+b')}, \frac{2bb'}{b+b'}\right)$.

7. $x^2 + y^2 = a^2$. 8. $\frac{hx}{a^2} + \frac{ky}{b^2} = 1$.

CCLVII.

1. (ii.) We can show that $(a+b)^{\frac{1}{2}} > \frac{1}{\sqrt{2}}(a^{\frac{1}{2}} + b^{\frac{1}{2}})$, etc.

2. (i.) $\dfrac{n(4n^2-1)}{3}$; (ii.) $\dfrac{1}{6}\left\{\dfrac{1}{10} - \dfrac{1}{(3n+2)(3n+5)}\right\}$;

(iii.) $\dfrac{1+x-(2n+1)x^n+(2n-1)x^{n+1}}{1-2x+x^2}$.

3. Equate coefficients of x^n, in the identity
$$\log(1-px)+\log(1-qx)=\log\{1-(p+q)x+pqx^2\}.$$

4.

5.

6. 60 yards.

7. $T=\dfrac{175}{3}(\sqrt{3}-1)$ lb. weight; $\mu=\dfrac{7(12-5\sqrt{3})}{69}$.

8. Inclined at an angle $\sin^{-1}\frac{1}{8}$ to the horizon.

CCLVIII.

1. $a-(a-b)x-bx^2+ax^3-(a-b)x^4-bx^5+\ldots\ldots$

2.

3. Assume $x=a\cos\theta$.

4. $s=\dfrac{ar_a}{r_a-r}$, $\tan\dfrac{A}{2}=\dfrac{r_a-r}{a}$, $(s-b)^2-a(s-b)+rr_a=0$.

5. (i.) Impossible; (ii.) $C=56°\ 28'\ 28''$, *or* $123°\ 31'\ 32''$, $b=43{\cdot}1548$ *or* $10{\cdot}0162$; (iii.) $C=33°\ 45'\ 47''$, $b=63{\cdot}996$.

6. £5 10s.

7. $\dfrac{x}{a}\cos\dfrac{\phi+\phi'}{2}+\dfrac{y}{b}\sin\dfrac{\phi+\phi'}{2}=\cos\dfrac{\phi-\phi'}{2}$.

8. $x+y=3$.

CCLIX.

1.

2. Every number is of form $5m$, $5m\pm1$, or $5m\pm2$.

3. Any number may be expressed in the scale of 2.

4. $c(\sec\alpha-\tan\alpha)+h$, $=c\tan\left(\dfrac{\pi}{4}-\dfrac{\alpha}{2}\right)+h$.

5. $nr^2 \tan \frac{\pi}{n}$; πr^2.

6. ·074689 inch from the centre, $= \frac{1}{13}$ inch nearly.

7. $500(2+\sqrt{11})$ foot pounds.

8. Moment of each couple = area of parallelogram.

CCLX.

1. (i.) $\frac{2}{x(x-1)}$; (ii). $-x$.

2. Every number is of form $7m$, $7m \pm 1$, $7m \pm 2$, or $7m \pm 3$.

3. Expand $\log(1-x)$.

4. (ii.) Follows from (i.).

5.

6. $r^2 - 2dr\cos(\theta - a) + d^2 - c^2$; $r_1 r_2 = \text{const.}$

7.

8. $y = mx + \frac{a}{m}$;

(ii.) tangents are $x - 3y + 18 = 0$, and $4x - 3y - 36 = 0$.

CCLXI.

1. (i.) n must be of form $5m$, $5m \pm 1$, or $5m \pm 2$;
(ii.) Assume $a = m^2 - n^2$, $b = 2mn$.

2.

3. 2.

4. They are obtained by writing $2\pi + \theta$, $4\pi + \theta$...... $2(n-1)\pi + \theta$, for θ.

5. 3·14159.

6. $9\frac{53}{98}$ tons.

7. In 1 second.

8. $2\frac{5}{12}$ lbs., in a direction cutting AB produced, $12\frac{1}{2}$ feet from B.

CCLXII.

1. $a_1 = \frac{2b + 2na - n(n+1)d}{2(n+1)}$,

$$a_r = \frac{2br + 2a(n-r+1) - dr(n+1)(n-r+1)}{2(n+1)},$$

$$d > \frac{2a(-b)}{n(n+1)} \text{ and } < \frac{2(b-a)}{n(n+1)}.$$

2. $p_n - 2ap_{n-1} - bp_{n-2} = 0$, and $q_n - 2aq_{n-1} - bq_{n-2} = 0$.

3. n^{th} term $= \dfrac{1}{\lfloor n-2} + \dfrac{3}{\lfloor n-1} + \dfrac{1}{\lfloor n}$.

4. (ii.) $\sin A + \sin B + \sin C = 4 \cos \dfrac{A}{2} \cos \dfrac{B}{2} \cos \dfrac{C}{2}$.

5. $x = (\cos n\theta + i \sin n\theta)^{\frac{1}{n}}$; apply De Moivre's Theorem. 6.

7. $a - b < c\sqrt{2}$. If $a - b = c\sqrt{2}$, the circles touch. 8.

CCLXIII.

1. 0.

2. H and A, 32 and 31; E and K, 12 and 9; C and G, 8 and 1. [63 can be put as the difference of two integral squares in 3 ways.]

3. Expression $= [2(1-x)^{-3} - 1 - \{(1-x)^{-2} - 1\}]$
$\times [2(1+x)^{-3} - 1 - \{(1+x)^{-2} - 1\}]$. 4.

5. (i.) $\dfrac{\cos \frac{n+1}{2} \alpha \sin \frac{n\alpha}{2}}{\sin \frac{\alpha}{2}}$; (ii.) $\cot \alpha - 2^n \cot 2^n \alpha$.

6. w poundals, $= \dfrac{w}{g}$ pounds weight. 7. $\frac{1}{2}vt$. 8.

CCLXIV.

1. 2. 3. $(1-x)^{-\frac{1}{x}} - 1$.

4. 5. 6. £466·22825.

7. $4ab$, where b, and $-b$ are the abscissæ of the two points.

8. $2x(x_1 - x_2) + 2y(y_1 - y_2) = x_1^2 + y_1^2 - x_2^2 - y_2^2$.

CCLXV.

1. 2. $-1 - \dfrac{1}{2^{n+1}}$.

3. (i.) $a^2b+b^2c+c^2a>3abc$; (ii.) $a^3+b^3>(a+b)(2ab-ab)$.

4. $e^{i\theta}=\cos\theta+i\sin\theta$.

5. (i.) $-\log\left(2\sin\frac{a}{2}\right)$; (ii.) $\dfrac{(-1)^{n-1}}{2}\tan\dfrac{\pi}{2n+1}$.

6. 7.

8. (i.) A circle, with centre $(-3, 2)$ and radius 5;
(ii.) An ellipse.

CCLXVI.

1. 2. (i.), (ii.) Arithmetic Mean > Geometric Mean.

3. By Mathematical Induction.

4. They form the Series—

$$2a\left(\cos\frac{\pi}{2n}+\cos\frac{2\pi}{2n}+\cos\frac{3\pi}{2n}+\ldots\ldots\right).$$

5. Use exponential values of *sine* and *cosine*.

6. $\dfrac{3-\sqrt{3}}{2}AB$. 7. $\frac{1}{4}$ second.

8. $\frac{88}{3}$ feet per second, at an angle 60° with the horizon.

CCLXVII.

1. (i.) Cf. Ex. CCLXVI. 3.

2. $a<b+c$, $\therefore a^2<ab+ac$, etc.

3. Both vary as $\sqrt{d}$. 4.

5. (i.) $\frac{1}{2}(\cos 2\theta-\cos 2^{n+1}\theta)$;

$$\text{(ii.)}\ \frac{3}{4}\cdot\frac{\cos\left(a+\frac{n-1}{2}\beta\right)\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$$

$$+\frac{1}{4}\cdot\frac{\cos\left(3a+\frac{3(n-1)}{2}\beta\right)\sin\frac{3n\beta}{2}}{\sin\frac{3\beta}{2}}.$$

6. Tangents at the ends of a focal chord intersect at right angles on the directrix.

7. $x^2+y^2-3ax-4ay=0$.

8. $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$; (ii.) If the polar passes through a fixed point, the pole lies on a fixed straight line.

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